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OPTIMISATION OF GAS TRANSPORT IN DISTRIBUTIVE GAS NETWORKS

SUMMARY

The article deals with the problem of network facilities location in the low-pressure networks. It is a "general m-median" problem. The demand of gas is defined both in the nodes and along the edge. The problem is solved by a heuristic algorithm. It searches for the optimal location of supply facilities in the network using Mini-sum criterion. The continuous demand on each link is replaced by a concentrated demand in the middle point of the link.

1. INTRODUCTION

In designing low-pressure gas networks, one of the problem is the selection of the number and location of the supply nodes (reduction stations). If an optimal gas network is to be obtained, we have to pay due attention to this problem. There are two networks, medium- and low-pressure, which are connected by reduction stations. Optimal solution is the solution in which the overall costs of construction and maintenance are minimal. The selection of the number and location of the supply nodes affects the construction and maintenance costs of both networks. The increase in the number of supply nodes means a decrease in the low-pressure network costs, and increase in the costs of medium-pressure network and reduction stations. How to provide a designer with a fast and quick way of determining the optimal number and position of the supply centres? The criteria need to be defined and software tools developed which will provide the designer with quick and simple decision-making.

The section 2 of this work, gives the criteria for allocating the given number of supply centres, and offers a suggestion of a method for calculating the optimal location. Section 3 analyses the influence of the number of supply centres on the weight of the pipeline. The calculation results of an example of a gas network are supplied, showing the influence of the selected number and location of the supply nodes on the optimum.

2. SELECTION OF THE OPTIMAL LOCATION OF A GIVEN NUMBER OF SUPPLY NODES IN A NETWORK

The optimal allocation of the given number of supply nodes in a network is calculated according to the Mini-sum criterion [1, 2]. The following holds in general:

$$d(u) = \sum \left[d(u,v); v \in V(G_n) \right] = \min$$

The task is to find node "u" for which the sum of the shortest paths d(u,v) in relation to all the other nodes "v" in the network G_n with n nodes $V(G_n)$ will be minimal. Generally, d(u) is called the distance of the node "u" in the network G_n, and the node with the minimal distance is "1-median of the network". If the members of each row in the n x n matrix D of the shortest paths are added, the row with the lowest sum in relation to the sums of other rows, determines the node "u". For a network with m supply nodes, there is, therefore, "m-median of the network". Depending on the type of the network and characteristics of transportation problem, in the Mini-sum criterion the shortest paths in the matrix D can be multiplied by a variable. In gas networks this means the gas flow along the edge [nm^3/h]. The characteristic of the low-pressure network is also that the demand can be located in the network node or along the edge, which makes the problem of searching for the optimal location of the supply nodes a more complex one and transforms it into a "general m-median" problem.

The edges with the given demand can be divided into the so-called common edges (branches) and marginal edges. Continuous demand along the edge is assumed. If this is not the case, we set the node at the position on the edge where the concentrated demand is greater. Figure 1 shows the diagram of the gas flow through common and marginal edges and the way of substituting the continuous demand in the edge by the concentrated demand at one point of the edge. It can

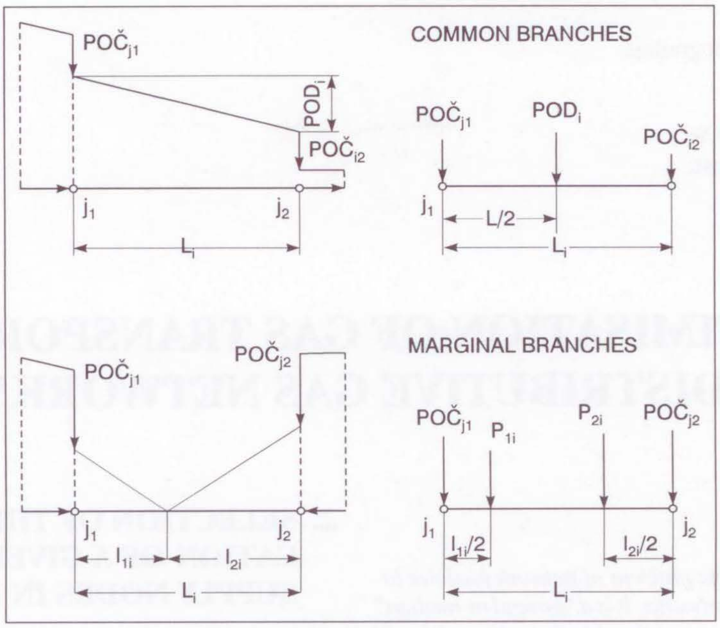

Figure 2

follow that the function of objective for the low-pressure gas network by using Mini-sum criterion is as follows:

$$f_c = \sum_{j=1}^{i\check{c}} PO\check{C}_j \, d_{kj} + \sum_{i=1}^{id} POD_i \left(d_{kj1} + \frac{L_i}{2} \right) +$$

$$+ \sum_{i=1}^{id} \frac{POD_i}{2} \left(d_{kj1} + d_{kj2} + \frac{L_i}{2} - \frac{\left(d_{kj1} - d_{kj2}\right)^2}{2\,L_i} \right)$$

The first term considers the demand "ič" of the $PO\check{C}_j$ nodes. The second term includes the demands of common edges POD_i, and the third includes the demands of marginal edges. There are "id" edges which are either common or marginal. The nodes j, j_1 and j_2 are located away from the supply facilities "k" by the shortest path d_{kj}, d_{kj1} and d_{kj2}. For the given number of supply facilities it is necessary to find that combination of nodes location for which the objective function value fc will be minimal. This cannot be solved by the linear programming methods [3], nor by methods of non-linear programming [5], due to the complexity of the objective function. The biggest problem lies in the fact that the flow in the network changes with the change in the position of one or more supply facilities, so that some marginal edges cease to be, and some other marginal edges appear.

If every marginal edge is divided into two fictitious edges of l_1 and l_2 lengths [m] $\left(L_i = l_{1i} + l_{2i}\right)$ and demands P_{1i} and P_{2i} [nm^3/h] $\left(POD_i = P_{1i} + P_{2i}\right)$, the objective function can be transformed into the following form:

$$f_c = \sum_{i=1}^{id+idg} GO_i \, L_i = \min$$

where idg is the number of marginal edges, and GO_i is the ideal flow through the i-th edge [nm^3/h] which is established if every demand in the network is supplied by the gas along the shortest path from the supply facilities. The marginal edges can be divided in several ways, both by the number and by the size of the fictitious edges. In the example for which the calculation results are later given, the marginal edges have been divided into two equal parts ($l_{1i} = l_{2i}$ and $P_{1i} = P_{2i}$).

The selection of the optimal location of the supply facilities in the network has been solved by the heuristic algorithm. The method has been developed for balancing the gas network by moving the i-th supply facility into the general 1-median of i-th network location. Figure 2 illustrates the convergence procedure. The pipe network has been replaced in the illustration by the area, and instead of looking for the general 1-median of the local part of the network, the centre of gravity of the form is sought which replaces the local area of the network. The Figure presents an example with three supply facilities, and the principle holds for any number of supply facilities. It can be seen in the Figure that the process converges very quickly. The procedure includes the following activities:

1. We make a random choice of m nodes in the network which are then called supply facilities. The nodes are selected by the random number generator.
2. For the combination of supply facilities, the shortest flow paths of gas flow to every demand in the network are calculated. We determine the belonging of every network node to its closest supply facility. The value of the objective function is calculated,

$$f_c = \sum_{i=1}^{id+idg} GO_i \, L_i,$$

for the whole network.

3. The network nodes are classified into m local regions, according to the supply facility from which they are supplied.
4. For every local region in the network we look for the general 1-median node in the following way (in a local region there are nlp nodes):
   - every node of the local region is set as the supply facility,
   - for a thus selected supply facility of the local region the ideal flow (shortest gas flow paths towards other nodes of the local region) are calculated, as well as the value of the objective function $\left(\sum \left(GO_m \, L_m\right) m = 1, nlp\right)$, which is valid in the local region.
   - from the set of nlp values of the objective function we select a node whose objective function value is minimal and this becomes the general 1-median of the local region.
5. The general 1-median nodes of the local regions are set as supply facilities of the network.
6. Return to 2: which is repeated until the supply facilities equal the general 1-median nodes, which is a solution for the initially randomly selected combination of supply nodes.
7. Return to 1: the whole procedure is repeated several times (in the enclosed example it was calculated 50 times) and out of the obtained solutions the one is selected for which the minimal objective function value is

$$f_c = \left(\sum \left(GO_i \, L_i\right) i = 1, id + idg\right).$$

Table 1.

$\Sigma(G0_i L_i)$	m	1	2	3	4
40735	2	16	36		
27590	2	16	70		
24677	2	70	108		
22144	2	61	106		
21592	2	60	98		
27484	3	4	16	67	
22632	3	26	34	59	
19303	3	53	60	98	
19160	3	53	59	98	
18198	4	40	67	68	102
18218	4	26	41	77	98
17348	4	47	52	76	106
17119	4	47	53	76	106
17102	4	53	58	76	106

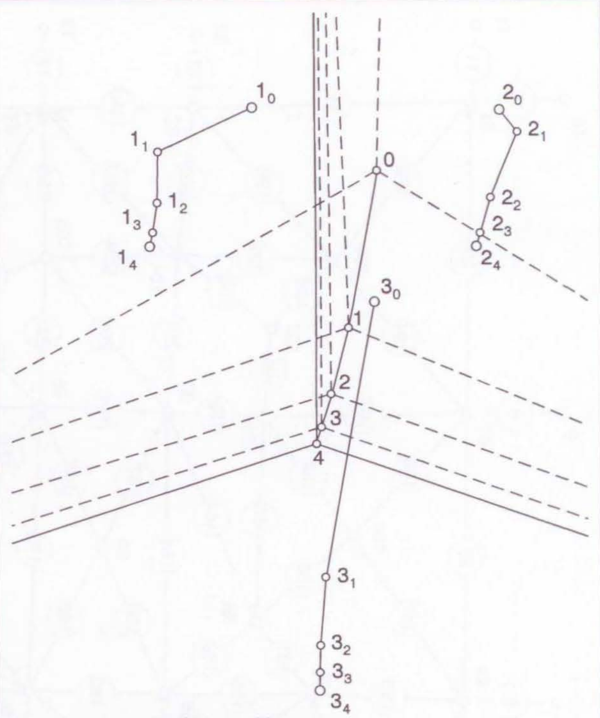

Figure 2

Table 1 presents the procedure for calculating the optimal position of the supply facilities for the example in Figure 3, and for m = 2, 3, and 4, and for the initial supply facilities listed as the first combination. The supply facilities written in one row of the table are the starting point for distributing the facilities into m local regions. For every local region the general 1-median is calculated, the node which is entered into the next row of the table. The results show the speed of convergence.

Table 2 shows the calculation procedure for the same example, but with the final results of 50 different starting combinations of the supply facilities, unlike Table 1, where the inter-results of every calculation step are entered for only one starting version.

3. ANALYSIS OF THE NETWORK WITH A DIFFERENT NUMBER OF SUPPLY NODES

The costs of construction and maintenance of a low-pressure network, considered separately from the medium-pressure network and reduction stations, decrease with the weight of the pipes. The same is true

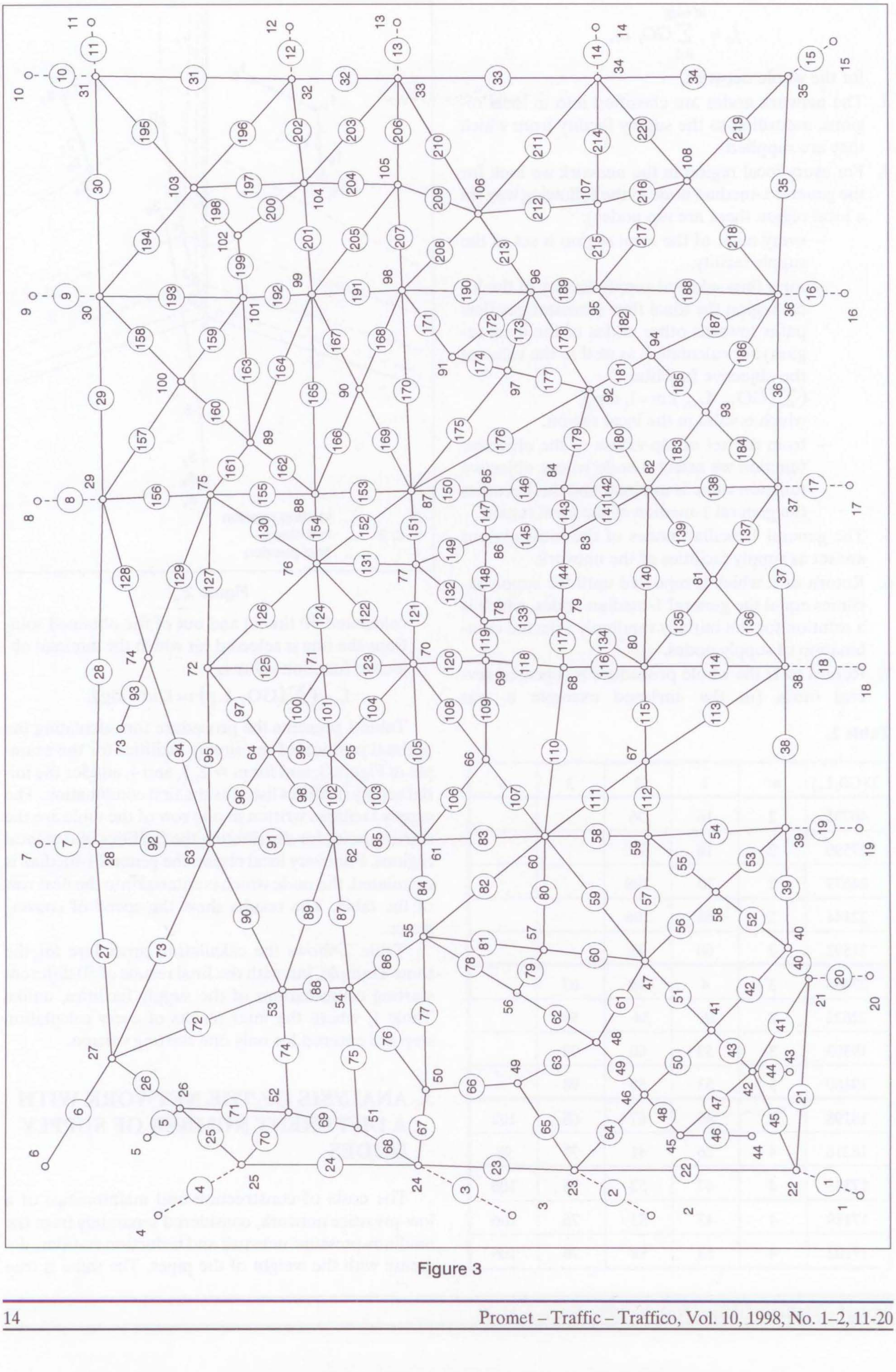

Figure 3

Table 2

The calculation of the objective function $fcilja = \left(\sum\left(GO_i\ L_i\right), i=1, id+idg\right)$, by using the method of moving the i-th supply facility into the general 1-median of the i-th network location. For each number of supply facilities (m=1-8) 50 iterations were calculated (full calculation). Out of all 50 results, only those combinations are listed which have the function value calculated until that moment.

Itera tion	fc /1000	Supply centres								Supply centre's gas quantity, $nm^3/(1000{*}h)$							
		1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
1	29479	70								24.9							
1	21592	60	98							13.1	11.8						
1	20525	31	60	98						1.7	13.1	10.1					
2	19210	47	63	98						7.6	5.9	11.5					
5	19160	53	59	98						6.4	7.1	11.3					
1	18094	31	53	59	98					1.7	6.4	7.1	9.6				
2	17611	42	53	80	98					3.3	6.2	5.0	10.4				
4	17584	41	53	82	99					4.4	6.1	6.2	8.2				
6	17102	53	58	76	106					4.6	6.4	6.3	7.6				
1	16579	31	41	63	80	98				1.7	4.5	5.6	4.8	8.3			
2	15989	40	54	87	103	108				4.5	6.3	6.9	3.5	3.7			
19	15913	42	53	80	99	108				3.3	6.2	5.2	7.4	2.8			
41	15824	42	53	80	89	106				3.3	5.5	4.5	5.9	5.6			
1	15327	31	41	60	63	87	108			2.6	3.7	4.4	4.5	5.9	3.8		
2	15113	21	24	38	62	99	108			2.9	2.8	3.0	5.4	7.8	3.0		
7	15078	32	40	53	76	92	108			3.2	5.0	5.3	5.3	3.7	2.4		
11	15000	31	36	40	53	76	106			1.7	2.4	5.2	5.3	5.9	4.3		
22	14808	32	41	53	81	88	108			2.6	4.4	5.0	3.1	6.5	3.2		
35	14776	32	38	41	53	76	108			3.2	3.1	4.0	4.6	6.2	3.8		
38	14665	32	41	53	75	80	95			2.7	3.8	5.0	4.5	4.4	4.5		
1	14789	8	31	36	40	53	76	106		0.5	1.7	2.4	5.2	5.3	5.5	4.3	
2	14482	21	26	34	49	62	82	101		2.5	1.4	2.8	2.2	5.3	5.2	5.4	
3	14294	20	31	38	47	53	88	95		0.6	2.5	3.0	3.5	5.0	5.6	4.7	
8	13991	10	41	53	75	80	105	108		0.8	3.8	5.0	4.3	4.5	3.9	2.5	
20	13901	25	32	41	62	75	80	95		2.1	2.7	3.8	4.1	4.2	3.4	4.5	
22	13750	31	38	42	53	76	98	108		1.7	3.7	2.8	5.1	5.5	3.6	2.5	
1	13877	18	29	32	40	53	70	82	108	0.4	2.7	3.2	3.7	4.9	3.9	3.0	3.1
3	13700	21	24	32	34	63	80	88	108	2.8	3.3	2.6	1.0	3.4	4.0	5.6	2.2
5	13655	23	27	31	59	75	83	98	108	2.9	2.8	1.7	4.9	4.3	2.5	3.3	2.4
9	13100	29	31	38	42	53	70	98	108	2.6	1.3	2.9	2.8	4.8	4.4	3.6	2.5
12	13076	21	24	31	63	75	80	98	108	2.8	3.3	1.7	2.9	3.6	4.5	3.6	2.5

also for the medium-pressure network. How do the number and location of the supply nodes affect the weight of the pipeline? This question can be answered by dimensioning an example of the gas network for a varying number and location of supply nodes. We shall use the gas network in Figure 3, in which 24,899 nm^3/h are distributed, and which has 108 nodes and 220 edges in the total length of 63,865 m. Table 3 presents

Table 3

Supply centres								T1 (Dst)	T1/T0	T0(Di)	(Gji*Li),i=1,id+idg		
											j=0	1	2
m = 1	2	3	4	5	6	7	8	kg	%	kg	nm^{3}m/1000h		
70								1458765	14.0	1279330	29479	29486	29302
84								1525319	11.5	1367540	32222	32213	31978
63								1689344	13.1	1493676	35400	35434	35186
105								1783779	12.1	1590938	38392	38397	38378
40								1808479	9.0	1659707	41254	41215	41317
42								1987104	15.1	1726749	44056	44056	44819
60	98							1146265	14.8	998273	21592	21595	21437
60	103							1242524	15.2	1078346	23812	23790	23613
51	94							1297543	14.1	1136935	25791	25793	25650
6	88							1416865	14.3	1239915	28114	28107	28012
26	60							1494849	14.3	1307486	30109	30081	29823
28	40							1589296	15.0	1382077	32296	32276	32173
53	59	98						1056036	17.2	900729	19160	19116	18889
29	60	104						1114830	15.4	966243	20942	20945	20809
37	45	71						1216133	16.7	1042016	22904	22927	22597
37	42	54						1307564	18.4	1104616	24815	24831	24755
38	59	66						1343414	13.6	1182332	26857	26850	26624
51	53	65						1404212	13.6	1236257	28679	28674	28638
53	58	76	106					956139	16.9	817844	17102	17108	16856
24	58	87	101					1027872	16.1	885644	18694	18680	18453
4	42	53	87					1091800	14.3	955145	20467	20482	20271
32	63	70	71					1140333	13.8	1002064	22105	22125	21834
22	32	44	57					1221091	14.2	1068885	24036	24031	23900
35	38	39	48					1303797	14.0	1144028	25686	25656	25568
76	47	53	95	102				894322	17.7	759637	15733	15740	15506
10	24	29	60	106				961045	18.7	809368	17016	17030	16880
10	30	34	62	82				1044196	18.1	883797	18749	18757	18605
25	38	44	49	76				1103646	18.3	933309	20137	20128	19831
18	29	38	54	73				1135657	14.5	992098	21833	21856	21776
33	72	90	95	101				1200938	14.1	1052092	23230	23218	22998

Table 3 (continued)

Supply centres								T1 (Dst)	T1/T0	T0(Di)	(Gji*Li),i=1,id+idg		
											j=0	1	2
m = 1	2	3	4	5	6	7	8	kg	%	kg	nm³m/1000h		
31	38	42	53	76	106			852531	19.6	712537	14732	14746	14512
25	42	62	75	81	99			895602	17.8	760587	15771	15811	15629
14	54	67	79	88	92			965705	17.5	822189	17169	17170	16904
7	20	33	55	67	105			1004862	15.5	870041	18633	18616	18422
21	42	58	59	98	107			1070268	14.5	935064	20126	20096	19843
6	17	22	58	79	85			1135024	15.8	979906	21577	21587	21340
32	41	53	75	80	98	108		799887	18.4	675596	13790	13762	13648
10	24	33	38	70	100	108		871021	18.5	735039	15195	15191	14972
10	18	33	48	75	79	106		932460	17.3	795108	16582	16575	16241
6	11	26	40	76	85	90		972159	15.8	839311	17871	17879	17649
2	12	20	42	88	93	103		1040444	16.2	895519	19066	19015	18810
42	51	60	63	82	88	103	106	766619	18.0	649471	13267	13296	13085
7	33	36	53	58	75	86	97	833172	18.4	703496	14554	14551	14467
13	24	64	67	84	89	99	101	888852	16.8	760899	15786	15791	15502
17	18	22	25	55	70	92	95	942413	18.0	798650	17000	16993	16832
2	13	16	65	79	91	93	99	1003938	16.6	861324	18309	18309	18112

the calculation results which are also presented in Figures 4 and 5. One row in the table represents one version of the network, which differ in number and position of the supply nodes. Using the algorithm described in Section 2, the supply nodes have been selected. For every number of supply nodes from 1 to 8, first the optimal version with fc=min was found. In the next step the version of supply nodes location is found for which the value of the objective function fc is approximately 10% greater than the value of the objective function of the optimal version. Every subsequent version has a characteristic that the value of its objective function is approximately 10% greater than the value of the objective function of the previous version. In the table, the versions are grouped according to the number of supply nodes, which are entered in the first eight columns of the table.

Optimal dimensioning of every version is done by a heuristic algorithm. Apart from satisfying the equation of continuity for every node of the network and the amount of flow for every edge, the algorithm also includes the criterion of function minimisation

The value of the function

$$f_c = \sum_{i=1}^{id+idg} \left(GO_i\, L_i\right) / v_i$$

will be minimal when two criteria are satisfied:

1. Flow of gas in the network so that every demand gets gas by the shortest path from the closest supply node => $(GO_i L_i)$=min [nm³ m/h]. Regarding dimension, this is equivalent to the transportation energy. The optimal solution is obtained with minimal transportation energy.
2. The selection of the flow speed along the i-th edge v_i according to the diagram of recommended maximum flow speeds v_{pi} in the function of the medium pressure in the edge, [6] page 134, $v_i = v_{pi}$.

Dimensioning is done for the ideal pipe diameters Di, which give the total network weight T0(Di), and with standard pipe diameters Dst which give the weight of the real network T1(Dst). The characteristic of the ideal solution is the minimal deviation of the realised ideal flow G_i from the theoretical hypothetical flow

$$GO_i \left(\sum \left(G_i\, L_i\, |GO_i - G_i| / GO_i\right) / \sum \left(G_i\, L_i\right) < 3\%\right)$$

as well as minimal deviations of the realised flow speed v_i compared to the maximal allowed recommended speed

$$v_{pi}\left(\sum\left(G_i\,L_i\,|v_{pi}-v_i|/v_{pi}\right)/\sum\left(G_i\,L_i\right)<3\%\right).$$

Transition from the ideal pipe diameters, in which pipes are not manufactured, to the commercial real standard pipe diameters stipulates the increase in the pipeline weight. A characteristic of these solutions is that the function value $\left(\sum\left(G_i\,L_i\right)\right)$ remains almost unchanged, and the increase in pipe diameter (pipeline weight) occurs to the detriment of the reduction in flow speed v_i.

The third group of data in the table includes the function value $\left(\sum\left(G_i\,L_i\right)\right)$ for the ideal hypothetical re-distribution of the gas flow in the network j=0, the solution of the network with ideal pipe diameters j=1, and the final real solution with standard pipe diameters j=2.

Figure 4 illustrates the answer to the question of how the change in the number and location of the supply nodes influences the gas network pipeline weight. Solutions for the same number of supply nodes, but

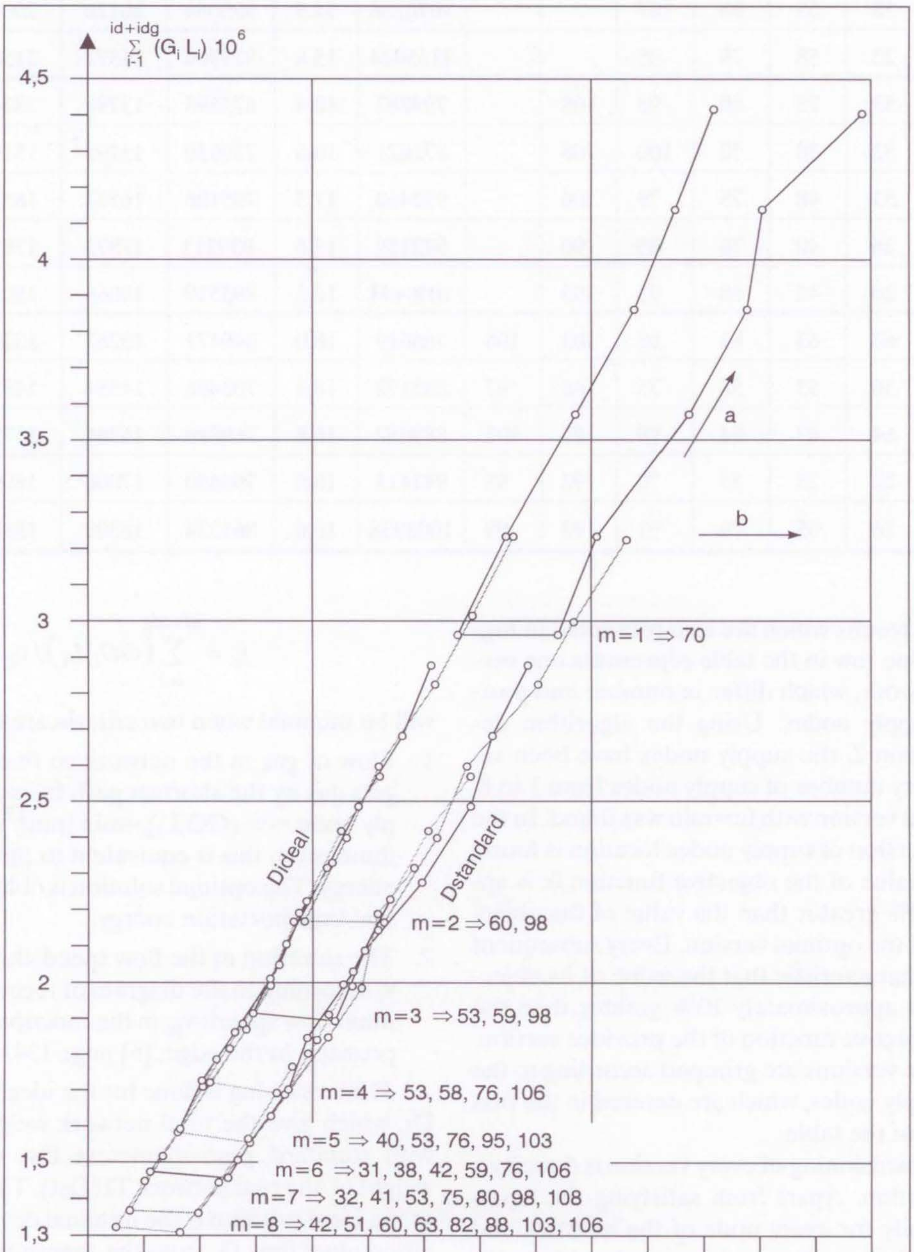

Figure 4

for a different combination of locations, have been linked by lines. This is clearly seen for m=1 supply nodes, whereas when the number of supply nodes increases, m lines overlap and become difficult to discern one from the other. For a given number of supply nodes m, the weight of the network with standard pipe diameters Dst, depend on two factors and they are:

- location of the supply nodes (moving the point in the diagram along the joining link upwards → a).
- average flow speed of gas through the gas pipeline (moving in the diagram to the right → b). Apart from the grading of the standard pipe diameters, this speed depends also on the quality of the network dimensioning procedure.

In solutions for Dst in the Figure, the maximally possible flow speeds of gas have been obtained within the given limitations (e.g. optimal version with two supply nodes has average speed of

$$\sum\left(v_{pi}\ L_i\right)/\sum L_i = 8{,}28\ m/s$$

$$\sum\left(v_{pi}\ L_i\right)/\sum L_i = 5{,}65\ m/s,$$

and the speed v_i never exceeds the maximal recommended speed $v_i\ v_{pi}$).

Figure 5 shows the change in the weight of the gas pipeline network and the values of the objective function fc on the number of supply nodes. The lowest points in the diagram for every number of supply nodes are the optimal solutions, and every subsequent point upwards has approximately 10% higher value fc than the previous point.

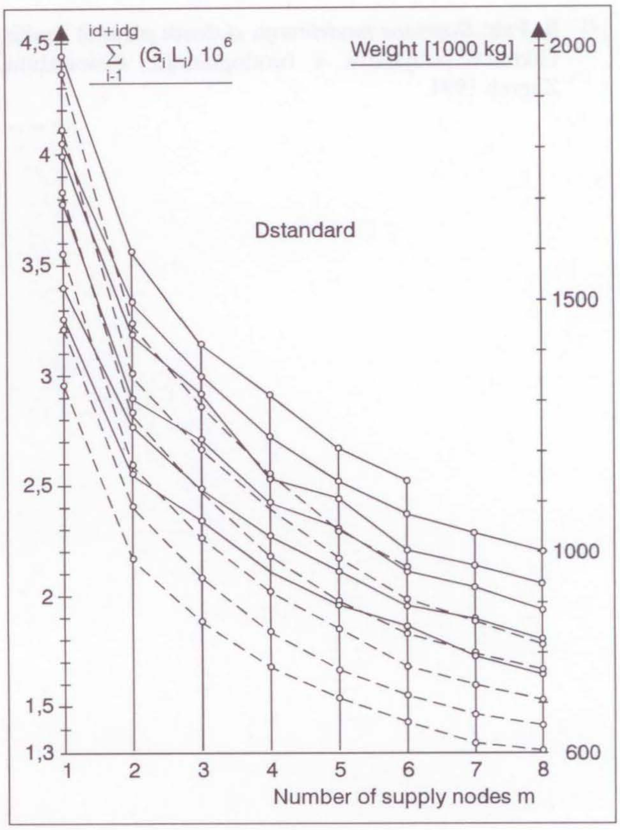

Figure 5

4. CONCLUSION

It can be concluded that it is very important to select the number and location of the supply nodes in an appropriate way. This, however, is not enough, but provides a potential which will be realised by the optimal solution of the pipeline network only by adequate dimensioning of the network. If we enter prices of the low-pressure network in Figure 5 instead of weight, and calculate the price of the medium-pressure network and the reduction stations in the same way, the optimum is obtained easily. The essence lies in the fact that we have the software which allows fast and simple defining and dimensioning, i.e. valorisation of the versions for solving the pipeline network, and which are the basis for finding the optimal solution. The input data are gathered easily and then used for any number and location of the supply nodes, with slight changes referring only to the input pressure in the network node. The described methodology should be used to process a greater number of characteristic examples of the gas network and to obtain numeric indicators which would simplify the decision-making in selecting the number and location of the supply nodes, delimiting the gas network onto low and medium pressure, calculating the price of the j-th demand for the network, etc.

SAŽETAK

OPTIMALIZIRANJE TRANSPORTA PLINA U DISTRIBUTIVNOJ PLINSKOJ MREŽI

U radu se analizira problem traženja optimalne pozicije opskrbnih centara u niskotlačnoj mreži. To je, ustvari, "generalni m-median" problem.Potrošači plina su smješteni i u čvorovima mreže i duž dionice. Problem je riješen heurističkim algoritmom, koji traži optimalnu lokaciju opskrbnih čvorova u mreži koristeći Mini-sum kriterij. Kontinuirana potrošnja duž dionice je zamijenjena koncentriranom potrošnjom u sredini dionice.

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