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QUEUING SYSTEM IN OPTIMIZATION FUNCTION OF THE PORT'S BULK UNLOADING TERMINAL

ABSTRACT

The paper demonstrates the application of the queuing theory in modelling the port's bulk cargo unloading terminal. A bulk cargo terminal can be observed as a queuing system defined by basic parameters: the rate of bulk cargo ship arrivals or quantity of bulk cargo and the rate of ship servicing i. e. quantity of bulk cargo, in an observed time unit. Appropriate indices of bulk cargo terminal operations are computed on the basis of these parameters. The unloading terminal is determined as a queuing system marked with M/M/1 by Kendall notation. The system is characterized by Poisson distribution of entity arrivals and exponentially distributed service time. Subsequently, the model set up will be tested on the real example of the unloading terminal of the bulk cargo port in Bakar. Through application of the proposed model it should be possible to make a decision on how to optimize the transhipment processes on the bulk discharging terminal to increase its efficiency. The obtained parameters and the calculated indices point to solid capacity employment rate in 2005 and the probability that the berth is unoccupied is relatively low. Indices show that the terminal traffic rate has dramatically improved taking into consideration the previous years.

KEY WORDS

stochastic processes, queuing system M/M/1, unloading terminal of bulk cargo port

1. INTRODUCTION

The necessity for studying processes in bulk cargo ports, which are the objects of research in this paper, results from the fact that many real processes characterized with unpredictability and changeability are called stochastic, because the parameters that determine these processes accept random values.

For the scientific research of the processes that take place in a port, the queuing theory will be used. It is one of the operational research methods dealing with the servicing processes of the entities that randomly arrive at the system and demand the service. By the mathematical models of queuing theory the interdependence between entity arrivals, waiting for service and their leaving the system has been determined, with the purpose of system functionality optimization. The basic features of the queuing phenomenon are the mass character and random property, because the demand for service and the service time are stochastic variables.

From the queuing theory viewpoint, a port's bulk cargo terminal features the following characteristics:

1. A bulk cargo terminal is an open system as ships are not an integral part of the system.
2. A bulk cargo terminal is a single or multi-channel system (depending on the number of berths) and, in this connection, ships on sea or at anchorage form queues for certain berths.
3. The number of ship arrivals as well as the duration of service i. e. length of ship's stay at berth is allocated according to certain theoretical distributions (most often based on Poisson's and Erlang's distribution order k, where *k* is a natural number). The service time of ship, together with the time spent queuing, represents the ship's stay at the terminal and is one of the more significant indices of port's bulk cargo terminal operations.
4. With regards to queuing discipline, a bulk cargo terminal is a system where servicing is most often carried out according to FIFO rule (first-come-first-served) but it is possible that there are certain ships which have priority in servicing.

There are many research papers in the field of queuing system type M/M/1 [2, 3, 7, 14]. Cullinane, Song, and Wang [4] apply mathematical programming approach to estimate container port production efficiency. Cullinane [5] investigated possible methods and their applications for productivity and efficiency modelling of ports and terminals. Whitt, Glynn and Melamed [11] apply customer arrival estimation and use time averaging to identify real based models' behaviour.

The theoretical features of the Poisson process are given as the base of the used type of queue. The main goal of this paper is to define the port's bulk cargo terminal as the queuing system and then to set up the appropriate model in which the unloading terminal is determined as a queuing system of type M/M/1. Subsequently, the model set up will be tested on the real example of bulk discharging terminal Bakar. Through application of the proposed model it should be possible to make a decision on how to optimize the transhipment processes on a bulk discharging terminal to increase its efficiency.

2. THEORETICAL FEATURES

The M/M/1 system is made of a Poisson arrival, one exponential (Poisson) server, FIFO (or not specified) queue of unlimited capacity and unlimited customer population. Note that these assumptions are very strong, not satisfied for practical systems (the worst assumption is the exponential distribution of service duration - hardly satisfied by real servers). Nevertheless, the M/M/1 model shows clearly the basic ideas and methods of the Queuing Theory. The next part summarizes the basic properties of the Poisson process and gives derivation of the M/M/1 theoretical model [14].

2.1 Poisson process

Define

$$p_n(t) = P[n \text{ arrivals in the time interval } (0,t)]. \quad (1)$$

Assumptions of the Poisson process are:

1) $p_1(h) = \lambda h + o(h)$, where λ is a constant, $t \geq 0$ and $\lim_{h\to 0} o(h) = 0$
2) $\lim_{h\to 0} p_{n>1}(h) = 0$
3) The above probabilities are independent, so they have "no memory" property.

The probability $p_n(t+h)$, $h \to 0$ could be expressed as

$$p_n(t+h) = p_n(t)[1-\lambda h] + p_{n-1}(t)\lambda h\,, \quad (1')$$

$$p_0(t+h) = p_0(t)[1-\lambda h]. \quad (1'')$$

The equations (1') and (1'') may be written in this way:

$$\frac{p_n(t+h)-p_n(t)}{h} = -\lambda p_n(t) + \lambda p_{n-1}(t)\,,$$

$$\frac{p_0(t+h)-p_0(t)}{h} = -\lambda p_0(t)\,. \quad (2)$$

Because of small h the terms on the left sides of (2) may be considered as derivatives:

$$\lim_{h\to 0}\frac{p_n(t+h)-p_n(t)}{h} = -\lambda p_n(t) + \lambda p_{n-1}(t)\,,$$

$$\lim_{h\to 0}\frac{p_0(t+h)-p_0(t)}{h} = -\lambda p_0(t)\,,$$

that is:

$$\frac{\mathrm{d}p_n(t)}{\mathrm{d}t} = -\lambda p_n(t) + \lambda p_{n-1}(t)\,,$$

$$\frac{dp_0(t)}{dt} = -\lambda p_0(t)\,. \quad (3)$$

Equations (3) represent a set of differential equations, with the solution

$$p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}\,, n=0, 1, 2, ... \quad (4)$$

Because of assumption 3) formula (4) holds for any interval $(s, s+t)$ from $\mathbf{R}_+$. In other words the probability of n arrivals during some time interval depends only on the length of this time interval (not on the starting time of the interval).

Probability of n arrivals during some time interval t is a discrete random variable associated with the Poisson process. Having the probabilities of random values (4), it is possible to find the mathematical expectation of the random variable N_t. Let E[X] be the mean (average) value, Var[X] the variance, and Std[X] the standard deviation of the random variable X:

$$E[N_t] = \sum_{n=0}^{\infty} n \frac{(\lambda t)^n}{n!} e^{-\lambda t} =$$

$$= \lambda t e^{-\lambda t} \sum_{n=1}^{\infty} \frac{(\lambda t)^{n-1}}{(n-1)!} = \lambda t e^{-\lambda t} e^{\lambda t} = \lambda t\,, t \geq 0\,. \quad (5)$$

Then (5) gives the interpretation of the constant λ, that is the average number of arrivals per time unit. That is why λ is called *arrival rate*.

$$\mathrm{Var}[N_t] = \sum_{n=0}^{\infty} n^2 \frac{(\lambda t)^n}{n!} e^{-\lambda t} - (\lambda t)^2 = \lambda t\,. \quad (6)$$

$$\mathrm{Std}[N_t] = \sqrt{\mathrm{Var}[N_t]} = \sqrt{\lambda t}\,. \quad (6')$$

Another random variable associated with the Poisson process is the random interval between two adjacent arrivals. Let T be the random interval. To find its distribution, let us express the *distribution function* $F(t)$:

$$F(t) = P[T < t] =$$
$$= P[\text{at least one arrival during the interval } t] =$$
$$= 1 - p_0(t) = 1 - e^{-\lambda t}\,.$$

Because the interval is a continuous random variable, it is possible to compute the *probability density* as a derivative of the distribution function:

$$f(t) = \mathrm{d}F(t)/\mathrm{d}t\,.$$

Subsequently, the distribution function of the random variable T is:

$$F(t)=\begin{cases}1-e^{-\lambda t} & t\geq 0\\ 0 & t<0\end{cases} \quad (7)$$

The distribution (7) is called *exponential distribution*, with parameter $\lambda > 0$. For the exponential distribution, *probability density* is:

$$f(t)=\begin{cases}0 & t\leq 0\\ \lambda e^{-\lambda t} & t>0\end{cases}. \quad (8)$$

If the random variable T is distributed by the exponential distribution with parameter $\lambda > 0$, then:

$$E[T]=\int_0^\infty t\cdot\lambda e^{-\lambda t}\,\mathrm{d}t=\frac{1}{\lambda},$$

$$Var[T]=E[T^2]-(E[T])^2=\frac{1}{\lambda^2},$$

$$\mathrm{Std}[T]=\sqrt{\mathrm{Var}[T]}=\frac{1}{\lambda} \quad (9)$$

Expression (9) gives another interpretation of constant λ. Its inverted value is the average interval between arrivals. Like the number of arrivals, the distribution of intervals between arrivals does not depend on time.

When applied to a service, the rate is called *service rate* (μ). The parameter μ is the average number of completed services per time unit (provided there are always customers waiting in the queue). Its inverted value $1/\mu$ is the average duration of the exponential service.

Define

$q_n(t)=P[n$ outcomes from berth during $(0,t)]$,

with assumptions:

1) $q_1(h)=\mu h+o(h),\ \lim_{h\to 0} o(h)=0,$

2) $\lim_{h\to 0} q_{n>1}(h)=0$

3) independence of time.

The variance and the standard deviation can be computed by replacing λ by μ in the formulae (9). Unlike arrival, exponential service is an abstraction that is hardly satisfied by real systems, because mostly it is very unlikely to have very short and/or very long services. Real service duration will be typically "less random" than the theoretical exponential distribution.

Another very important parameter of queuing systems is the ratio ρ of the arrival and the service rates called *traffic rate* (sometimes called *traffic intensity* or *utilisation factor*), representing the ratio between the arrival and service rate:

$$\rho=\frac{\lambda}{\mu} \quad (10)$$

The value of ρ shows how "busy" the server is. It is obvious, that for $\rho\geq 1$ the queue will grow permanently. Therefore, the basic condition of the system stability is $\rho<1$, for the cases with the unlimited customer population.

2.2. M/M/1 system

The M/M/1 system is made of a Poisson arrival (arrival rate λ), one exponential server (service rate μ), unlimited FIFO (or not specified queue), and unlimited customer population. Since both arrival and service are Poisson processes, it is possible to find probabilities of various states of the system, which are necessary to compute the required quantitative parameters. System state is the number of customers in the system. It may be any nonnegative integer number. Let

$p_n(t)=P[n$ customers in the system at time $t]$.

As with the Poisson process, by using the assumptions 1) and 2), it is possible to express the probability $p_n(t+h), h\to 0$ in this way:

$$p_n(t+h)=p_n(t)[1-\lambda h][1-\mu h]+p_n(t)\cdot\lambda h\cdot\mu h+$$
$$+p_{n-1}(t)\cdot\lambda h[1-\mu h]+p_n+1(t)[1-\lambda h]\cdot\mu h$$
$$p_0(t+h)=p_0(t)[1-\lambda h]+p_1(t)[1-\lambda h]\cdot\mu h. \quad (11)$$

The above equations may be written in the following way, where the terms with h^2 are omitted because they are relatively very small ($h\to 0$):

$$\frac{p_n(t+h)-p_n(t)}{h}=$$
$$=-(\lambda+\mu)p_n(t)+\lambda p_{n-1}(t)+\mu p_{n+1}(t)$$
$$\frac{p_0(t+h)-p_0(t)}{h}=-\lambda p_0(t)+\mu p_1(t). \quad (12)$$

Because of small $h, h\to 0$, the terms at the left sides of (12) may be considered as derivatives:

$$\frac{\mathrm{d}p_n(t)}{\mathrm{d}t}=-(\lambda+\mu)p_n(t)+\lambda p_{n-1}(t)+\mu p_{n+1}(t),$$
$$\frac{\mathrm{d}p_0(t)}{\mathrm{d}t}=-\lambda p_0(t)+\mu p_1(t), \quad (13)$$

Equations (13) represent a set of differential equations called *Kolmogorov Differential Equations*. Their solution is a set of equations showing how each probability changes with time. Because of the third assumption (stationary) after some transition period the system will become stable. The $p_n(t)$ is probability depending on time interval t and the number of ships n, but here we deal with dependence on n, while

$$\frac{\partial p(n,t)}{\partial t}=0.$$

Of course, the state will permanently change, but the probabilities of various numbers of customers in the system will be constant, independent of time. So, the functions $p_n(t)$ become constants p_n.

From (12) differential equations (13) are obtained and the derivates are:

$$\lim_{t\to\infty}\frac{\mathrm{d}}{\mathrm{d}t}p_n(t)=\frac{\mathrm{d}}{\mathrm{d}t}\lim_{t\to\infty}p_n(t)=\frac{\mathrm{d}}{\mathrm{d}t}p_n=0.$$

Constant functions have zero derivatives, so the set (13) becomes a set of algebraic equations for the stable state:

$$\mu p_{n+1}-(\lambda+\mu)p_n+\lambda p_{n-1}=0,\ \mu p_1-\lambda p_0=0\,. \quad (14)$$

Dividing the equations (14) with μ, this set of equations is obtained that contains only parameter ρ - the *traffic rate*:

$$p_{n+1}=(1+\rho)p_n-\rho\cdot p_{n-1},\ p_1=\rho\cdot p_0\,. \quad (15)$$

In the set (15) p_1 is expressed by p_0. By inserting p_1 and p_0 to the equation for p_2, it follows:

$$p_2=(1+\rho)p_1-\rho p_0=(1+\rho)\rho p_0-\rho p_0=\rho^2 p_0\,.$$

Similarly, p_2 and p_1 may by used to express p_3, etc. which gives the general formula for p_n:

$$p_n=\rho^n p_0=(\lambda/\mu)^n p_0\,. \quad (16)$$

The value p_0 can be computed by using the obvious requirement, that the sum of all probabilities must be equal to 1:

$$\sum_{n=0}^{\infty}p_n=p_0\sum_{n=0}^{\infty}\rho^n=p_0\frac{1}{1-\rho}=1,$$

giving

$$p_0=1-\rho\,. \quad (17)$$

In (17) the sum of geometric progression has been used with coefficient $\rho<1$. From (17) it is also obvious that the traffic rate ρ must be less than 1, otherwise the sum of probabilities would not be 1 (even not limited):

$$0\le\rho<1\Rightarrow\sum_{n=0}^{+\infty}\rho^n=\frac{1}{1-\rho}\,.$$

Inserting (17) to (16) gives the general formula for probability of n customers being in the stable system:

$$p_n=\rho^n(1-\rho)=\left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right). \quad (18)$$

The equations (18) represent a very important result used later to obtain all the characteristics of the M/M/1 system. The relatively simple derivation of (18) was enabled by the Poisson process assumptions used when expressing the probabilities in (11). The equations (18) may be used directly to express these probabilities:

- P[service idle] = P[not queuing on arrival] = $=p_0=1-\rho=1-\lambda/\mu$
- P[service busy] = P[queuing on arrival] = $1-p_0=$ $=\rho=\lambda/\mu$
- P[n customers in the system] = $p_n=\rho^n(1-\rho)$
- P[n or more customers in the system] = ρ^n
- P[less than n customers in the system] = $1-\rho^n$.

The proof for P[n or more customers in the system] is:

$$\sum_{m=n}^{\infty}p_m=\sum_{m=n}^{\infty}\rho^m(1-\rho)=\sum_{k=0}^{\infty}\rho^{n+k}(1-\rho)=$$
$$=\rho^n(1-\rho)\sum_{k=0}^{\infty}\rho^k=\rho^n(1-\rho)\frac{1}{1-\rho}=\rho^n\,.$$

Now the formula (18) will be used to obtain quantitative characteristics of the M/M/1 system. (18) actually represents a distribution of a discrete random variable *number of customers in the system* (probabilities of all possible random values). So it is possible to compute the mean value, which is the average number of customers in the system:

$$L=E[n]=\sum_{n=0}^{\infty}np_n=\sum_{n=0}^{\infty}n\rho^n(1-\rho)=$$
$$=\sum_{n=0}^{\infty}n(\rho^n-\rho^{n+1})=$$
$$=1(\rho-\rho^2)+2(\rho^2-\rho^3)+3(\rho^3-\rho^4)+...=$$
$$=\rho(1+\rho+\rho^2+\rho^3+...)=$$
$$=\rho\frac{1-\rho^n}{1-\rho}=/\lim_{n\to\infty}\rho^n=0/=\frac{\rho}{1-\rho},$$

$$L=\frac{\rho}{1-\rho}=\frac{\lambda}{\mu-\lambda}\,. \quad (19)$$

Formula for geometric progression has been used in (19). Note again, that ρ must be less than 1, otherwise the number of customers in the system will grow permanently. In a similar way it is possible to find the average number of customers in the queue. Assume that service of customer is not a part of the queue. So n customers in the system means the queue length is n-1.

$$L_Q=\sum_{n=1}^{\infty}(n-1)p_n=\sum_{n=1}^{\infty}np_n-\sum_{n=1}^{\infty}p_n=L-(1-p_0)=$$
$$=\frac{\rho}{1-\rho}-(1-(1-\rho))=\frac{\rho}{1-\rho}-\rho$$

$$L_Q=\frac{\rho^2}{1-\rho}=L\rho\,. \quad (20)$$

The average queue length is a parameter that represents the quality of service especially from the user's point of view. Other parameters are the average time W spent in the system and the average time W_Q spent in the queue. These parameters are necessary to compute costs caused by customers waiting for service. *Little's formula* gives relations between the mentioned values:

$$L=\lambda W,\qquad L_Q=\lambda W_Q \quad (21, 22)$$

To justify (21), assume that the average time spent in the system is W. During this time the average number of newcomers is $W\lambda$, because λ is the arrival rate (average number of arrivals per time unit). So at the instant a customer is leaving the systems, it sees (on the average) $W\lambda$ customers left in the system. Because in the stable state the average number of customers in the system is L, the above formula follows. Similar justification can be given for L_Q and W_Q. The average number of arrivals during the time spent in the queue is λW_Q that is the average number left in the queue when leaving the queue, which is L_Q. Using the Little's formula (21) these formulae are obtained:

$$W=\frac{L}{\lambda}=\left(\frac{1}{1-\rho}\right)\frac{1}{\mu}=\frac{1}{\mu-\lambda}=\frac{W_Q}{\rho},$$

$$W_Q=\frac{L_Q}{\lambda}=\left(\frac{\rho}{1-\rho}\right)\frac{1}{\mu}=\frac{\rho}{\mu-\lambda}=W\rho \tag{23}$$

There is another relationship between the average time spent in the system and the average time spent in the queue, because the total spending time is made of the time in the queue (that may be zero) and the time of the service:

$$W=W_Q+\frac{1}{\mu} \tag{24}$$

Formula (24) can be obtained from (23). Using the Little's formula, it is possible to express a similar relation between L and L_Q by multiplying both sides of (24) by λ:

$$L=L_Q+\frac{\lambda}{\mu}=L_Q+\rho=L_Q+\rho=L_Q+L_{SERV} \tag{25}$$

$L_{SERV}=\rho$ is the average number of customers being served and must be less than one. For a customer it is also important to know the probability, that the time spent in the system (or in the queue) will be greater than a certain value. Together with the queue length this probability represents a quality of service from the user's point of view.

The above expressions will be used in the next part of this paper for modelling the port's unloading bulk cargo terminal in Bakar, defined as queuing system M/M/1/∞.

3. ANALYTIC MODEL OF UNLOADING BULK CARGO TERMINAL

In order to set up the model M/M/1 it is necessary to define the basic parameters for an unloading bulk cargo terminal as a queuing system. These are: average number of bulk cargo ships (or quantity of bulk cargoes) which arrive at the terminal in an observed time unit and average number of bulk cargo ships (or quantity of bulk cargoes) which can be serviced in a time unit at the terminal. On the basis of these parameters, one can compute indices of unloading bulk cargo terminal operations and make decisions on optimal capacity using results of the exhibit model.

3.1 The unloading bulk cargo terminal

An unloading bulk cargo terminal is defined as a queuing system with the following structure: bulk cargo ships are entry units which form (or not) queues (depending on the immediate situation) waiting to be serviced (unloading bulk cargoes) at the berth and, having been serviced, leave the system.

The unloading terminal as a service system is characterized by the following facts:

- It is not possible to anticipate the arrival time of ships at the terminal as it depends on route, speed of ship in knots, weather, organization of maritime transportation processes and other reasons.
- It is not possible either to accurately predict service time of the ship, i. e. duration of trans-shipment, as it depends on ship deadweight, quantity of bulk cargoes, capacity and technology of trans-shipment facilities, weather, organization of port trans-shipment processes etc.

As a result of these factors, there is irregular berth employment. If the number of bulk cargo ships arriving is greater than the berth capacity, i. e. the number of ships which can be serviced in a unit of time, then the ships queue or, conversely, if there are fewer ships, they do not queue and berth capacity is not completely employed.

A change in the number of berths impacts on the increased or reduced values of certain bulk cargo terminal indices. Through an increase in the number of berths, the number of ships in the queue and at the terminal, as well as the waiting time and length of ship's stay at the terminal, is reduced and berth unemployment is increased.

A decision on the optimal number of berths of a bulk cargo terminal depends on the previously set criteria of optimization, e. g. percentage of berth employment, ship's length of time spent in queue, number of ships in queue or waiting costs and berth unoccupancy, i. e. that chosen criterion seems the most significant for efficient operations of a bulk cargo terminal.

The efficiency of the bulk cargo terminal is very often determined by means of an index on length of time of the ship's stay at the terminal (ship's length of time spent in queue and duration of ship service) and it is augmented either by increasing the number of berths or curtailment of average service time. However, growth in the number of berths increases the likelihood that berths will be vacant which, in turn, means that berth unoccupancy will be greater. Similarly, a curtailment in ship service time may affect the quality of service as well as the reduction in the number of ship arrivals.

From the queuing theory viewpoint, an unloading bulk cargo terminal has the following characteristics [12, p. 54, 55]:

- terminal is an open system as ships are not an integral part of the system,
- the bulk cargo port Bakar has two specialized quays (loading and unloading terminal) as servicing places for which on sea or at anchorage ship queuing lines are formed,

- the number of ship arrivals as well as the duration of the service i. e. length of ship's stay at berth is allocated according to certain theoretical distributions (most often based on Poisson's distribution). The service time of ship, together with the time spent queuing, represents the ship's stay at the terminal and is one of the most significant indices of unloading bulk cargo terminal operations,
- mutual help between loading and unloading terminals does not exist,
- with regards to queuing discipline, an unloading bulk cargo terminal is a system where servicing is most often carried out according to the FIFO rule (first come-first served) but it is possible that there are certain ships which have priority in servicing.

Course of ship arrivals is stationary Poisson course with the following properties [8, p. 495]:

- time independence, in arbitrary short time probability to arrive more than one ship is very small, i. e. ships enter the port separately,
- "no memory" property, arrivals of ships are independent,
- stationary, intensity of a ship course is time independent since it is a constant value dependent only on the length of the observed period.

Based on the queuing problem classification, the unloading bulk cargo terminal is a system which permits an unlimited number of ships to wait in a queue, most frequently using Poisson's distribution for ship arrivals and duration of service, i. e. with the symbol $M/M/1/\infty$.

The basic parameters of an unloading bulk cargo terminal are the ship arrival rate λ and the rate of service μ.

For a chosen unloading bulk cargo terminal system, parameter λ represents the average number of bulk carriers or quantity of bulk cargoes which arrive at a terminal during an observed time unit (e. g. during a year, month or day).

If the time which elapses between two consecutive ship arrivals is available, an arithmetical mean which represents the average interval between two consecutive ship arrivals ($\bar{t}_{arr}$) can be computed. This interval is, in fact, the reciprocal value of the ship arrival rate:

$$\bar{t}_{arr} = 1/\lambda, \text{ or } \lambda = 1/\bar{t}_{arr}.$$

The service rate can be explained analogously. For a chosen system, the unloading bulk cargo terminal represents the average number of bulk carriers or quantity of bulk cargoes which can be serviced in a time unit at certain berths.

If the number of ships which can be serviced during an observed time unit is unknown and only duration of service per ship is known, then the arithmetical mean pattern represents the average service duration per ship ($\bar{t}_{serv}$) and this time is the reciprocal value of the service rate:

$$\bar{t}_{serv} = 1/\mu, \text{ or } \mu = 1/\bar{t}_{serv}.$$

The parameter μ represents the accommodative capacity of one berth and multiplicand $S \cdot \mu$, where S is the number of berths, accommodative capacity i. e. unloading bulk cargo terminal capacity.

The arrival rate and service rate quotient represents berth employment rate or traffic rate ρ:

$$\rho = \lambda/\mu.$$

If $\lambda > \mu$, one berth is insufficient as the employment rate is greater than 100%. In this event, the number of berths should be increased until the service system stability condition that the system employment coefficient $p/S<1$ has been satisfied.

In practice, parameter values λ and μ are determined on the basis of empirical data or assessment depending on the goal and subject of research.

Bulk cargo port in Bakar consists of unloading and loading bulk cargo terminals. In this paper the unloading bulk cargo terminal, as part of the port of Rijeka, has been analyzed. This terminal is assigned to trans-ship various types of bulk cargoes, iron ore, coal, bauxite, phosphate. However, in the last few years coal represents the majority of cargo being handled in that port.

Unloading bulk cargo terminal has maximum degree of utilization for cargoes with bigger specific gravity, for example iron ore. The limiting possibility for expanding port capacities, as far as forwarding is concerned, rests on the number of stationed wagons per day. The amount of cargo transported by wagons equals 7,000 - 8,000 tons a day, and the maximum capacity of the wagon distribution centre is 14,000 tons. The facility capacities are presented in Table 1.

It should be noted in Table 1 that crane No. 1 was dismantled in 2004 and crane No. 3 was installed in 2002. However, this new crane has not yet been used for ship discharging, except for the test period, and will not be considered in the calculations in this paper.

Since the trans-shipment process consists of several technological operations (weighing of cargo, transport of cargo with conveyor from storage to ship, ship loading), the theoretical maximum capacity of loading terminal includes the maximum capacity of every single equipment that takes part in trans-shipment (cranes, conveyors, distribution station, weighbridge, storage, wagons). The theoretical maximum capacity implies the maximum capacity of the equipment with the minimum capacity in trans-shipment chain.

Terminal capacity is the maximum terminal capacity reduced by the cargo that has not been transhipped during breaks, i. e. breaks caused by mechanical failures of equipment, breaks caused by maintenance and

Table 1 - Facility capacities of bulk cargo terminal Bakar

<table>
<tr><td colspan="5">Quay Podbok</td></tr>
<tr><td>length: 394 m</td><td colspan="2">maximum depth: 18.5 m</td><td colspan="2">ship: max 160 000 DWT</td></tr>
<tr><td colspan="5">Storage</td></tr>
<tr><td rowspan="2"></td><td rowspan="2">length × width (m)</td><td colspan="2">capacity in tons</td><td rowspan="2">Equipment</td></tr>
<tr><td>iron ore</td><td>coal</td></tr>
<tr><td>Podbok</td><td>330 x 27</td><td>300,000</td><td>100,000</td><td>discharging equipment with conveyor</td></tr>
<tr><td>Dobra</td><td>340 x 19</td><td>80,000</td><td>25,000</td><td>conveyor</td></tr>
<tr><td>Plato</td><td>160 x 36</td><td>80,000</td><td>26,000</td><td>unequipped</td></tr>
<tr><td colspan="5">Cranes</td></tr>
<tr><td></td><td>capacity in t/h</td><td colspan="2">year of manufacture</td><td>quantity</td></tr>
<tr><td>discharging equipment no.1</td><td>800</td><td colspan="2">1967</td><td>1</td></tr>
<tr><td>discharging equipment no.2</td><td>1,600</td><td colspan="2">1978</td><td>1</td></tr>
<tr><td>discharging equipment no.3</td><td>3,000</td><td colspan="2">2001</td><td>1</td></tr>
<tr><td>loading equipment</td><td>600</td><td colspan="2">2001</td><td>1</td></tr>
<tr><td>storage conveyor</td><td>500</td><td colspan="2">1967</td><td>1</td></tr>
<tr><td colspan="5">Conveyors</td></tr>
<tr><td>capacity for iron ore</td><td colspan="4">1,600 t/h</td></tr>
<tr><td>capacity for coal</td><td colspan="4">1,300 t/h</td></tr>
<tr><td colspan="5">Capacities according to cargo type in tons</td></tr>
<tr><td></td><td colspan="2">Ore</td><td colspan="2">Coal</td></tr>
<tr><td>single storage capacity in tons</td><td colspan="2">400,000</td><td colspan="2">150,000</td></tr>
<tr><td>tech. capacity ship-storage for crane 45+16 t/shift</td><td colspan="2">8,167</td><td colspan="2">3,268</td></tr>
<tr><td>tech. capacity ship-storage for crane 45 t/shift</td><td colspan="2">–</td><td colspan="2">2,400</td></tr>
<tr><td>tech. capacity storage-wagon for storage gantry crane in t/shift</td><td colspan="2">2,500</td><td colspan="2">–</td></tr>
<tr><td>theoretical max. capacity in tons</td><td colspan="2">4,960,000</td><td colspan="2">3,000,000</td></tr>
<tr><td>real capacity in tons</td><td colspan="2">3,500,000</td><td colspan="2">2,000,000</td></tr>
<tr><td>technological-market capacity in tons – unloading terminal</td><td colspan="4">2,000,000</td></tr>
</table>

Source: Port of Rijeka [13]

cleaning of facility, working breaks and time spent on ship mooring and unmooring.

Finally, the quantity of cargo trans-shipped in port doesn't depend only on equipment, transport and storage capacities, but also on external factors. These are:

- transport of cargo in port and out of port that depends on railway flow rate, flow rate of the railway hub and inland storages,
- cargo demand,
- breaks caused by weather or strikes.

Technological-market capacity of the terminal includes the above factors, and is calculated taking into account the terminal capacity and several-year record of cargo flow.

3.2 Queuing model M/M/1 of the port's bulk cargo unloading terminal Bakar

The procedure of unloading cargo from ships consists of several technological processes:

- unloading of cargo from a ship with port discharging equipment on conveyors,
- transport of cargo by conveyors from shore cranes to port storage or to loading weighbridge for wagons.

Technological process of cargo unloading stipulates that duration of ship service depends on:

- the capacity of port cranes,
- the capacity of conveyor belts,
- the capacity of port storage,
- the disposability of wagons,
- ship construction,
- weather conditions, etc.

From statistical data of the Port of Rijeka it follows that the arrival rate of cargo (iron ore and coal) λ is 1,462,489 tons for the year 2005.

It can be seen from Table 1, that the yearly capacity of unloading terminal, representing service rate μ of the unloading terminal, amounts to 2,000,000 tons (technological-market capacity). These data take into account the trans-shipment capacity of discharging equipment and capacity of conveyors (whose yearly capacity separately regarded is well over), storage capacity, disposability of wagons and working breaks.

The parameters for the observed model M/M/1/∞ are:

- cargo arrival rate

$$\lambda=1{,}462{,}489 \text{ t/year}$$

- cargo service rate

$$\mu=2{,}000{,}000 \text{ t/year}$$

Unloading bulk cargo terminal operation indices are computed according to the appropriate queuing theory formulae (see section 2.2) and placed into Table 2:

- traffic rate

$$\rho=\frac{\lambda}{\mu}=0.7312$$

- the probability that there is no ship/cargo at the terminal, i. e. the berth is unoccupied

$$P_0=1-\rho=0.2688$$

- the average quantity of cargo in queue

$$L_Q=\frac{\rho^2}{1-\rho}=1.9896 \text{ tons}$$

- the average quantity of cargo at the terminal (in queue and being serviced)

$$L=\frac{\lambda}{\mu-\lambda}=\frac{\rho}{1-\rho}=2.7208 \text{ tons}$$

- the average quantity of cargo which is just being serviced

$$L_{serv}=L-L_Q=\rho=0.7312$$

- the average queuing time of cargo, i. e. queuing time of cargo prior to being serviced

$$W_Q=\frac{L_Q}{\lambda}=\left(\frac{\rho}{1-\rho}\right)\frac{1}{\mu}=\frac{\rho}{\mu-\lambda}=1.36\cdot 10^{-6} \text{ year} =$$
$$=0.01192 \text{ hour} = 11.92 \text{ h/1000t}$$

- the average time of cargo's stay at the terminal, i. e. queuing time of cargo and duration of its unloading

$$W=\frac{L}{\lambda}=\left(\frac{1}{1-\rho}\right)\frac{1}{\mu}=\frac{1}{\mu-\lambda}=1.86\cdot 10^{-6} \text{ year} =$$
$$=0.0163 \text{ hour} = 16.3 \text{ h/1000t}$$

- the average duration of cargo service - cargo unloading

$$W_{serv}=W-W_Q=\frac{1}{\mu}=5\cdot 10^{-7} \text{ year} =$$
$$=0.004 \text{ hour} = 4.4 \text{ h/1000t.}$$

Table 2 - Computed unloading terminal operation indices for years 2002 and 2005

Indices	Unit	2002 λ_1=702 341 t/year	2005 λ_2=1 462 489 t/year
λ	ton/year	702 341	1 462 489
μ	ton/year	2 000 000	2 000 000
ρ	-	0.35	0.7312
P_0	-	0.65	0.2688
L_Q	ton	0.19	1.9896
L	ton	0.54	2.7208
L_{serv}	ton	0.35	0.7312
W_Q	hour/1000 t	2.40	11.9200
W	hour/1000 t	6.75	16.3000
W_{serv}	hour/1000 t	4.40	4.4000

It can be seen that the cargo arrival rate in relation to cargo service rate, traffic rate ρ is 73%, which points to solid capacity employment rate in 2005, and the probability that the berth is unoccupied amounts to 27%. That is a very good result, especially considering the traffic rate of three years ago amounting to only 35%.

The above is substantiated by the average quantity of cargo at the terminal of 2.72 tons. That includes almost 2 tons of cargo in queue and 0.73 tons of cargo at service.

The increase in the cargo arrival at the unloading terminal results in longer average time of cargo's stay at the terminal - 16.3 hour/1000 tons of cargo.

4. CONCLUSION

In this paper we have assumed that the bulk unloading terminal in Bakar behaves as a queuing model M/M/1 which proves useful in modelling of capacity employment and results in realistic indices of terminal's behaviour. The results can be used in the interpretation of terminal's workload and as a base for decision support in the creation of the future planning and investment policies.

Through statistical data analysis on the number of ship arrivals per day and month of a chosen bulk cargo terminal, it has been established that no significant dependence exists in the sequence of daily arrivals of bulk cargo ships, i. e. that arrivals are statistically random. An analogous conclusion is obtained by the statistical analysis of the duration of bulk cargo ship service.

From the previous conclusion it follows that the number of ship arrivals and the duration of service can be taken as random variables and, in addition, the empirical distribution of those variables approximated with the appropriate theoretical distributions. The queuing theory can be applied in such cases for computing indices of unloading bulk cargo terminal operations.

The Port of Bakar has two specialized terminals for transhipment of bulk cargo - loading and unloading terminal. The unloading terminal is defined as the queuing system M/M/1, which means that the ships' arrivals are Poisson distributed, service time is exponentially distributed and the terminal has one quay, representing the service place. Appropriate parameters as well as indices are calculated.

The exploitability of the terminal in 2005 has drastically changed in contrast to the year 2002, and amounted to 73%. Still, there is space for better use of the existing resources. This is especially true taking into account the capacities of discharging facility itself. It should be noted that crane no. 3 with capacity of 3,000 t/hr has not been considered in the calculations since it is out of use; otherwise, the results for terminal would have been even less favourable.

The terminal is over-capacitated and in the near future there is no need to invest in the capacity enhancement and facilities expansion. A strategy should be developed to attract new cargoes in order to keep the existing facility busy.

The research that follows will take into consideration the behaviour of the associated loading terminal in Bakar which is assumed to be of queuing system type M/D/1. Finally, the correlation of the results and interoperability of those terminals will be made.

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SAŽETAK

SUSTAV OPSLUŽIVANJA U FUNKCIJI OPTIMIZACIJE TERMINALA ZA ISKRCAJ RASUTOG TERETA

U radu je istražena primjena teorije redova čekanja u modeliranju lučkog iskrcajnog terminala za rasute terete. Terminal za rasute terete može se definirati kao sustav masovnog opsluživanja s parametrima: intenzitet toka dolazaka brodova za rasute terete ili količine rasutog tereta i intenzitet toka opsluživanja brodova za rasute terete odnosno količine rasutog tereta u promatranoj vremenskoj jedinici. Na temelju ovih parametara izračunati su odgovarajući pokazatelji funkcioniranja iskrcajnog terminala za rasute terete. Iskrcajni terminal definiran je kao sustav masovnog opsluživanja oznake M/M/1 prema Kendallovoj notaciji. Ovu vrstu reda čekanja karakterizira Poissonova razdioba toka dolazaka jedinica i duljina vremena opsluživanja prema eksponencijalnoj razdiobi. Postavljeni model testira se na primjeru iskrcajnog terminala luke za rasute terete u Bakru, na temelju kojega se može utvrditi funkcioniranje terminala u svrhu donošenja odgovarajućih odluka za njegovo daljnje poslovanje. Dobiveni parametri i izračunati pokazatelji ukazuju na zadovoljavajuću iskoristivost kapaciteta u 2005 godini a vjerojatnost da je pristan nezauzet je relativno niska. Pokazatelji upućuju na to da je stopa uposlenosti terminala značajno porasla uzevši u obzir prethodne godine.

KLJUČNE RIJEČI

stohastički procesi, sustav masovnog opsluživanja M/M/1, iskrcajni terminal luke za rasute terete

LITERATURE

[1] **D. Barković**: "*Operacijska istraživanja*", Ekonomski fakultet Osijek, Osijek, 2001

[2] **S. J. Bose**: "*An Introduction to Queuing Systems*", Chapter 1, Kluwer/Plenum Publishers, 2002

[3] **J. D. Camm, J. R. Evans**: "*Management Science: Modeling, Analysis and Interpretation*", South-Western College Publishing, Cincinnatti, OH, 1996

[4] **K. Cullinane, D. W. Song, T. Wang**: "*The Application of Mathematical Programming Approaches to Estimating Container Port Production Efficiency*", Journal of Productivity Analysis, Springer Netherlands. Vol. 24. No. 1. 2005, pp. 73-92

[5] **K. Cullinane**: "*The Productivity and Efficiency of Ports and Terminals: Methods and Applications*", Grammenos, C. T. eds., The Handbook of Maritime Economics and Business, LLP, London, 2003

[6] Z. Ivković: "*Uvod u teoriju verovatnoće, slučajne procese i matematičku statistiku*", Građevinska knjiga, Beograd, 1970

[7] J. A. Lawrence, B. A. Pasternack: "*Applied Management Science, a Computer-Integrated Approach for Decision Making*", John Wiley and Sons, New York, 1998

[8] M. Lazić: "*Primena metoda teorije masovnog opsluživanja u analizi vremena zadržavanja brodova u luci Koper*", Saobraćaj, Vol. 28, No. 3, 1981, pp. 493-499

[9] H. C. Tijms: "*Stochastic Models, an Algorithmic Approach*", John Wiley and Sons, New York, 1994

[10] E. Wentzel, L. Ovcharov: "*Applied Problems in Probability Theory*", Mir Publishers. English translation, 1986

[11] W. Whitt, P. W. Glynn, B. Melamed: "*Estimating Customer and Time Averages*", Operations Research, Vol. 41, No. 2, 1993, pp. 400-408

[12] Z. Zenzerović: "*Teorija redova čekanja, Stohastički procesi II dio*", autorizirana predavanja, Pomorski fakultet u Rijeci, Rijeka, 2003

[13] Port of Rijeka, Bulk cargo terminal Bakar, Rijeka, 2005

[14] http://staff.um.edu.mt/jskl1/simweb/mm1.htm

[15] http://www.cs.vu.nl/obp/queueing/MDc.pdf

[16] http://www.cs.vu.nl/~franx/Mdctran11.pdf