SERĐO KOS, D. Sc.
ZDENKA ZENZEROVIĆ, D. Sc.
Pomorski fakultet u Rijeci
Studentska 2, 51000 Rijeka, Republika Hrvatska

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MODELLING THE TRANSPORT PROCESS IN MARINE CONTAINER TECHNOLOGY

ABSTRACT

The paper introduces a mathematical problem that occurs in marine container technology when programming the transport of a beforehand established number of ISO containers effected by a full container ship from several ports of departure to several ports of destination at the minimum distance (time in navigation) or at minimum transport costs. The application of the proposed model may have an effect on cost reduction in container transport thereby improving the operation process in marine transport technology. The model has been tested by using a numerical example with real data. In particular, it describes the application of the dual variables in the analysis of optimum solution.

KEY WORDS

container traffic, transport problem, linear programming, dual problem

1. INTRODUCTION

Since the costs of cargo transport from the place of production to the storage area, to the processing sector and finally to the consumer have a considerable effect on the price rise of the product, it is economically proper to try to achieve reduction in costs as much as feasible, but at the same time, from the carrier's standpoint, to effect the transport with maximum possible profit. In both cases cargo transport should be optimized no matter whether the criteria of optimization refer to transport costs or profit made, i.e. income realized.

Marine container transport technology combines three essential components [2, page 2]: the container as a technical means for storing merchandise, the port container terminal as an area provided with specific facilities and infrastructures for the loading, discharging and transhipment of containers and the container ship as a navigable means for the conveyance of containers from the port of departure to the port of destination. The transport process in marine container technology is accomplished by the suitable integration of the named components and, with regards to its complexity, it is necessary to monitor as well as to improve the functioning of the whole process from the technical, technological, economical, organizational, etc., standpoint.

An improvement in the transport process within marine container technology is possible, among other things, by optimizing the transport plan of containers in a given geographical area.

The object of this paper is to demonstrate how the transport process in marine container technology can be optimized by the use of pertinent quantitative methods, i.e. by establishing the most profitable transport variant concerning transport costs or incomes. To attain this object the method of linear programming has been applied.

2. DEFINITION OF THE PROBLEM

The container is a receptacle used for storing different items of general cargo intended for conveyance to the same destination. However, in practice it frequently occurs that the port of destination of the containers may not be at the same time the port of departure of the same units filled up with new merchandise. For this reason the technology of marine containers must cope with the problem of carrying empty containers to the port where the refilling will be effected. Therefore, the transport of empty containers is not only a technical and technological inconvenience but an economical disadvantage as well because such carriage increases the operational costs of the ship operator.

The primary assumption in marine container technology is that a full container ship in service should have three sets of containers at disposal:

- one set of loaded containers being directly on board;
- one set of empty containers being at disposal at the ports of loading of a given geographical area;
- one set of empty containers being at disposal at the ports of discharge of a given geographical area.

The term 'set of containers' refers to the nominal TEU-capacity relative to the container ship.

The problem dealt with in this paper is defined as follows: to program the transport of a number of ISO containers by full container ships from several ports of departure to several ports of destination with the minimum distance (time in navigation) or at minimum transport costs.

The solution of the problem involves the transport plan of containers from the ports of departure to the ports of destination taking into account the number of containers at the ports of departure and the number of containers required at the ports of destination with the objective of effecting the transport with the minimum total distance or at minimal transport costs

3. FORMULATION OF THE MATHEMATICAL MODEL

The problem of container transport can be illustrated with the aid of appropriate tables: the Matrix of Transportation (Table 1) and the Matrix of Costs, or Distances, (Table 2):

Table 1 - Matrix of Transportation

Ports of departure	Ports of destination				Number of ISO containers
	B_1	B_2	...	B_n	
A_1	x_{11}	x_{12}	...	x_{1n}	a_1
A_2	x_{21}	x_{22}	...	x_{2n}	a_2
.		.			.
.		.			.
.		.			.
A_m	x_{m1}	x_{m2}	...	x_{mn}	a_m
Number of ISO containers	b_1	b_2	...	b_n	

Table 2 - Matrix of Costs or Distances by Sea

Ports of departure	Ports of destination			
	B_1	B_2	...	B_n
A_1	c_{11}	c_{12}	...	c_{1n}
A_2	c_{21}	c_{22}	...	c_{2n}
.		.		
.		.		
.		.		
A_m	c_{m1}	c_{m2}	...	c_{mn}

where:

A_i – ports of departure, $i = 1,2, \dots ,m$

B_j – ports of destination, $j = 1,2, \dots ,n$

m – total number of departure ports

n – total number of destination ports

c_{ij} – distance by sea (or time in navigation), or unit costs for the transport on the distance between i-th port of departure and the j-th port of destination,
$i = 1,2, \dots ,m$, $j = 1,2, \dots ,n$

x_{ij} – number of ISO containers carried on the distance from i-th port of departure to j-th port of destination,
$i = 1,2, \dots ,m$, $j = 1,2, \dots ,n$

a_i – available number of ISO containers at i-th port of destination, $i = 1,2, \dots ,m$

b_j – demand for containers at j-th port of destination,
$j = 1,2, \dots ,n$.

The mathematical model for solving the problem in marine container technology is as follows:

$$\begin{aligned} \text{Min } Z = {} & c_{11}x_{11} + c_{12}x_{12} + \dots + c_{1n}x_{1n} + \\ & + c_{21}x_{21} + c_{22}x_{22} + \dots + c_{2n}x_{2n} + \\ & \dots \\ & + c_{m1}x_{m1} + c_{m2}x_{m2} + \dots + c_{mn}x_{mn} \end{aligned} \tag{1}$$

with constraints

$$\begin{aligned} x_{11} + x_{12} + \dots\dots + x_{1n} &= a_1 \\ x_{21} + x_{22} + \dots\dots + x_{2n} &= a_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots & \\ x_{m1} + x_{m2} + \dots\dots + x_{mn} &= a_m \end{aligned} \tag{2}$$

$$\begin{aligned} x_{11} + x_{21} + \dots\dots + x_{m1} &= b_1 \\ x_{12} + x_{22} + \dots\dots + x_{m2} &= b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots & \\ x_{1n} + x_{2n} + \dots\dots + x_{mn} &= b_n \end{aligned} \tag{3}$$

$$x_{11}, x_{12}, \dots, x_{mn} \geq 0 \, . \tag{4}$$

From the system of equations exposed above it is noticeable that it is a problem of linear programming with the $(m+n)$ equation and $(m \cdot n)$ variable. The equations (2) and (3) form an equation system composed of independent equations $(m+n-1)$. For this reason the solution of the given transport problem must comprise $(m+n-1)$ non-negative values x_{ij}. If the solution of the problem comprises less than $(m+n-1)$ non-negative values x_{ij}, then such a solution is degenerative, in which case it is necessary to obtain a non-degenerative solution by applying the appropriate procedure.

The mathematical model for the solution of the transport problem in marine container technology is, in summarized form, as follows:

Criteria function

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{5}$$

with constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2, \dots ,m \tag{6}$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,\dots,n \tag{7}$$

$x_{ij} \geq 0;\quad i = 1,2,\dots,m\ ,\ j = 1,2,\dots,n\ .$

To be conveniently correlated the equations (6) and (7) should fulfil the following condition:

$$\sum_{i=1}^{m} a = \sum_{j=1}^{n} b_j\ .$$

The transport problem in marine container technology in which the total number of ISO containers dispatch is equal to the number of ISO containers receipt represents a closed transport problem.

When the number of containers at the ports of departure exceeds or is less than the number of containers at the ports of destination then it is the case of open transport problem in marine container technology.

The solution of the transport problem in marine container technology by using the simplex method is rather time-consuming; consequently, some specialized methods can be applied to cope with the problem. These are:

- methods requiring program initialization with further upgrading until the optimum program is achieved. For program initialization the following are available: the "North-West Corner Rule" method, the minimum cost method, the Vogel method, etc., and for upgrading and achieving the optimum solution: the "Stepping Stone Method", the MODI--method, etc.
- methods not requiring program initialization. These methods gradually upgrade their program until the optimum solution is achieved. Among these the best known is the Ford-Fulkerson method.

For any problem of linear programming, and this is the case of transport problems, the dual problem can be formulated. The value of dual variables are useful for analyzing the optimum solution, in particular analyzing the influence of the change the containers number at the port of departure or at the port of destination on the minimum distance or minimum transport costs.

Thanks to the development of computerization the solution of many problems from operations research has been facilitated by the application of pertinent computer programs. Because of its simplicity the use of the personal computer QSB program can be recommended [3].

Detailed explications concerning the application of methods and means of solutions are given in specific textbooks dealing with operations research [2], [6], [1].

4. NUMERICAL EXAMPLE

To illustrate the mentioned mathematical model an example with real data has been chosen.

In a given geographical area served by the shipping carrier "X" empty containers must be carried from the ports of departure A_1, A_2, A_3 and A_4 to the ports of destination B_1, B_2, B_3 and B_4.

At the ports of departure the following quantities of containers are available for carriage: 700, 500, 400 and 400 TEU, respectively.

The demand for containers at the ports of destination is the following: 560, 380, 620 and 440 TEU, respectively.

The distances by sea between the ports of departure and destination in nautical miles are given in Table 3:

Table 3 - Distances by sea between the ports of departure and destination in NM

Ports of departure	Ports of destination			
	B_1	B_2	B_3	B_4
A_1	1368	2736	3648	4104
A_2	912	3192	2280	4560
A_3	456	5016	5472	5928
A_4	1824	6384	6840	7296

The shipping carrier "X" runs a container ship that achieves in navigation a service speed of 19 knots, and the time in navigation between the ports expressed in days, amounts to:

Table 4 - Days in navigation between the ports of departure and destination

Ports of departure	Ports of destination			
	B_1	B_2	B_3	B_4
A_1	3	6	8	9
A_2	2	7	5	10
A_3	1	11	12	13
A_4	4	14	15	16

The task consists in working out a plan of container transport from the ports of departure to the ports of destination aiming at the minimum total distance, or minimum time in navigation.

For the given problem the mathematical model runs as follows:

Min $Z =$

$$\begin{aligned} &1368x_{11} + 2736x_{12} + 3648x_{13} + 4104x_{14} + \\ &912x_{21} + 3192x_{22} + 2280x_{23} + 4560x_{24} + \\ &456x_{31} + 5016x_{32} + 5472x_{33} + 5928x_{34} + \\ &1824x_{41} + 6384x_{42} + 6840x_{43} + 7296x_{44} \end{aligned} \tag{8}$$

with constraints

$$\begin{aligned} x_{11}+x_{12}+x_{13}+x_{14}&=700\\ x_{21}+x_{22}+x_{23}+x_{24}&=500\\ x_{31}+x_{32}+x_{33}+x_{34}&=400\\ x_{41}+x_{42}+x_{43}+x_{44}&=400 \end{aligned} \tag{9}$$

$$\begin{aligned} x_{11}+x_{21}+x_{31}+x_{41}&=560\\ x_{12}+x_{22}+x_{32}+x_{42}&=380\\ x_{13}+x_{23}+x_{33}+x_{43}&=620\\ x_{14}+x_{24}+x_{34}+x_{44}&=440 \end{aligned} \tag{10}$$

$$x_{ij} \geq 0,\ i=1,2,3,4,\ j=1,2,3,4. \tag{11}$$

The transport problem of ISO containers is solved by means of "North-West Corner Rule" and MODI methods. According to the optimum program the transport of ISO containers should be arranged as follows:

- from the port of destination A_1 to be carried: 380 TEU to port B_2, 120 TEU to port B_3, 200 TEU to port B_4,
- from the port of destination A_2:500 TEU to port B_3,
- from the port of destination A_3: 400 TEU to port B_1,
- from the port of destination A_4: 160 TEU to port B_1 and 240 TEU to port B_4.

The value of the optimum transport plan amounts to: Min Z = 5 663 520 TEU-NM, or Min Z = 12 420 TEU-days.

The optimum solution is visually shown in Fig. 1:

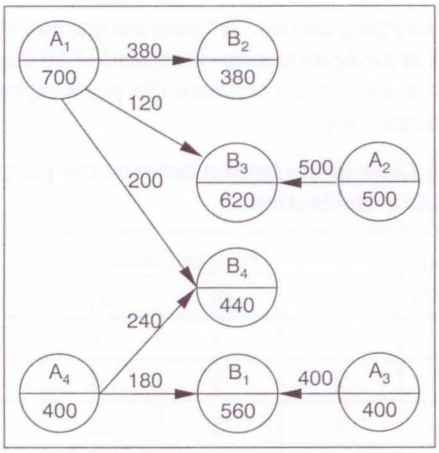

Figure 1 - Optimum transport plan of containers

The mathematical model of the dual problem of container transport runs as follows:

$$\text{Max } W = 700u_1+500u_2+400u_3+400u_4+560v_1+380v_2+620v_3+440v_4 \tag{12}$$

with constraints

$$\begin{aligned} u_1+v_1&\leq 1368\\ u_1+v_2&\leq 2736\\ u_1+v_3&\leq 3648\\ u_1+v_4&\leq 4104 \end{aligned} \tag{13}$$

$$\begin{aligned} u_2+v_1&\leq 912\\ u_2+v_2&\leq 3192\\ u_2+v_3&\leq 2280\\ u_2+v_4&\leq 4560 \end{aligned} \tag{14}$$

$$\begin{aligned} u_3+v_1&\leq 456\\ u_3+v_2&\leq 5016\\ u_3+v_3&\leq 5472\\ u_3+v_4&\leq 5928 \end{aligned} \tag{15}$$

$$\begin{aligned} u_4+v_1&\leq 1824\\ u_4+v_2&\leq 6384\\ u_4+v_3&\leq 6840\\ u_4+v_4&\leq 7296\,. \end{aligned} \tag{16}$$

The dual problem (12)-(16) is solved by the simplex method; however, the optimum solution can be obtained by solving the primal by the application of the previously stated methods (MODI-method) that enables obtaining the following values of dual variables:

$u_1=0$, $u_2=-1368$, $u_3=1824$, $u_4=3192$,
$v_1=-1368$, $v_2=2736$, $v_3=3648$, $v_4=4104$.

The positive values of dual variables that refer to the port of departure, i.e. values u_1, indicate the amount of increase in total distance expressed in NM, or number of TEU-days, and with this the total costs of the container transport, if to the number of shipped containers from i-th port of departure one TEU unit is added. Conversely, the negative values u_i indicate that the addition of one TEU unit to the number of containers at i-th port of departure may affect the reduction of total distance and total transport costs for the amount shown by the values of dual variables.

Similar conclusions are valid for the positive or negative values of dual variables v_j that indicate the quantity of increase or decrease in costs of container transport, if the need, i.e. demand for TEU at the port of destination j-th, or vice versa, is increased by one unit.

It needs emphasizing that in the analysis of dual variables the increase referring to one departure (or destination) and the decrease to the second departure (or destination) is the same figure to avoid disturbing the balance between supply and demand of the transport matrix.

For example: if at port A_1 the number of containers drops to 600 units, and at port A_2 there is an increase to 600 units, with all the other elements in the model remaining unchanged, the alteration will cause an increase in total distance of 136,800 NM, thereby the minimum distance amounting to 5,526,720 TEU-NM. Namely, the rise in number a_2 of 100 TEU causes an increase of 100·1368 TEU-NM, and the decrease in number of containers a_1 of 100 TEU affects in no way the function Z, as $u_1=0$. Or, if the demand from B_1 drops to 400 containers, and there is an increase in demand from port B_4 to 600 containers, then based on the values of dual variables there follows that the alterations will have an effect on the increase in distance to the amount of 875,520 TEU-NM (160·1368 + 160·4104). As a result the total distance will be 6,539,040 NM.

5. CONCLUSION

From the mathematical viewpoint the problem of marine container technology is defined as a problem requiring an assessment of minimal value of criteria function with a specific number of structural variables x_{ij}, mutually correlated by linear functions, with limitations in the form of linear equation or non-equations.

From the technological and organizational viewpoint it is a process of optimum distribution of a limited number of units determined by a set of objectives such as minimum sea distance, minimum days in navigation or minimum costs.

In order to solve the transport problem in marine container technology various methods can be employed: some require initializing the program being further upgraded until the optimum program is achieved and other methods not requiring initializing the program with gradual program upgrading until the optimal solution is found. For the purpose, adequate computer programs can be used as well.

The presented mathematical model is basic for making suitable business decisions that regard the transport of ISO containers by full container ships in a given geographic area served by a shipping operator running a container carrier fleet.

Of particular significance is the validity of the model in the fluctuating conditions of shipping business on the market; by analysis of the dual variables several solutions of the given problem are possible in cases of change in the supply and demand for containers.

SERĐO KOS, D. Sc.
ZDENKA ZENZEROVIĆ, D. Sc.
Pomorski fakultet u Rijeci
Studentska 2, 51000 Rijeka, Republika Hrvatska

SAŽETAK

MODELIRANJE PRIJEVOZNOG PROCESA MORSKE KONTEJNERSKE TEHNOLOGIJE

U ovom je radu prikazan matematički model za jedan problem koji se pojavljuje u morskoj kontejnerskoj tehnologiji kada treba programirati prijevoz unaprijed određenog broja ISO kontejnera morskim potpuno kontejnerskim brodovima iz više luka ukrcaja u više luka iskrcaja uz minimalne udaljenosti (ili vrijeme u plovidbi), maksimalan prihod ili minimalne troškove prijevoza. Korištenjem predloženog modela moguće je utjecati na sniženje troškova prijevoza kontejnera i time poboljšati funkcioniranje prijevoznog procesa morske transportne tehnologije. Model je testiran na numeričkom primjeru s realnim podacima. Posebno je prikazana primjena dualnih varijabli u analizi optimalnoga rješenja.

KLJUČNE RIJEČI

kontejnerski promet, transportni problem, linearno programiranje, dualni problem

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