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AIRPORT SYSTEM CONTROL IN CONDITIONS OF DISCRETE RANDOM PROCESSES OF TRAFFIC FLOW

SUMMARY

The article presents a system approach to air traffic operation and control. A mathematical model of the system has been developed, for the case when the input/output functions are discrete random processes. A solution for a special example of input functions has been calculated and analysed.

1. INTRODUCTION

Air traffic system is a very complex dynamic system. In creating a theoretical mathematical model, we would have to take into account an extremely large number of variables and their interrelationships. However, with methods of logical and methodological decomposition, a traffic system may be divided into a finite set of simpler subsystems, which are then studied and analysed separately [4].

In this article we are interested in the airport subsystem within the framework of the air traffic system. We are going to deal with it as a system of aircraft, passenger, cargo, luggage and postal operations. All the necessary activities are carried out by airports, which are organised as business companies ([3], [4], [9]). Airports have become complex technological and organisational structures, which follow the laws of dynamic systems, and for this reason we have to adopt a scientific approach to managing them.

2. A THEORETICAL MODEL OF THE AIRPORT SYSTEM CONTROL

The components of air traffic (airports, airlines and passengers) are functionally connected via airports. These connections appear in pairs: airport-airlines, airport-passengers and airlines-passengers. Technological-production processes in all traffic systems including airport systems are specific in that the production and consumption of traffic services are simultaneous. One special characteristic of traffic systems is that pairs of elements within the systems occur together: technological operations - services and traffic flow - users.

The operational technological-production level of an airport system consists of three subsystems which are typical of all production systems: the subsystem of demand, which is shown as the flow of passengers, luggage and cargo, the subsystem of production, which is shown as a technological process, and the subsystem of stock - facilities which is shown as infrastructure, terminals and technical resources.

A mathematical model of control for this system will be structured around the theoretical model of control of linear stationary systems. For this model the regulation circuit is given in Fig. 1 ([6], [7]).

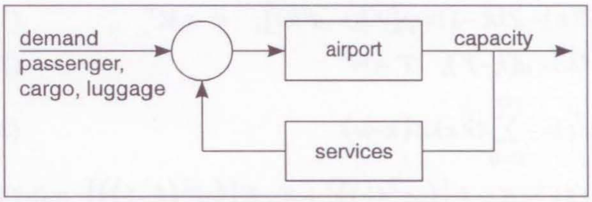

Figure 1 - Regulation circuit of airport system

In developing the model we will restrict ourselves to dynamic linear system where the input is a random process with known statistical properties. The system provides the output which is, due to the condition of linearity, also a random process. These processes could be continuous or discrete. The model and its solving for continuous processes is obtained in [4] and [7]. So we will set up the mathematical model for discrete stochastic processes.

The optimisation model of dynamic system regulation is determined by the system and by the optimality criterion. The system as regulation circuit generally consists of a regulator, the object of regulation, feedback, input and output information [5] (See Fig. 2).

We will restrict ourselves to dynamic linear discrete system where the input is a random process with known statistical properties.

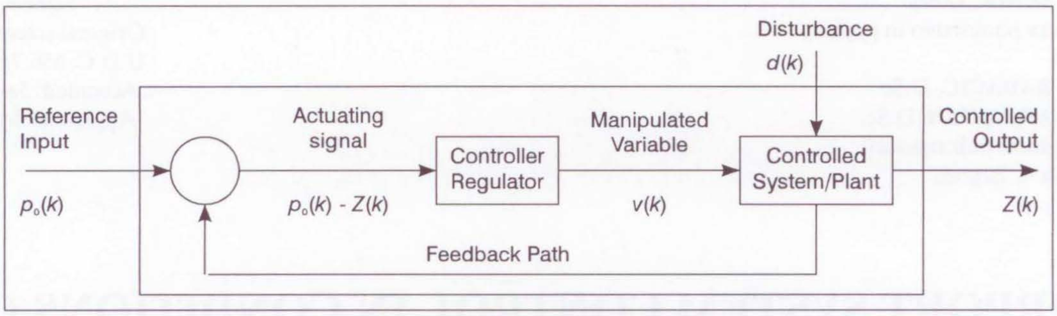

Figure 2 - A regulation circuit

Let us denote:

Z – activated facilities (resources) at given moment,

u – the amount of services performed (production) at a given moment,

d – the demand for services at a given moment,

T – time elapsed between the moment the data are received and the carrying out of a service,

Q – crierion function, complete costs,

K_Z – constant coefficient, dependent on activated resources, derived empirically,

K_u – constant coefficient, dependent on performed services and derived empirically.

In the situation of discrete functions it is: $Z=Z(k)$, $u=u(k)$, $d=d(k)$ for $k\in\{0,1,2,\ldots\}$.

The system will be modelled with equations ([4], [5]):

$$Z(k)-Z(k-1)=\psi\left[v(k)-d(k)\right],\quad \psi\in\mathbf{R}^{+} \tag{1}$$

$$v(k)=u\left(k-T\right),\quad T\in\mathrm{N} \tag{2}$$

$$u(t)=-\sum_{\kappa=0}^{\infty}G(\kappa)Z\left(k-\kappa\right) \tag{3}$$

$$Q(k)=K_Z\,E\left\{\left(Z^2(k)\right)\right\}+K_u\,E\left\{\left(u^2(k-1)\right)\right\}\ \min \tag{4}$$

$G(k)$ is the regulation function which, with optimum regulation, needs to be defined in such a way that the demand for minimum total costs will be met. We are looking for a system control with minimum operation costs $Q(k)$. The total costs (4), whose minimum we are looking for, is expressed with mathematical expectation (mean value) of the square of random variables $Z(k)$ and $u(k)$. Here K_Z and K_u are constant factors which give greater or smaller weight to individual costs. Both factors have been determined empirically and are known.

Equations (1)-(4) form a stationary stochastic linear model of control where the minimum of criteria function is calculated on the basis of Wiener's filter. This means that we will obtain the searched-for solution using Wiener-Hopf equation for these cases.

Indicating z-transforms:

$$\mathrm{Z}\{Z(k)\}=Z(z),\ \mathrm{Z}\{v(k)\}=v(z),$$

$$\mathrm{Z}\{d(k)\}=d(z),\ \mathrm{Z}\{u(k)\}=u(z)$$

and applied to equations (1)-(3):

$$Z(z)=\frac{\psi z}{z-1}\left(v(z)-d(z)\right),\quad \psi\in\mathbf{R}^{+} \tag{5}$$

$$v(z)=z^{-T}\,u(z) \tag{6}$$

$$u(z)=-G(z)Z(z) \tag{7}$$

The searched-for optimum control operator $G(z)$ is obtained from optimal cascade compensation operator $W_{\mathrm{opt}}(z)$ with the formula:

$$G_{\mathrm{opt}}(z)=\frac{\mathrm{W}_{\mathrm{opt}}(z)}{1-\mathrm{W}_{\mathrm{opt}}(z)G_f(z)} \tag{8}$$

Cascade compensation operator $W_{\mathrm{opt}}(z)$ is obtained as a solution to Wiener-Hopf equation for discrete functions.

Finally, the obtained functions are transformed into time zone with inverse z-transform [4].

3. AN EXAMPLE

Let us take an example for discrete dynamic system with situation in which the function of demand has the autocorrelation in the form:

$$R_{dd}(k)=\xi^2 a^{|k|},\quad \xi>0,\quad 0<\alpha<1 \tag{9}$$

From Wiener-Hopf equation the following is obtained ([4], [5], [6]):

$$W_{\mathrm{opt}}(z)=\frac{z-1}{z\left(z-z_1\right)}\left[C_1\left(z-a\right)-C_2\,a^{T+1}\left(z-1\right)\right] \tag{10}$$

where

$$C_1=\frac{\psi^2 K_Z z_1}{K_u\left(1-a\right)\left(1-z_1\right)} \tag{11}$$

$$C_2=\frac{\psi^2 K_Z z_1}{K_u\left(1-a\right)\left(1-a z_1\right)} \tag{12}$$

$$x=\frac{\psi^2 K_Z+2K_u}{K_u}=\frac{\psi^2 K_Z}{K_u}+2>2 \tag{13}$$

$$z_1=\frac{x-\sqrt{x^2-4}}{2}<1 \tag{14}$$

From (5), (7), (8) and (10) it is obvious that:

a) the facilities involved comply with function (facilities that were operating):

$$Z(z)=\psi\left(\frac{U_1}{z^T}+\frac{U_2}{z-z_1}-1\right)\frac{z\cdot \mathrm{d}(z)}{z-1} \quad (15)$$

where

$U_1=C_1-C_2a^{T+1}$ and,

$$U_2=C_1(z_1-a)-C_2a^{T+1}(z_1-1) \quad (16)$$

b) the services performed comply with function

$$u(z)=\left[C_1(z-a)-C_2a^{T+1}(z-1)\right]\cdot\frac{\mathrm{d}(z)}{z-z_1} \quad (17)$$

With inverse z- transform we obtain these functions in the time area:

$$Z_{\mathrm{opt}}(k)=Z^{-1}\{Z(z)\}=$$

$$=U_1D(k-T)+U_2\sum_{\kappa=1}^{\infty}z_1^{\kappa}D(k-T-\kappa)-D(k) \quad (18)$$

and

$$u_{\mathrm{opt}}(k)=Z^{-1}\{u(z)\}=U_1d(k)+U_2\sum_{\kappa=1}^{\infty}z_1^{\kappa}d(k-\kappa)= \quad (19)$$

$$=U_1d(k)+U_2\left[z_1d(k-1)+z_1^2d(k-2)+z_1^3d(k-3)+\ldots\right]$$

where $D(k)$ is the total demand in a given time interval with changeable upper boundary:

$$D(k)=Z^{-1}\{D(z)\}=Z^{-1}\left(\frac{z}{z-1}\cdot d(z)\right)=\sum_{\kappa=1}^{k}d(k-\kappa) \quad (20)$$

$$=\psi(d(k)+d(k-1)+\ldots+d(1)+d(0))$$

4. DISCUSSION

These data and results include parameters T, a and A, which have influence on values of functions and on the results of control. Those parameters are involved in the constants U_1, U_2, C_1, C_2 and z_1, which are defined by (11), (12), (13) and (14).

I. According to constants U_1 and U_2 there are three possibilities:

1. $(U_1=0)\wedge(U_2=0)$

In this case the system is degenerated completely. From (18) and (19) it is

$Z_{\mathrm{opt}}(k)=-D(k)$

$u_{\mathrm{opt}}(k)=0$

The production of services equals zero which means the system doesn't work. The needs for capacities are only registered and equal the common demand in the given time interval. The system of equations $(U_1=0)\wedge(U_2=0)$ is possible only for $a=1$, but in (9) there is condition $0<a<1$, which means this situation is not possible.

2. $(U_1=0)\wedge(U_2\neq0)$

In this case there is a very unpleasant situation because the optimal solution is

$$u_{\mathrm{opt}}(k)=U_2\sum_{\kappa=1}^{k}z_1^{\kappa}d(k-\kappa)$$

$$Z_{\mathrm{opt}}(k)=U_2\sum_{\kappa=1}^{k}z_1^{\kappa}D(k-\lambda-\kappa)-D(k)$$

The supply of airport services satisfies only the demand from the past and never that from the present. It is the same with capacities. A situation like this is not optimal because there would be permanent delays in the system operation. From the equation

$U_1=C_1-C_2a^{T+1}=0$

we get

$$T=\frac{\log\dfrac{1-az_1}{a(1-z_1)}}{\log a}$$

Because $0<a<1$ it is: $\log\dfrac{1-az_1}{a(1-z_1)}>0$ and $\log a<0$, so $T<0$. From the definition of delay (2) it is clear that $T>0$. The condition $(U_1=0)\wedge(U_2\neq0)$ is impossible, too.

3. $(U_1\neq0)\wedge(U_2=0)$

In this case the optimal solution is

$u_{\mathrm{opt}}(k)=U_1d(k)$

$Z_{\mathrm{opt}}(k)=U_1D(k-T)-D(k)$

The production of services satisfies only the present demand. It is possible to use the capacities with delay T only for the present demand. Also this control is not optimal because it doesn't satisfy all the needs of the system in whole time period of its operation.

From $U_2=C_1(z_1-a)-C_2a^{T+1}(z_1-1)=0$ it is

$$T=\frac{\log\dfrac{(a-z_1)(1-az_1)}{a(1-z_1)^2}}{\log a}$$

Because of the value of parameters a and z_1 we would get $T<0$. The condition $(U_1\neq0)\wedge(U_2=0)$ is impossible, too.

The system will be controlled optimally, when the constants U_1 and U_2 are not equal zero. This condition is true for $0<a<1$.

II. According to parameters K_u and K_Z there are two possibilities:

1. $K_u>K_Z$,
2. $K_u<K_Z$.

Because (13) and (14) expression

$$\sum_{\kappa=1}^{\infty}z_1^{\kappa}d(k-\kappa)=z_1d(k-1)+z_1^2d(k-1)+z_1^3d(k-1)+\ldots$$

is convergent faster for $K_u > K_Z$ than for $K_u < K_Z$. That means the production of services depends on the demand in the given moment more than on the past demand. This is why we will cover the demand with extra capacities.

In the second case $(K_Z < K_u)$ the storing and activating of extra capacities is very expensive and we have to cover the demand with present capacities i.e. present production of services.

5. CONCLUSION

A theoretical mathematical model of system control can be used also in a traffic system and in all their subsystems. Input-output signals are either continuous or discrete functions. For air traffic operation many conditions have to be fulfilled. During the control process a great deal of information must be processed, which can only be done if transparent and properly developed information system is available. During the operation of the airport an enormous amount of data is used. The solutions i.e. optimal control functions depend on many numerical parameters. All data and numerical analyses can only be processed into information for control if high quality and sophisticated software and powerful hardware are available.

POVZETEK

UPRAVLJANJE SISTEMA ZRAČNE LUKE PRI DISKRETNIH SLUČAJNIH PROCESIH PROMETNEGA TOKA

V članku je predstavljen sistemski pristop k upravljanju letališča. Kreiran je matematični model sistema za primer, ko so vhodno/izhodne funkcije diskretni slučajni procesi. Izračunana in analizirana je rešitev za poseben primer vhodnih funkcij.

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