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# A HEURISTIC APPROACH TO SATELLITE LINK CAPACITY PLANNING APPLIED IN MOBILE NETWORKS

*ABSTRACT*

*The efficient heuristic algorithm for sizing of N satellite capacity types (on LES side of link) in mobile networks during exploitation is being developed, minimizing the total expansion cost. Using the network flow approach and the extreme flow theory many expansion solutions (sub-problems) are eliminated from further calculation because they consist at least of one flow that cannot be a part of optimal expansion sequence.*

*At first, the heuristic algorithm is compared with algorithm based on exact approach. In all numerical test-examples the best possible result is achieved. After that we developed and tested some algorithm options, using various limitations for capacity state values. It is obvious that all heuristic options are very effective and for some cases they are capable to find the best possible result but with significant savings. It means that our heuristic approach can be successfully applied to short-term or medium-term satellite network planning with finite number of discrete time periods. Only if we apply adequate heuristic solution can we ensure both significant improvement of QoS (Quality of Service) and minimal capacity expansion cost.*

*KEY WORDS*

optimal capacity expansion, satellite link capacity planning, mobile satellite networks, QoS in mobile satellite networks

## 1. INTRODUCTION

In mobile satellite network any operator LESO (*Land Earth Station Operator*) usually has access to more than one capacity type, usually two or three satellite links, or even more. It could be done through capacities of one or more partners LESs (*Land Earth Station*). Traffic from mobiles can be routed through different standards of communication in the same network, through different satellite links (if more than one satellite is visible), or through various channel types on the same satellite link. Anyway, multiple capacity types (called facilities) are in firm correlation. Taking into account continuous increase of traffic demands periodical expansions of capacity are necessary, but new constructions could be very expensive.

The capacity expansion problem with shortages (CEPS) exists in planning of satellite link capacity over time to satisfy given demands at the lowest possible cost. The main task is how to increase satellite link capacity rationally, but at the same time to improve QoS. New construction can be a simple solution with positive influence on QoS, but the total cost rises extremely. On the other hand, traffic demand increments for any capacity type at any time period can be negative, so the capacity conversion from one capacity type to another can be a good solution, too.

The expansion problem for three and more different capacity types or multiple satellite links, with allowed shortages and possible conversions between them (in both directions) is very hard to solve. Such problem exists especially in huge and complex networks, so the intensive research is still subject of many scientific papers. Some of the most important papers with significant contribution in this field have been written by Lee & Luss [4], Luss [5] and [6], Rajagopalan [7], Suk-Gwon & Gavish [8], but many other authors are still investigating the problem. In special circumstances of satellite link expansion of mobile networks we applied this knowledge to the model for CEPS, but some improvements of the algorithm implementation have been made.

For example, the application of CEPS can be in the capacity planning of satellite network Inmarsat Ventures, initially developed for maritime communications. Today, this mobile satellite system is still the most important mobile network in the maritime world, although it ensures services to other types of mobile users as well. Communication links are made through four geo-stationary satellites, the AOR-E (*Atlantic Ocean Region - East*), AOR-W (*Atlantic Ocean Region - West*), POR (*Pacific Ocean Region*) and IOR (*Indian Ocean Region*).

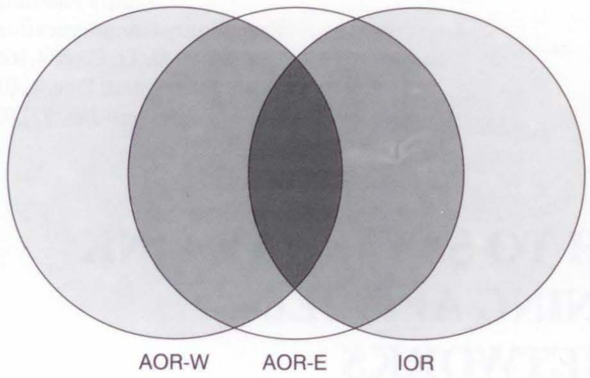


**Figure 1 - An example of various coverage regions in mobile satellite network Inmarsat where capacities of different satellite links are in correlation.**

LES *(Land Earth Station)* usually has access to more than one satellite, two or three. Each geographic area has a different number of visible satellites, which influences the complexity of the optimization problem; see Fig 1. It is necessary to solve the expansion problem for a particular geographic area, dependent on satellite coverage. The traffic demands can be routed over different satellites but each satellite link has specific traffic characteristics, channel availability and QoS that influences LESO position in the world market. LESO has to decide to what amount and at which moment the capacity expansion has to be done for the appropriate satellite link, to ensure minimal expansion cost and fulfillment of traffic demands.

The sizing of satellite link capacity can be done through expansion or through reduction. The expansions have to be limited only for positive values, because in telecommunications capacity disposals (reductions) of transmission equipment are not acceptable. Reduction could be appropriate only for long-term planning where new technology replaces the old. They are often initiated due to high holding cost of idle capacity when traffic demands decrease over successive periods. Instead of reduction, conversion of channel capacities from one satellite link to another or from one channel type to another could be a much more appropriate option.

The expansion model is formulated in section 2. Possible strategies of single period expansion problem (SPEP) are explained in section 3. Further, certain properties of optimality are derived and basic heuristic algorithm solution is developed. In section 4 heuristic algorithm is tested through many numerical examples and compared with results of algorithm based on the exact approach. In section 5 different algorithm options are developed and compared with the base heuristic approach.

## 2. MODEL FOR CEPS

The satellite link capacities (called *facilities*) are expanded over time to serve $N$ demand types. Facility $i$ for $i = 1, 2, \dots, N$ is designed primarily to serve demand of type $i$, but it can be converted to satisfy demand $j$ $(j \neq i)$. Conversions normally require certain physical modifications with some cost. Once converted from $j$ to $i$, the capacity becomes an integral part of facility $j$, but it can be rearranged to its original type at any time with some cost. In this capacity expansion problem conversions are permitted in both directions. This fact drastically increases the complexity of the problem.

During exploitation we can change the transmission capacity through expansion and conversion on one hand, but we can permit the idle capacities or shortages (from one time period to another) on the other. They can be used as stand-alone strategies or combined together. If both strategies are necessary they are not substitutes but complements. The objective is to find an expansion (or/and conversion) policy that minimizes the total cost incurred over a finite planning horizon of $T$ periods. The capacity-sizing policy consists of timing and sizing decisions for new construction and conversion.

The expansion problem can be represented by a flow diagram. Figures 2 and 4 show examples of the *Multi-Commodity Single Source Multiple Destination Network*, and the problem can be treated as the *Minimum Cost Network Flow Problem*. The extreme flow theory enables separation of these extreme flows which can be part of an optimal expansion solution from those which cannot be; see Luss [5] and [6]. With such heuristic approach we enable optimal result, using significantly less effort in programming and algorithm development. Figure 2 gives a network flow representation of CEPS for $N$=3 and $T$=4. Common node "O" is the source for capacity expansions, and the $i$-th row of nodes represents the capacity state in $T$ periods for capacity type $i$. The links represent the expansions, conversions, idle capacities and capacity shortages.

In the model the following notation is used:

- $i, j$ and $k$ – indices for capacity type. The $N$ facilities are not ranked, just present different capacity types from $1, 2, \dots, N$.
- $t$ – indices for time period. The planning horizon consists of $T$ periods $(t = 1, \dots, T)$.
- $u, v$ – indices for time period in sub-problem, $0 \leq u, \dots, v \leq T$.
- $r_{it}$ – increment of demands of capacity type $i$ for additional capacity in period $t$. For convenience, the $r_{it}$ are assumed to be integer.

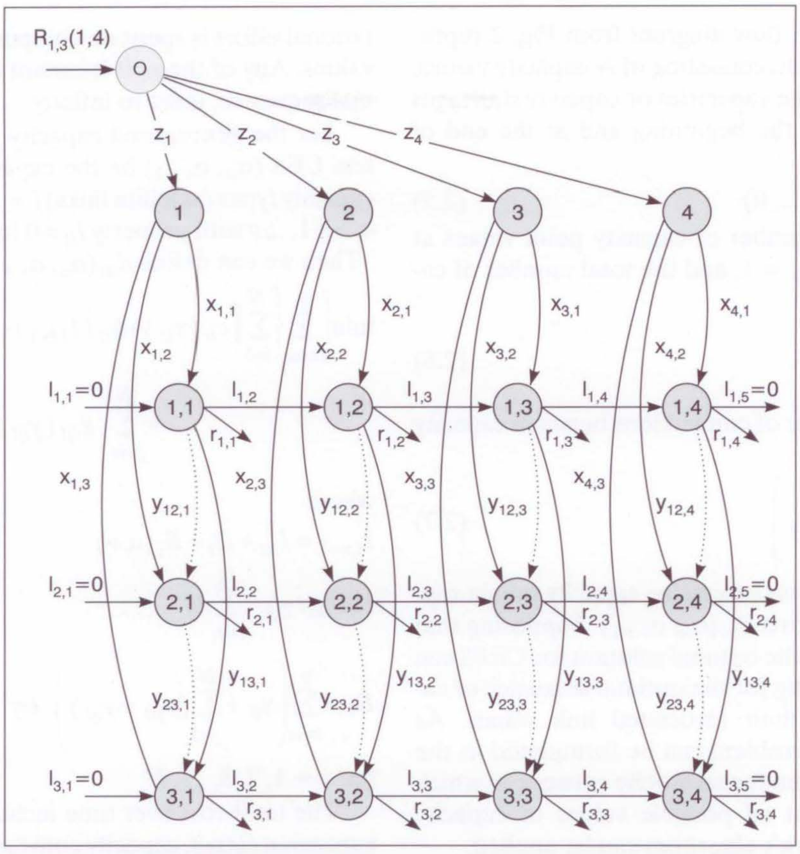


**Figure 2 - A network flow presentation of capacity expansion problem with shortages (CEPS)**

The demand can also be satisfied by converted capacity from any capacity type $k$, with $k \neq 0$.

- $I_{it}$ – the amount of excess or lack capacity of capacity type $i$ at the beginning of period $t$ (or, equivalently, at the end of period $t$-1). We assumed that initially there is no idle capacity or capacity shortages, $I_{i1} = 0$ .
- $kI_i$ – the lowest step of possible capacity change for capacity type $i$. In numerical examples given in Chapter 4 we set $kI_i = 10$.
- $x_{it}$ – the expansion ($x_{it} \geq 0$) of capacity type $i$, allowed at the beginning of period $t$.
- $y_{ijt}$ – the amount of capacity converted immediately after the beginning of period $t$. The positive value implies that conversion is in referent direction if it satisfies $i < j$; $y_{ijt} = - y_{jit}$. Negative value means that conversion is in opposite direction $i < j$. Converted capacity becomes an integral part of new capacity type and can be rearranged to its original type at any point in time. For any combination between capacity types conversion is possible. Any restriction can be imposed by upper bound set to 0.
- $z_t$ = the total expansion for all the capacity types in period $t$ , i.e., $z_t = \sum_{i=1}^{N} x_{it}$

The traffic demand increments, capacity expansions, and capacity conversions occur instantaneously, immediately after beginning of each period.

The CEPS problem can be formulated as follows:

$$\min\left(\sum_{t=1}^{T}\left\{\sum_{i=1}^{N}\left[c_{it}(x_{it})+h_{it}(I_{i,t+1})+ + \sum_{j=i+1}^{N} g_{ijt}(y_{ijt})\right]+p_t(z_t)\right\}\right) \tag{2.1}$$

so that we have:

$$I_{t+1} = I_{it} + x_{it} + \sum_{j=1}^{i-1} y_{jit} - \sum_{j=i+1}^{N} y_{ijt} - r_{it} \tag{2.2}$$

$$I_{it} = I_{i,T+1} = 0 \tag{2.3}$$

for $t = 1, 2, .., T; i = 1, 2, .., N; j = i + 1, .., N$ $(i<j)$

## 2.1. Definition of capacity point

Generalizing the concept of capacity state in period $t$, in which capacity of each satellite link is known in defined bounds and which at least one capacity $I_{it} \leq 0$, we define as *a capacity point*. In (2.4) $\alpha_t$ denotes the vector of capacities $I_{it}$ for all satellite capacity types (satellite links) in period $t$, and we call it capacity point.

$$\alpha_t = (I_{1t}, I_{2t}, \ldots , I_{Nt}) \tag{2.4}$$

Each column on flow diagram from Fig. 2 represents a capacity point, consisting of $N$ capacity values. (2.5) implies that idle capacities or capacity shortages are not allowed at the beginning and at the end of planning horizon.

$$\alpha_1 = \alpha_{T+1} = (0, 0, ..., 0) \tag{2.5}$$

Let $C_t$ be the number of capacity point values at period $t$, $C_1 = C_{T+1} = 1$, and the total number of capacity points is:

$$C_p = \sum_{i=1}^{T+1} C_t \tag{2.6}$$

The total number of connections between capacity points is:

$$N_d = \sum_{i=1}^{T} C_i \cdot \left[ \sum_{j=i+1}^{T+1} C_j \right] \tag{2.7}$$

Links between two successive capacity points represents minimum costs $d_{uv}(\alpha_u, \alpha_{v+1})$. Supposing that all links are known, the optimal solution for CEPS can be found by searching for the optimal sequence of capacity points and their associated link values. As shown in Figure 3 problem can be formulated as the shortest path problem for an acyclic network in which the nodes represent all possible values of capacity points. Then Dijkstra's algorithm can be applied.

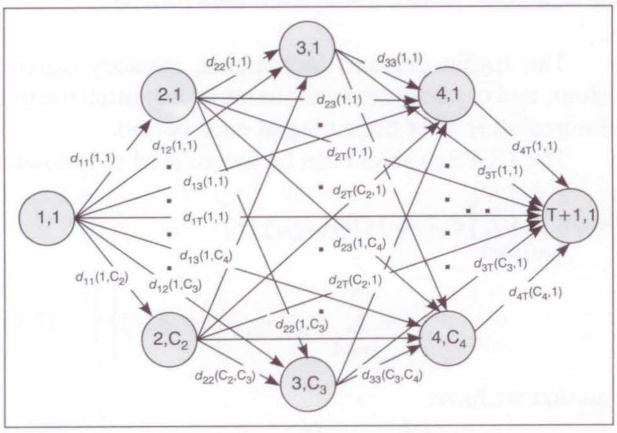


**Figure 3 - The expansion problem can be solved as the shortest path formulation**

It is very important to reduce that number of capacity points and that can be done through imposing of appropriate capacity bounds or by introduction of adding constraints.

## 2.2. Solving of sub-problem (CES)

We denoted the associated value between two capacity points, that represents minimum cost $d_{uv}(\alpha_u, \alpha_{v+1})$ as CES (*Capacity Expansion Sub-problem*). In CEPS we have to find many cost values $d_{uv}(\alpha_u, \alpha_{v+1})$ that emanate two capacity points, from each node ($u$, $\alpha_u$) to node ($v+1$, $\alpha_{v+1}$) for $v > u$. Most of the computational effort is spent on computing the sub-problem values. Any of them, if it cannot be a part of the optimal sequence, is set to infinity.

Let the generalized capacity expansion sub-problem CES ($\alpha_u$, $\alpha_{v+1}$) be the expansion problem for $N$ capacity types (satellite links) $i = 1, 2, .., N$ over period $u, u+1, .., v$ with property $I_{it} \neq 0$ for $t = u+1, u+2, ..., v$. Then we can define $d_{uv}(\alpha_u, \alpha_{v+1})$ as follows:

$$\min \left( \sum_{t=u}^{v} \left\{ \sum_{i=1}^{N} \left[ c_{it}(x_{it}) + h_{it}(I_{i,t+1}) + \sum_{j=i+1}^{N} g_{ijt}(y_{ijt}) \right] + p_t(z_t) \right\} \right) \tag{2.8}$$

where

$$I_{i,v+1} = I_{iu} + D_i - R_i(u,v) \tag{2.9}$$

$$R_i(t_1, t_2) = \sum_{t=t_1}^{t_2} r_{it} \tag{2.10}$$

$$D_i = \sum_{t=u}^{v} \left( x_{it} + \sum_{j=1}^{N} (y_{jit} - y_{ijt}) \right), \; i \neq j \tag{2.11}$$

for $i = 1, 2, 3, ..., N$

The total cost over time includes cost for capacity expansion $c_{it}(x_{it})$, capacity conversion cost $g_{ijt}(y_{ijt})$, idle capacities and capacity shortages cost $h_{it}(I_{i,t+1})$, and strategic penalty costs in form of joint set-up cost $p_t(z_t)$. Capacity expansion cost is often represented by fix-charge cost functions, and joint set-up costs are often given as a constant.

We assume that all costs functions are concave and non-decreasing, reflecting economies of scale, and they can be changed in time.

To compute the sub-problem value $d_{uv}$ it is convenient to describe the problem as a single source multi-facility and multi-destination network, represented by a flow diagram as we can see in Fig. 4. At each node $(i,t)$ there is an external demand increment $r_{it}$, possibly negative. The nodes are connected by links, where each flow represents the construction (expansion), or conversion, or idle capacity and capacity shortage. The flow on each link can be in any direction, and the link direction in Figure 4 indicates referent positive flow.

In contrast to the case with $r_{it} \geq 0$, optimal construction and conversion may take place in any period $t$, so the algorithm for optimal CES has to find three periods for changes of three capacity types over whole interval $u \leq (t_1, t_2, t_3) \leq v$. But we assume that all changes are made in the same period, that is very appropriate for real application. So, the algorithm is searching for unique period $t$ between two periods $u \leq t \leq v$ to minimize the equation (2.8). Such limitation reduces the computation complexity significantly without influence on optimal expansion sequence.

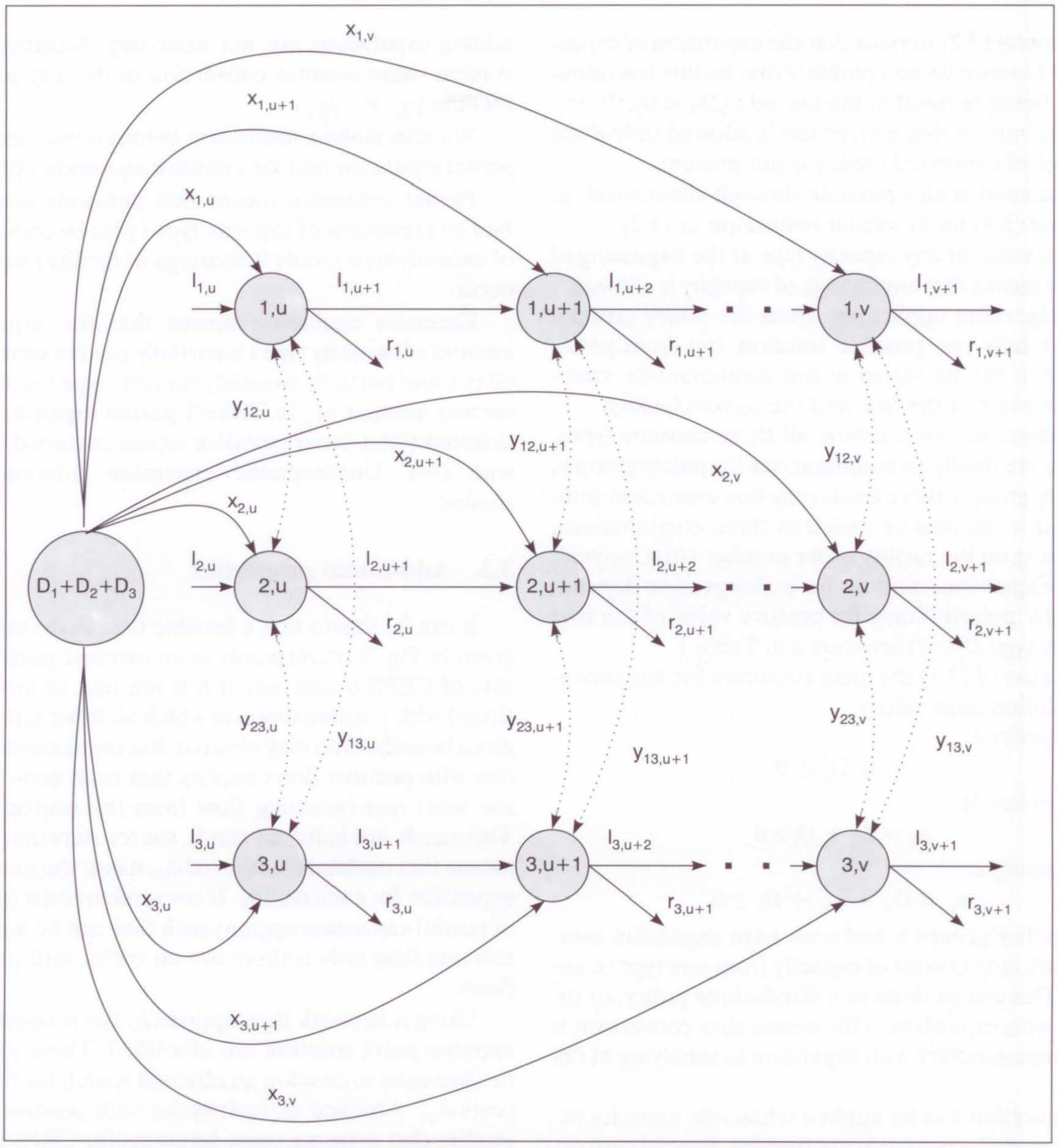


**Figure 4 - A network flow representation of a sub-problem for N=3**

## 3. POSSIBLE STRATEGIES OF SPEP

The approach described in chapter above requires solving repeatedly a certain single period problem. Let $SPEP_{ij}$ $(t, D_i, \ldots D_j)$ be a *Single Period Expansion Problem* associated with period $t$ for capacity type $i$, $i+1$, ..., $j$ and the corresponding values of *capacity change intention* $D_i, D_{i+1}, \ldots, D_j$. For detailed explanation of various SPEP type-solutions depending on conversion strategy, see [2].

Solving $SPEP_{13}$ for three different capacity types we have many expansion solutions divided into three different strategies (policy groups):

a) capacity changes of one capacity type are not connected with changes of others;

b) capacity changes of two capacity types depend on each other, but change of the third is independent;

c) capacity changes of all three capacity types depend on each other (see Table 1).

### 3.1. Basic properties

From three policy groups many different expansion solutions can be derived, depending on $D_i$ polarity. Lots of them are not acceptable and are not part of optimal sequence. For this problem the acceptable expansion solution has to satisfy some basic properties:

$$x_{it} \geq 0 \tag{3.1}$$

$$x_{it} \cdot D_{it} \geq 0 \tag{3.2}$$

$$y_{ij,t} \cdot D_{it} \leq 0 \tag{3.3}$$

$$y_{ij,t} \cdot D_{jt} \geq 0 \tag{3.4}$$

Constraint (3.1) implies that only positive expansions are allowed.

Property (3.2) implies that the expansion of capacity type $i$ cannot be acceptable if that facility has intention of being reduced in the period $t$ ($D_{it} < 0$).

(3.3) implies that conversion is allowed only if the shortage of converted facility is not present.

Expansion is also possible through conversion, so (3.3) and (3.4) imply similar restriction as (3.2).

Zero value of any capacity type at the beginning of period $t$ means that any change of capacity is allowed.

In algorithm application from the policy group a we have only one possible solution, but from policy group b, it can be varied in two combinations, swapping the place of the first and the second facility.

Further, we can combine all three capacity types, so there are finally six combinations for policy group b. In policy group c there exists only one expansion solution, but it can also be varied in three combinations, depending on the facility order number (first, second, third). Expansion solutions for policy group c depending on $D_i$ polarity (only for positive value of the first capacity type $D_f \geq 0$) are shown in Table 1.

Because of (3.1) the main condition for any expansion solution must satisfy:

for policy a:

$$x_{it} = D_f \geq 0$$

for policy b:

$$x_{it} = D_f + D_s \geq 0$$

for policy c:

$$x_{it} = D_f + D_s + D_t \geq 0$$

In policy groups b and c we have expansion solutions with conversions of capacity from one type to another. This can be done as a stand-alone policy, or together with expansion. This means that conversion is just complementary with expansion in satisfying of demands.

Conversion can be applied when idle capacity occurs or negative demands are present. Special case occurs when conversion $y_{ij}$ eliminates both: demands for capacity type i, and idle capacity for capacity type j, so adding expansions are not necessary. Negative conversion value denotes conversion in the opposite direction: $y_{ij,t} = - y_{ji,t}$.

We also make a distinction between two cases: *the partial expansion* and *the excessive expansion* options.

Partial expansion means that demands are satisfied by expansion of capacity type $i$ plus by conversion of capacity type $j$, only if shortage of facility $j$ does not occur.

Excessive expansion means that the expansion amount of capacity type $i$ is partially used to expand facility $i$, and partially to satisfy capacity type $j$, with conversion amount $y_{ij}$. In Table 1 partial expansions are denoted with (-) and excessive expansions are denoted with (+). Unacceptable expansion solutions are shaded.

## 3.2. Additional properties

It can be shown that a feasible flow in the network given in Fig. 2 corresponds to an extreme point solution of CEPS if and only if it is not part of any cycle (loop) with positive flows, in which all flows satisfy the given bounds. One may observe that the absence of cycles with positive flows implies that each node has at the most one incoming flow from the source node. This result holds for all single source networks. This means that optimal solution of $d_{uv}$ has at the most one expansion for each facility. If conversions exist (as part of partial expansion option) such flow can be a part of extreme flow only if there are no cycles with positive flows.

Using a network flow approach, the properties of extreme point solution are identified. These properties are used to develop an efficient search for the link costs $d_{uv}$. Absence of such cycles with positive flows implies that extreme point solutions for CEPS satisfy the following properties:

$$I_{it} \cdot x_{it} \leq 0, \qquad (3.5)$$

**Table 1 - Possible expansion solutions from strategy c.**

| Expansion solutions from strategy c. | $D_f \geq 0$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_s > 0$ $D_f > 0$ | $D_s < 0$ $D_f > 0$ | | | | $D_s < 0$ $D_f < 0$ | | $D_s > 0$ $D_f < 0$ | | | |
| $D_s + D_t$ | $> 0$ | $> 0$ | $< 0$ | | $= 0$ | $< 0$ | | $> 0$ | $< 0$ | | $= 0$ |
| $\frac{\lvert D_f \rvert}{\lvert D_s \quad D_f \rvert}$ | - | - | $\geq 1$ | $< 1$ | - | $\geq 1$ | $< 1$ | - | $\geq 1$ | $< 1$ | - |
| $X_f = D_f + D_s + D_t$ | Exp(-) | Exp(+) | Exp(-) | Red | Exp ($X_f = D_f$) | Exp(-) | Red | Exp(+) | Exp(-) | Red | Exp ($X_f = D_f$) |
| $Y_{fs}$ | $D_s$ | _ | $D_s + D_t$ | $D_s + D_t$ | _ | $D_s$ | $D_s$ | $D_s + D_t$ | _ | _ | _ |
| $Y_{st}$ | _ | $-D_s$ | $D_t$ | $D_t$ | $-D_s$ | _ | _ | $D_t$ | $-D_s$ | $-D_s$ | $-D_s$ |
| $Y_{ft}$ | $D_t$ | $D_s + D_t$ | _ | _ | _ | $D_t$ | $D_t$ | _ | $D_s + D_t$ | $D_s + D_t$ | _ |

$I_{it} \cdot y_{ij,t} \geq 0,$ (3.6)

$I_{jt} \cdot y_{ij,t} \leq 0,$ (3.7)

$I_{jt} \cdot x_{it} \cdot y_{ij,t} = 0, \text{ if } x_{it} \cdot y_{ij,t} \neq 0$ (3.8)

$I_{it} \cdot I_{jt} \cdot y_{ki,t} \cdot y_{kj,t} = 0, \text{ if } y_{ki,t} \cdot y_{kj,t} \neq 0$ (3.9)

for: $i,k,j = 1, 2, 3 \; ; i \neq k \neq j \; ; \; t = 1, \ldots, T$

Properties (3.5) to (3.9) imply that the capacity of any capacity type is increased at a given period through an expansion or by conversions only if it does not make cycles with positive flows.

(3.5) and (3.6) imply that the capacity of any capacity type can be increased by an expansion or by a conversion only if there is no idle capacity at the beginning of that period.

(3.7) implies that capacity can be reduced by conversion only if there is no capacity shortage at the beginning of that period.

(3.8) implies that incoming flow of facility, going to be converted (reduced) in partially or excessive expansion solution has to be zero. If not, cycles with positive flows can occur; which is shown by a diagram; see Fig. 5.

Property (3.9) is used for simultaneous multi-conversion solution from policy group c. Only one incoming flow of converted (reduced) facility can exist, but expansion is not allowed. In the case we have expansion with simultaneous conversions incoming flows of both facilities have to be zero, which is proposed in adding condition (3.8).

Any acceptable $SPEP_{13}$ expansion solution for any CES has to satisfy properties (3.5) - (3.9), so many expansion solutions that are not part of optimal sequence could be eliminated from further computing.

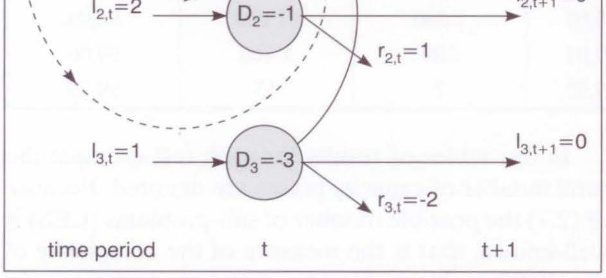


**Figure 5 - An example of single period expansion solution $SPEP_{13}$ that is not a part of extreme solution. Dotted lines mark a cycle with positive flows from the common source. Both capacity values at the beginning of the period t must be zero**

The optimal solution of any sub-problem (CES) is obtained by the following:

1) For each expansion solution from the policy group: (a), (b) and (c) find the optimal values of $t$, which minimize $d_{uv}$ as given by equation (2.8), while satisfying properties (3.1) - (3.4) and (3.5) - (3.9);
2) Choose as optimal expansion solution the best of those found in step 1.

If none of the optimal solutions satisfies all properties we set $d_{uv} = \infty$.

## 4. NUMERICAL EXAMPLES AND THE EFFICIENCY OF HEURISTIC ALGORITHM

Many numerical examples were solved for $N = 3$, $T = 6$ and results obtained by heuristics are compared with results obtained by algorithm based on exact approach, that is, calculating all possible expansion solutions for each sub-problem.

Traffic demands are defined for each time period with the number of channel capacity units for each capacity type (satellite link). Values are given in relative amount, increasing or decreasing over the planning horizon. Also, they are non-linear which is more appropriate for real application.

a ($r_1 = -10$, $r_2 = 10$, $r_3 = 0$, $r_4 = 10$, $r_5 = -10$, $r_6 = 10$)
b ($r_1 = 0$, $r_2 = 10$, $r_3 = 0$, $r_4 = -10$, $r_5 = 0$, $r_6 = 10$)
c ($r_1 = 10$, $r_2 = -10$, $r_3 = 0$, $r_4 = 20$, $r_5 = 10$, $r_6 = 10$)
d ($r_1 = 10$, $r_2 = 10$, $r_3 = 0$, $r_4 = 0$, $r_5 = 10$, $r_6 = 0$)

Tables 2 and 3 exhibit results for some numerical test-examples with combinations of traffic demands given above. Bounds for idle capacities and capacity shortages are not defined, the same as for expansion and conversion values.

In testing we used the capacity expansion cost function as follows:

$$c_{it}(x_{it}) = f_i^{\,t-1}(A_i + B_i x_{it}^{a_i}) \tag{4.1}$$

where $A_i$ is a fixed charge cost per expansion of capacity type $i$, $B_i$ is a variable cost per expansion unit, $a_i$ represents the factor of concavity, and $f_i$ is the discount factor. For our test-examples we set: $A_1$=3000, $B_1$=25, $a_1$ = 0,9, $A_2$=1000, $B_2$=20, $a_2$=0,85, $A_3$=2000, $B_3$=30, $a_3$=0,95. For all periods and for all facilities we use the same value $f$ = 0,9.

All holding costs of excess capacities and shortage penalty costs show linear growth:

$$h_{it}(I_{i,t+1}) = f_i^{\,t-1} H_i \, I_{i,t+1} \tag{4.2}$$

For examples shown in Table 2. :

$H_i = 400$ ; for all $I_{it} > 0$

$H_i = 300$ ; for all $I_{it} < 0$ ; $i = 1, 2, 3$

For examples shown in Table 3. :

$H_i = 500$ ; for all $I_{it} > 0$

$H_i = 10$ ; for all $I_{it} < 0$ ; $i = 1, 2, 3$

**Table 2 - Results of heuristics tested and compared on numerical test-examples**

| Demand | | | Excessive expansions allowed | Heuristic algorithm approach | The best solution found | Number of capacity points | Number of acceptable sub-problems | Savings in perc. (%) |
|---|---|---|---|---|---|---|---|---|
| $r_{1t}$ | $r_{2t}$ | $r_{3t}$ | | | | | | |
| a | b | b | Yes | Exact | 12 479,53 | 699 | 87 557 | – |
| | | | | BasicH | 12 479,53 | 699 | 47 432 | 45,83 |
| | | | | AH | 12 915,01 | 463 | 14 134 | 83,86 |
| | | | | RH | 12 915,01 | 421 | 12 471 | 85,76 |
| | | | | PH | 12 915,01 | 109 | 1 005 | 98,85 |
| | | | | TH | 18 059,49 | 7 | 11 | 99,99 |
| | | | No | Exact | 13 692,25 | 699 | 83 725 | – |
| | | | | BasicH | 13 692,25 | 699 | 44 447 | 46,91 |
| | | | | AH | 14 763,38 | 463 | 14 742 | 82,39 |
| | | | | RH | 14 763,38 | 421 | 7 522 | 91,00 |
| | | | | PH | 14 763,38 | 109 | 1 544 | 98,16 |
| | | | | TH | 20 497,00 | 7 | 11 | 99,99 |
| c | b | d | Yes | Exact | 5 232,77 | 2 451 | 890 543 | – |
| | | | | BasicH | 5 232,77 | 2 451 | 332 560 | 62,66 |
| | | | | AH | 5 232,77 | 1 673 | 145 447 | 83,67 |
| | | | | RH | 5 232,77 | 1 417 | 91 637 | 89,71 |
| | | | | PH | 5 232,77 | 256 | 3 855 | 99,57 |
| | | | | TH | 5 232,77 | 7 | 21 | 99,99 |
| | | | No | Exact | 14 488,12 | 2 451 | 819 038 | – |
| | | | | BasicH | 14 488,12 | 2 451 | 305 168 | 62,74 |
| | | | | AH | 15 137,09 | 1 673 | 138 964 | 83,,3 |
| | | | | RH | 15 137,09 | 1 417 | 89 257 | 89,1 |
| | | | | PH | 15 137,09 | 256 | 3 855 | 99,53 |
| | | | | TH | 15 137,09 | 7 | 21 | 99,99 |
| a | a | c | Yes | Exact | 7 809,72 | 2 122 | 478 772 | – |
| | | | | BasicH | 7 809,72 | 2 122 | 250 390 | 47,70 |
| | | | | AH | 8 227,97 | 1 228 | 94 751 | 80,21 |
| | | | | RH | 8 227,97 | 1 090 | 66 453 | 86,12 |
| | | | | PH | 8 227,97 | 289 | 4 160 | 99,13 |
| | | | | TH | 19 085,19 | 7 | 15 | 99,99 |
| | | | No | Exact | 12 392,37 | 2 122 | 442 030 | – |
| | | | | BasicH | 12 392,37 | 2 122 | 211 491 | 58,96 |
| | | | | AH | 13 229,07 | 1 228 | 86 788 | 80,37 |
| | | | | RH | 13 229,07 | 1 090 | 61 600 | 86,06 |
| | | | | PH | 13 229,07 | 289 | 4 160 | 99,06 |
| | | | | TH | 22 879,92 | 7 | 15 | 99,99 |

Conversion cost does not depend on $y_{it}$. It is the constant value:

$$g_{it}(y_{ij,t}) = f_i^{\,t-1} G_i \qquad (4.3)$$

For all examples $G = 100$; for $y_{ij,t} \neq 0$ and $i = 1,2,3$. In numerical examples from Table 2 the joint set-up cost: $p_t(z_{,t}) = 0$.

For all these cost functions:

$$c_{it}(0) = h_{it}(0) = g_{it}(0) = p_t(0) = 0 \qquad (4.4)$$

In our tables of results for each test-example the total number of capacity points are denoted. Because of (2.7) the possible number of sub-problems (CES) is well-known, that is the measure of the complexity of the CEPS-problem. Also, for each test-example in the tables we gave the number of acceptable sub-problems, satisfying basic and additional properties of optimal flow. Savings in percents, shown in the last column, are the value of algorithm efficiency in compari-

**Table 3 - results of heuristics tested and compared on numerical test-examples**

| Demand $r_{1t}$ | Demand $r_{2t}$ | Demand $r_{3t}$ | Excessive expansions allowed | Heuristic algorithm approach | The best solution found | Number of capacity points | Number of acceptable sub-problems | Savings in perc. (%) |
|---|---|---|---|---|---|---|---|---|
| a | b | c | Yes | OSH | 1 809,88 | 1617 | 276 630 | - |
| | | | | BasicH | 1 809,88 | 1617 | 148 986 | 46,14 |
| | | | | AH | 3 265,19 | 1147 | 73 207 | 73,54 |
| | | | | RH | 3 265,19 | 1009 | 53 144 | 80,79 |
| | | | | PH | 3 265,19 | 265 | 3 943 | 98,57 |
| | | | | TH | 3 265,19 | 7 | 21 | 99,99 |
| | | | No | OSH | 4 873,21 | 1617 | 254 700 | - |
| | | | | BasicH | 4 873,21 | 1617 | 135 437 | 46,82 |
| | | | | AH | 7 092,18 | 1147 | 70 429 | 72,35 |
| | | | | RH | 7 092,18 | 1009 | 51 464 | 79,79 |
| | | | | PH | 7 092,18 | 265 | 3 943 | 98,45 |
| | | | | TH | 7 092,18 | 7 | 21 | 99,99 |
| b | a | d | Yes | OSH | 1 972,64 | 1259 | 267 100 | - |
| | | | | BasicH | 1 972,64 | 1259 | 142 670 | 46,59 |
| | | | | AH | 2 442,96 | 565 | 25 266 | 90,54 |
| | | | | RH | 2 442,96 | 526 | 21 109 | 92,10 |
| | | | | PH | 2 442,96 | 115 | 1 445 | 99,46 |
| | | | | TH | 2 442,96 | 7 | 21 | 99,99 |
| | | | No | OSH | 3 829,13 | 1259 | 247,156 | - |
| | | | | BasicH | 3 829,13 | 1259 | 123 390 | 50,07 |
| | | | | AH | 8748,13 | 565 | 24 529 | 90,08 |
| | | | | RH | 8748,13 | 526 | 20 716 | 91,62 |
| | | | | PH | 8748,13 | 115 | 1 445 | 99,42 |
| | | | | TH | 8748,13 | 7 | 21 | 99,99 |
| a | a | c | Yes | OSH | 6 221,59 | 2122 | 621 668 | - |
| | | | | BasicH | 6 221,59 | 2122 | 247 968 | 60,11 |
| | | | | AH | 9 718,46 | 1228 | 74 965 | 87,94 |
| | | | | RH | 9 718,46 | 1090 | 52 367 | 91,58 |
| | | | | PH | 9 718,46 | 289 | 4 160 | 99,33 |
| | | | | TH | 17 750,09 | 7 | 15 | 99,99 |
| | | | No | OSH | 9 072,39 | 2122 | 579 960 | - |
| | | | | BasicH | 9 072,39 | 2122 | 211 369 | 63,50 |
| | | | | AH | 14 719,56 | 1228 | 71 868 | 87,61 |
| | | | | RH | 14 719,56 | 1090 | 51 138 | 91,18 |
| | | | | PH | 14 719,56 | 289 | 4 160 | 99,28 |
| | | | | TH | 21 538,47 | 7 | 15 | 99,99 |

son with previous algorithm option, proportionally reflected on computation time.

The heuristic solution, that in all test-examples can obtain the best possible result (near-optimal expansion sequence), is denoted with BasicH (*Basic Heuristic option*); see Tables 2 and 3. The computation effort required to solve one sub-problem is $O(N^2T)$. The number of all possible $d_{uv}$ values depends on the total number of capacity points and requires the effort of $O(T^3N^4R_i^{2(N-1)})$. If there are no limitations on capacity state values the complexity of such heuristic approach is pretty large and increases exponentially with $N$.

## 5. DIFFERENT HEURISTIC OPTIONS

In real application it is very important to ensure good QoS for capacity availability, so we have to introduce some limitations on capacity state value. With different limitations we tested some options of our heuristic algorithm:

a) **Sum of capacity state values inside of each capacity point is positive**. Such heuristic approach is denoted with AH *(Acceptable Heuristic option)*. For example: in capacity planning of multiple transmission links it is unacceptable to allow total shortage on all links at the same time. This means that traffic demands must be satisfied anyway; in the worst case through migration of traffic from one capacity type to another, where idle capacity exists;

b) **Only one capacity state value inside of each capacity point can be negative, but the total sum of capacity on three capacity types (satellite links) is positive**. This heuristic option is denoted with RH (*Real Heuristic option*). In real application the capacity shortage for only one capacity type (facility) can be easily satisfied by sufficient (idle) capacities on others;

c) **Only non-negative capacity values are allowed.** Heuristics that allows only non-negative capacity state values is denoted with PH (*Positive Heuristic option*). It means that capacity shortages are not possible;

d) **Only null capacity values are allowed.** A trivial heuristic solution that allows only zero values in capacity point is denoted with TH *(Trivial Heuristic option)*.

Introducing appropriate limitations can ensure good QoS for capacity availability and reduction of algorithm complexity, but anyway it is much over $O(N^2T^3)$, that is for a trivial solution (TH)

## 6. CONCLUSION

Through many numerical examples we can see the efficiency of our algorithms shown in Tables 2 and 3. The basic heuristic option (BasicH) is able to find the best result (expansion sequence) and provide the minimal cost for all numerical test-examples, same as the algorithm based on exact approach.

For each test-example we gave the total number of capacity points (the number of possible sub-problems), that shows the complexity of the CEPS problem. In the next column of our tables we have the number of acceptable sub-problems, satisfying basic and additional conditions. We can see that the basic heuristic algorithm for all solved examples has significant complexity savings, on average over 50%, which is proportionally reflected on computation time savings.

Further, we compared the efficiency of multi-facility capacity expansion algorithm in four different options, shown in Tables 2 and 3. The comparison of their efficiency given in saving percents can be seen through numerical examples. For few examples all algorithm options found the best expansion sequence, providing the minimal cost, regardless of the heuristic

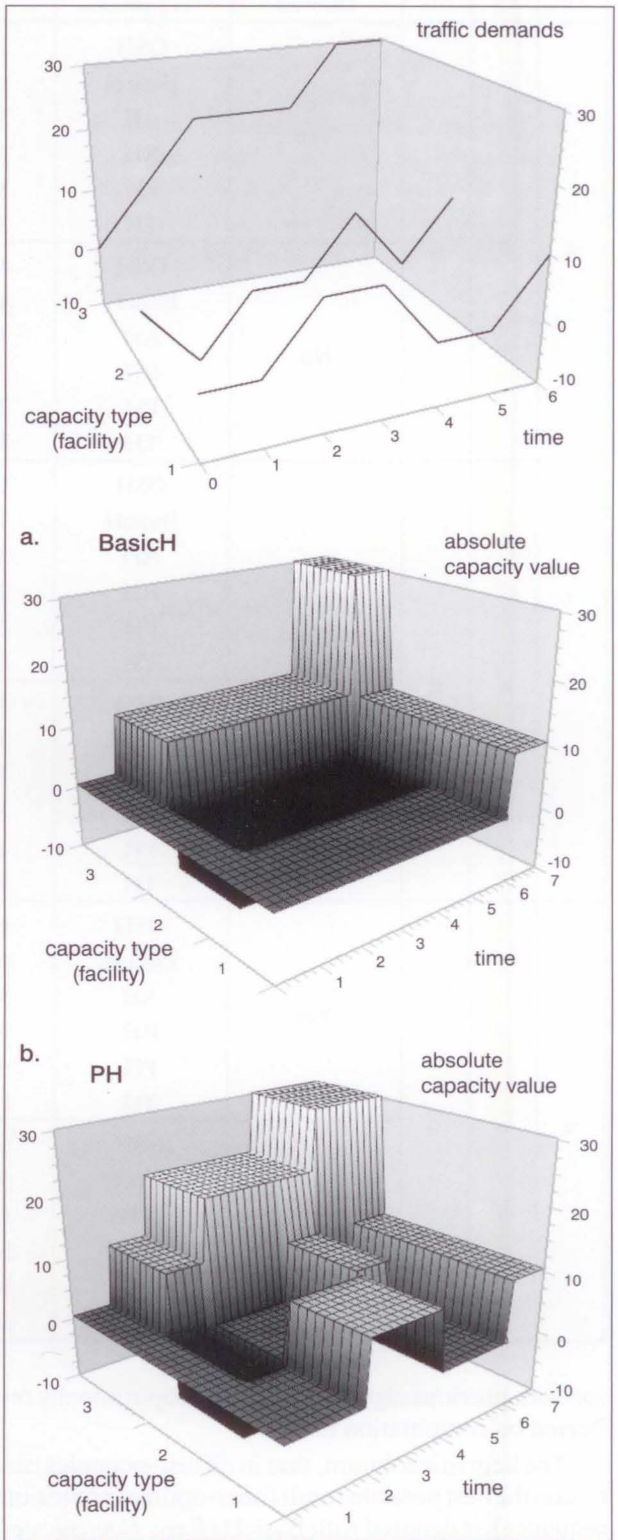


**Figure 6 - The optimal solution b (with PH), with limited capacity point values, has better influence on QoS, but minimal expansion cost rises. Test-example is for traffic demands combination b a d from Table 3.**

approach option we use. Significant deterioration of result is present only in trivial heuristic option (TH).

It is obvious that all heuristic options are very effective, capable of finding the best possible result. It means that this heuristic approach can be successfully applied to short-term or medium-term satellite network planning with finite number of discrete time periods, taking care of the allowed capacity values for adequate problem. Only in that case can we ensure significant improvement of QoS for capacity availability, together with minimal expansion costs.

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## *SAŽETAK*

### *HEURISTIČKI PRISTUP U PLANIRANJU KAPACITETA NA SATELITSKOM LINKU S PRIMJENOM U MOBILNIM MREŽAMA*

*U radu je razvijen efikasan heuristički algoritam za optimalno dimenzioniranje više vrsta kapaciteta na satelitskom linku prema određenoj obalnoj postaji, minimizirajući troškove. Razvijajući optimizacijski model preko mreže tokova, a koristeći postupak mrežnog planiranja, precizno su određene karakteristike ekstremnih tokova i ispitane sve mogućnosti proširenja za jednovremenski problem. Teorija ekstremnih tokova omogućuje eliminiranje onih tokova koji nisu dio optimalne sekvencije proširenja, čime se značajno ubrzava postupak optimizacije.*

*Prvo se osnovni heuristički pristup uspoređuje s egzaktnim algoritmom, a u svim test-primjerima ostvaruje najbolji mogući rezultat. Dalje je razvijeno nekoliko varijanti algoritma s obzirom na način definiranja kapacitivne točke. Istraživanje je pokazalo da su sve opcije algoritma vrlo efikasne, s mogućnošću postizanja optimalnog ili blizu-optimalnog rješenja, ali uz značajne uštede u vremenu računanja. To znači da se ovaj heuristički algoritam i njegove opcije mogu uspješno primjenjivati za kratkoročno i srednjoročno planiranje kapaciteta na satelitskom linku.*

## *KLJUČNE RIJEČI*

*optimalno proširenje kapaciteta, planiranje kapaciteta satelitskog linka, mobilne satelitske mreže, QoS (kvaliteta usluge) u mobilnim satelitskim mrežama*

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