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TOWARD THE THEORY OF TRAFFIC FLOW ORGANISATION

ABSTRACT

Every traffic network can be reorganised, i.e. the traffic flows may be redirected and guided along other routes. Mathematical calculation of the number of intersecting points, merging and diverging of traffic can determine the current volume of conflicting traffic flows in the network. The aim of redirecting traffic flows is to obtain lower intensity of intersecting flows, indicating at the same time better organisation of traffic flows. The work presents a model of traffic flow intersections on an isolated road section. It also provides mathematical formulas for calculating the number of collision points for the same and a different number of entrances and exits (nodes). The problem is further developed for the case which searches for the number of conflicting points of traffic flows with two-way traffic. A mathematical formula has been found for the same number of entries and exits (nodes, or sources and sinks) of traffic flows.

KEY WORDS

traffic flows, direction, collision points, intersecting, mathematical formula

1. INTRODUCTION

For optimal traffic flow it is necessary to observe the relations between the traffic flows, in order to act therapeutically. The advantages of solving urban traffic in such a way are multiple: the number of traffic accidents is reduced, the traffic network throughput capacity increased as well as the average vehicle speed, environmental pollution is reduced and the investments necessary for the infrastructure are lower (because the existing infrastructure is optimally used), and the costs of vehicle exploitation both of private and public urban traffic are lower.

The relations between traffic flows at intersections are one of the causes of their reduced throughput capacity. Avoiding unnecessary collisions and reducing the interrupting traffic flows are one of the possible important factors that favourably contribute to the increase in the intersection throughput capacity.

Relations between traffic flows in the form of unnecessary conflicting points occur at intersections, and they are caused by organisation and direction of traffic flows in the network. Therefore, any procedure in reorganising the traffic flows has to be based on a closely studied existing status and the analysis of the possibilities to change it. Thus, one may practically speak about the management of traffic flows.

Relations between traffic flows in the network are especially complex in urban parts of the traffic network. Therefore, these need to be described in the best possible manner with as few parameters as possible. The conflicts are classified as passing, intersecting, weaving, merging and diverging of traffic flows.

Intersecting of traffic flows occurs when two at-grade traffic flows that are not parallel "face" each other (i.e. come into conflict, pass one through the other).

2. DEFINITION OF THE PROBLEM

Every traffic network can be reorganised, that is, the directions of traffic flows can be changed and guided along other routes. By mathematically calculating the number of points of intersecting, merging and diverging, it is possible to determine the current volume of conflicting flows in the network. The redirection of traffic flows means that a lower intensity of flow intersections needs to be established, indicating at the same time better quality of organising traffic flows.

In order to make the problem theoretically better understood, Figure 1 shows a part of the town in which two very busy traffic flows intersect and cause standstills at points N_1 and N_2 and how they do not intersect any more after flow redirection.

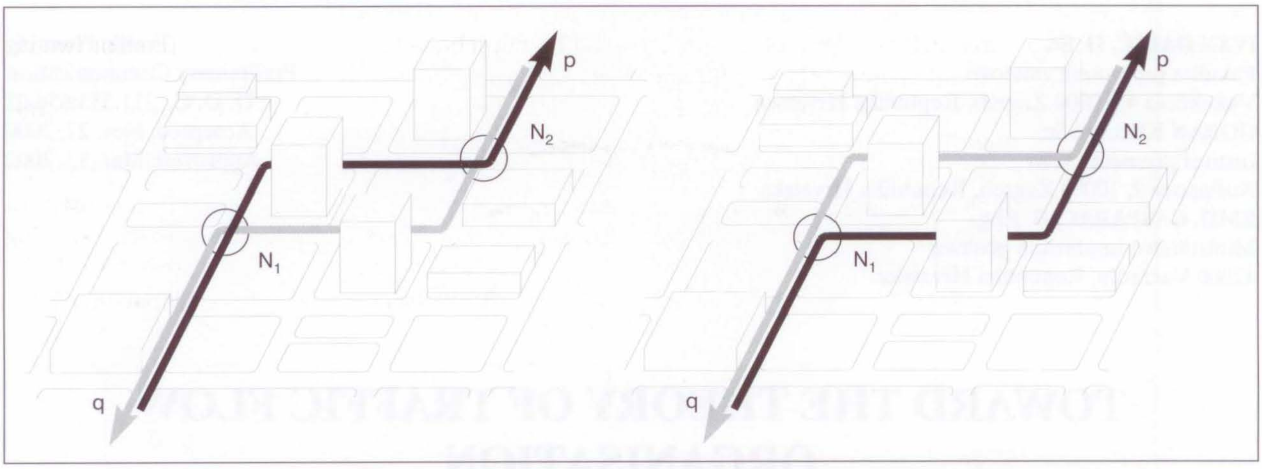

Figure 1 - Example of unnecessary intersecting of traffic flows and its elimination

3. DEVELOPING A MODEL OF IDEAL NUMBER OF TRAFFIC FLOW INTERSECTING POINTS

Traffic flows are optimal when the number of their intersecting points is the least possible compared to the ideal model of traffic flow in the network, i.e. when intersecting, diverging and merging of the flows is reduced to the actual minimum. Therefore, the following question arises: how to develop a model containing the minimum number of traffic flow intersecting points in a network? Since such a model contains the minimum number of intersecting points, it can be called also the model of ideal number of intersecting points and it can represent the standard of flow intersections. Figure 2 presents traffic flows in a section of a street network. The origin and target of movement of a certain traffic flow within the zone boundaries are designated by letters (A, B, C, D, E and F).

Some of the represented streets are one-way streets. The intensity of traffic flows are not important for setting the model of minimum number of intersecting points, and are therefore represented by the same line thickness.

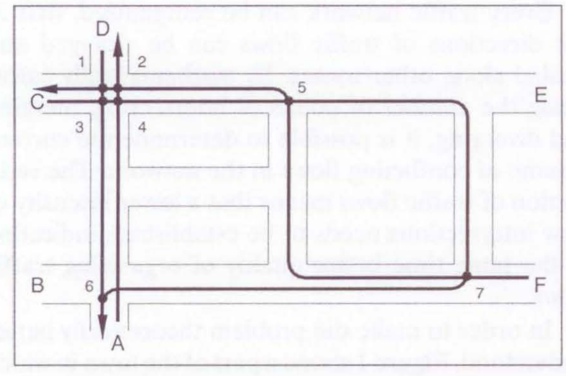

Figure 2 - Traffic flows in a section of a street network with seven intersecting points (no merging or diverging)

The numbers indicate observation points of flow intersections. Figure 3 shows a model of ideal number of intersecting points. Only two traffic flows intersect (FC and AD). Since the street connecting points A and D is a two-way street, there are two traffic flows in the Figure.

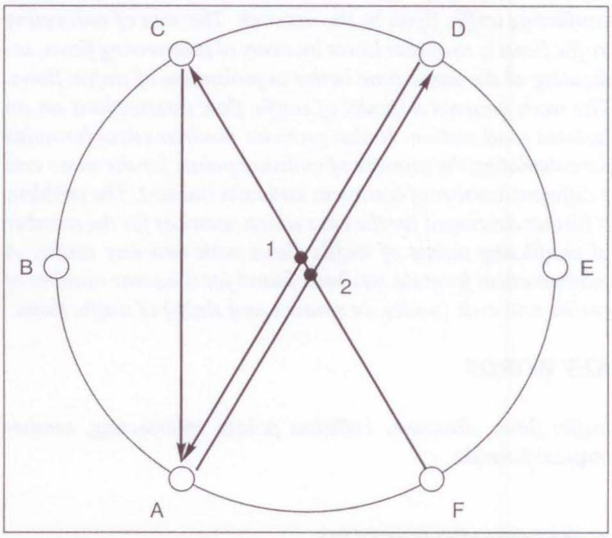

Figure 3 - Model of ideal number of intersecting points with only two intersecting points of traffic flows (also with no merging or diverging)

According to the model of ideal number of traffic flow intersecting points it is obvious that the traffic flows in Figure 2 are not optimal and that their intersecting can be reduced to only two points. For the sake of simplicity, the traffic flow diverging and merging points have not been indicated, since it is important to indicate the main intersecting points.

The intensity of intersections at conflicting points is determined by mathematical methods of determining the volume of traffic flow intersections. Based on research (2) three methods for measuring collision (intersecting, merging and diverging) intensity are set for the street network.

(1) *The method of collision area between traffic flows* is the product of traffic intensity at the conflicting points. The traffic flow itself needs to be converted into the car unit equivalent (3) (EJA). Traffic flows "p" and "q" do not yield any conflicting intensity if one of the flows equals zero. In order to be able to measure the conflict intensity in vehicle units per hour, the expression needs to be corrected, and then it can be defined as the method of the square root of the conflict area:

$$I_{PR(t)} = \sum_{1}^{N_{pr}} \sqrt{pq} \quad \left[\frac{\text{veh.}}{\text{h}}\right] \tag{1}$$

(2) *Method of summing up traffic flows* at the point of collision; the disadvantage is in the fact that collision exists even when one of the flows equals zero, therefore the formula includes also an additional condition:

$$I_{PR(t)} = \sum_{1}^{N_{pr}} (p+q), \forall p, q > 0 \quad \left[\frac{\text{veh.}}{\text{h}}\right] \tag{2}$$

(3) *Method of minimal flow at the collision point,* at which the intensity of collision represents the minor traffic flow:

$$I_{PR(t)} = \sum_{1}^{N_{pr}} \min(p, q) \quad \left[\frac{\text{veh.}}{\text{h}}\right] \tag{3}$$

4. DEVELOPING A MODEL OF TRAFFIC FLOW INTERSECTING ON AN ISOLATED ROAD SECTION

Previous considerations of conflicting traffic flows have mainly referred to measuring the intensity of intersecting, diverging and merging at intersections. However, certain interactions between traffic flows exist also on isolated road sections (4) (streets, fast urban roads, motorways, etc.). Figure 4 shows weaving of traffic flows on a three-lane road. Two flows intersect (these can be replaced by two mergings and two divergings /points 1 and 2/) as well as one merging and one diverging (3 and 4) of traffic flows.

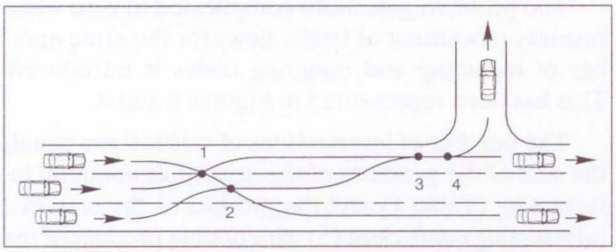

Figure 4 - Weaving of traffic flows on an isolated road section

Only some combinations are shown that can occur when three traffic flows interweave on an isolated road section between e.g. two intersections or two diverging roads from the highway.

Since focus is on the turbulence (weaving) of traffic flows between intersections, the starting point is the theoretical possibility that on a one-way flow between two intersections the combined flow consists of N_m individual sequences (platoon) of vehicles arriving from N_m intersection approach and move towards the adjacent closest intersection with N_n departures (Figure 5).

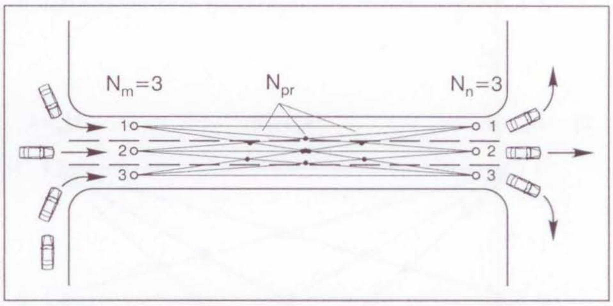

Figure 5 - Weaving of traffic flows between intersections with N traffic lanes and flows

During the initial phase traffic flows are taken without their intensity in order to determine only their routes and relations among the flows. The starting point is the assumption that at the one-way exit from an intersection there are at least N_m traffic lanes (number of entries), and at the entry to the adjacent intersection there are at least N_n traffic lanes (number of exits), as many as there are exits from the intersection.

For simplicity reasons of studying the problem the starting point is that $N_m = N_n$ and that the traffic flow from every entry to intersection N_m diverges into N_n flows, the same as there are exits at the next intersection.

If U-turns at approach to intersection are excluded, then the problems in practice are much simpler. Then, as a rule, N_m and N_n can amount to a maximum of 1 to 4 entries (if also the intersections with five entries are excluded from the consideration).

This problem can be further simplified by determining the number of traffic flows between a group of N_m origins towards the group of N_n destinations.

This excludes communication within traffic flows of group N_m and within group N_n. Such case occurs in practice in different forms, as e.g. delivery of goods from N_m cities on one coast (sea or rivers) towards N_n cities on another coast.

Theoretically, there is a need to determine the number of traffic flows between N_m origins towards N_n destinations and the number of intersecting points between these flows, but with the assumption that the traffic flows follow the shortest routes.

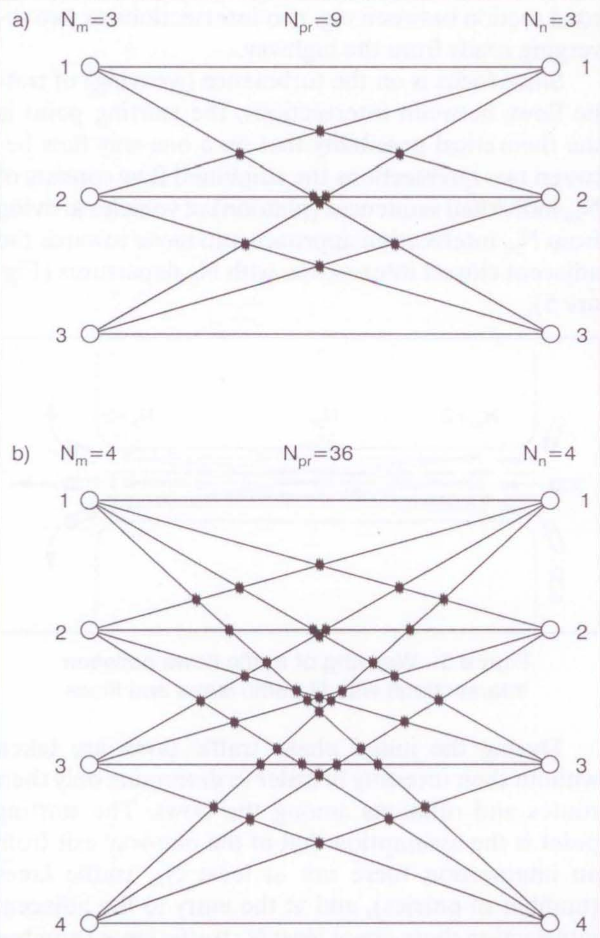

Figure 6 - The number of intersecting points of traffic flows for $N_m = N_n = 3$ and $N_m = N_n = 4$

In order to simplify further study of this issue, the following is taken into account:

$$N_m = N_n \tag{4}$$

The total number of traffic flows is obtained by the following forms:

$$N_t = N_m N_n \tag{5}$$

$$N_t = N_m^2 \tag{6}$$

Based on Figure 6 it is obvious that there are nine intersecting points for three origin and destination points, and 36 intersecting points for four origin and four destination points. One traffic flow has no intersecting points, and for two traffic flows the intersecting can occur at only one point. Table 1 presents the number of intersecting points N_{pr} for a defined number of origins (destinations) N_m.

The number of intersecting points N_{pr} between the traffic flows of the group of origin N_m is obtained by means of the following expression:

$$N_{pr} = \frac{N_m}{2}(N_m - 1)(1+2+3+\ldots N_m - 1) \tag{7}$$

$$N_{pr} = \frac{(N_m - 1)^2 N_m^2}{4} \tag{8}$$

This expression can be written also in another form:

$$N_{pr} = \frac{N_m}{2}(N_m - 1)\sum_{i=1}^{N_m - 1} i \tag{9}$$

Based on the expression (9) the following Table 1 was formed.

Table 1 - The number of intersecting points N_{pr} for a certain number of origins, i.e. destinations of Nm traffic flows.

N_m	N_{pr}
1	0
2	1
3	9
4	36
5	100
6	225
7	441
8	784
9	1296
10	2025

For the case in which:

$$N_m \neq N_n \tag{10}$$

the problem is presented by Figure 7.

Mathematical formula for calculating the number of conflicting points in case of unequal number of incoming and outgoing nodes is:

$$N_{pr} = \frac{N_m}{2}(N_m - 1)\sum_{i=1}^{N_n - 1} i \tag{11}$$

According to this expression, Table 2 was formed, containing the total number of intersecting points of traffic flows for combinations 1x1 to 6x6 inputs and outputs.

The problem gets more complicated in case when two-way movement of traffic flows for the same number of incoming and outgoing nodes is introduced. This has been represented in Figures 8 and 9.

The number of intersections of traffic flows equals the sum of the products of the quadruple common intersecting (Table 1) and the product of the marginal intersecting coefficient (5) with double product of the number of incoming (outgoing) node.

The marginal intersecting coefficient of traffic flows forms the string:

$$k_{rub} = 0, 1, 3, 6, 9, 12 \ldots 3 \cdot (N_m - 2) \tag{12}$$

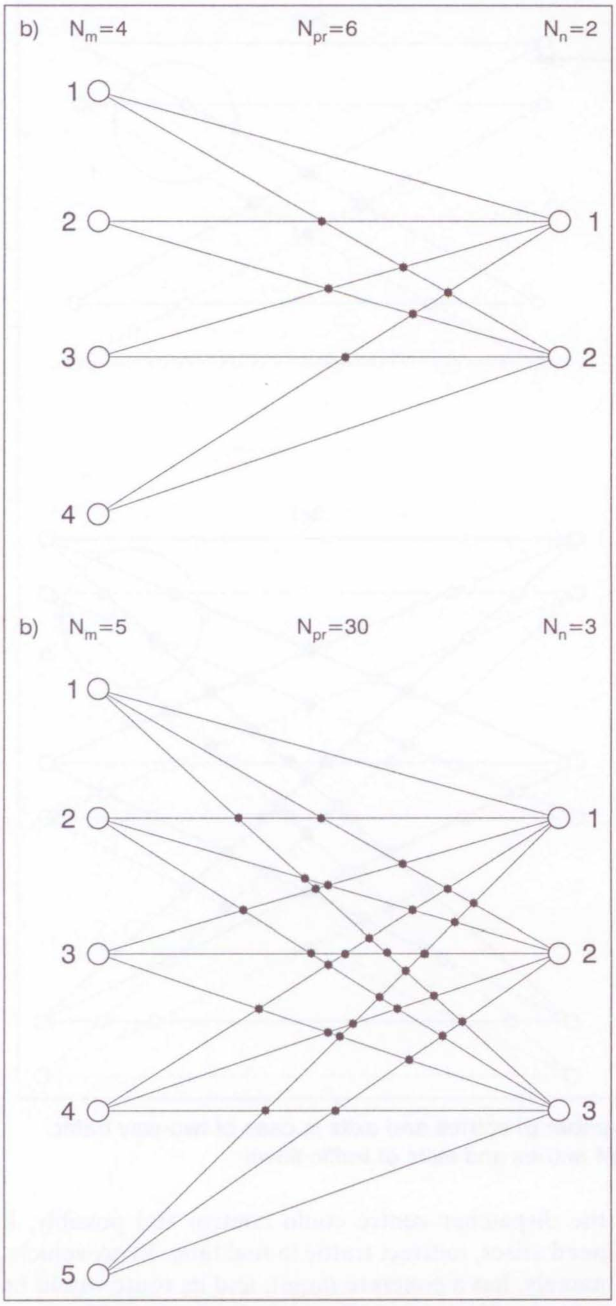

Figure 7 - The number of conflicting points for $N_m \neq N_n$ (4;2 and 5;3)

For traffic matrices of 1×1 to 5×5, the number of intersecting amounts to:

$$
\begin{array}{ll}
1\times1 & 4\cdot0+1\cdot0=0 \\
2\times2 & 4\cdot1+1\cdot4=4+4=8 \\
3\times3 & 4\cdot9+3\cdot6=36+18=54 \\
4\times4 & 4\cdot36+6\cdot8=144+48=192 \\
5\times5 & 4\cdot100+9\cdot10=400+90=490
\end{array} \tag{13}
$$

It follows that:

$$N_{pr}=\left[4\cdot\frac{N_m}{2}(N_m-1)\sum_{i=1}^{N_m-1} i\right]+k_{rub}\cdot 2N_m \tag{14}$$

Table 2 - The number of conflicting points N_{pr} for a selected number of origins N_m i.e. destinations N_n of traffic flows

N_m	N_n	N_{pr}
1	1	0
2	1	0
	2	1
3	1	0
	2	3
	3	9
4	1	0
	2	6
	3	18
	4	36
5	1	0
	2	10
	3	30
	4	60
	5	100
6	1	0
	2	15
	3	45
	4	90
	5	150
	6	225

Arrangement of the expression (32) yields:

$$N_{pr}=3(N_m-2)\cdot 2N_m+2N_m(N_m-1)\sum_{i=1}^{N_m-1} i \tag{15}$$

$$N_{pr}=2N_m\left[3(N_m-2)+(N_m-1)\sum_{i=1}^{N_m-1}\right] \tag{16}$$

The final expression for calculating the number of intersecting points of traffic flows with the same number of entries and exits (nodes) in two-way flows take the following form:

$$N_{pr}=2N_m\left[k_{rub}+(N_m-1)\sum_{i=1}^{N_m-1} i\right] \tag{17}$$

5. CONCLUSION

The model of intersecting traffic flows on an isolated road section has been presented and mathematical formulas have been determined for calculating the number of intersecting points for the equal and different number of entries and exits (nodes). The problem is further applied to the case of determining the number of intersecting points of two-way traffic flows. A mathematical model has also been suggested for the

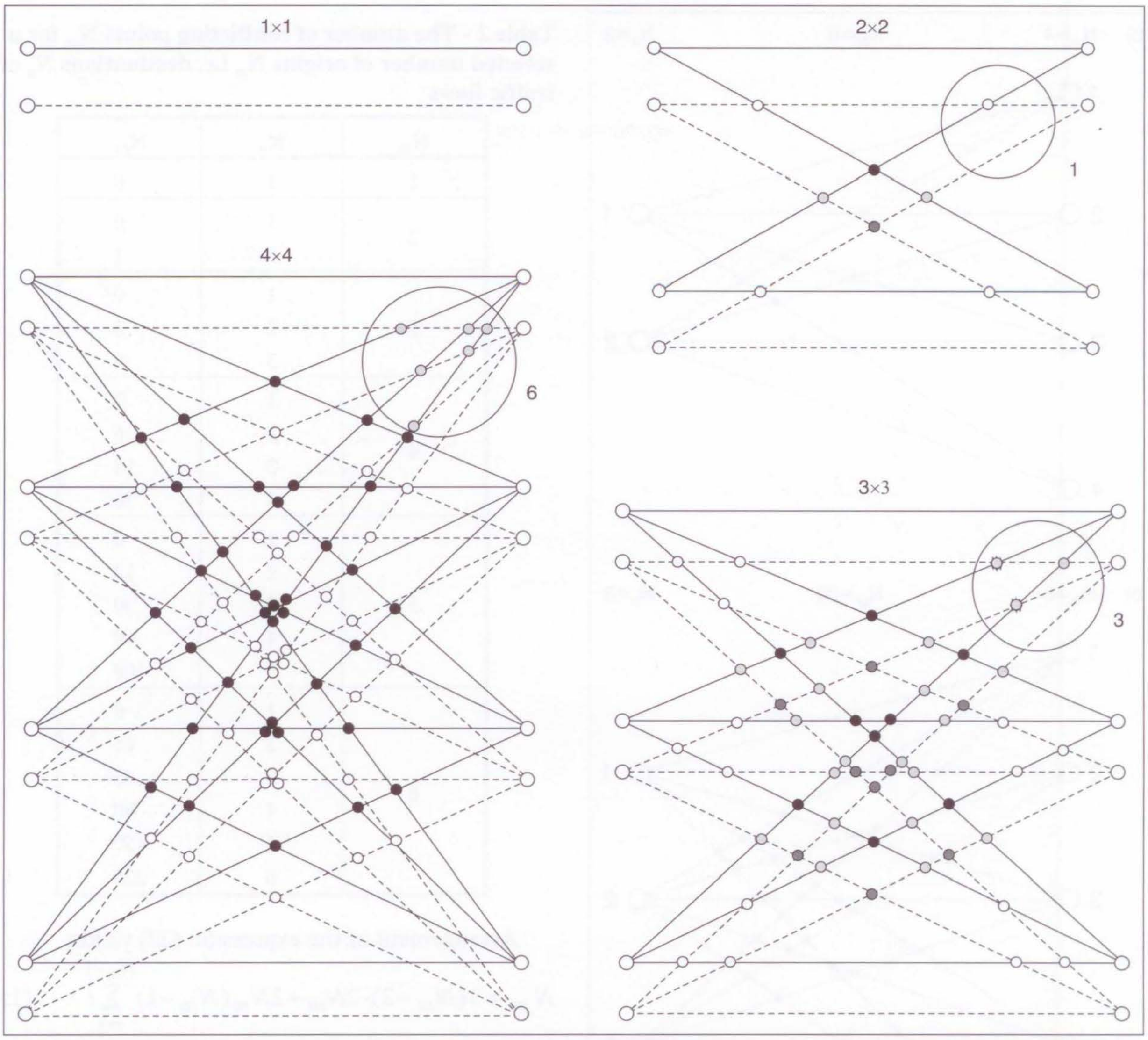

Figure 8 - The intersecting traffic flows for the same number of entries and exits in case of two-way traffic for the cases of 1x1, 2x2, 3x3 and 4x4 entries and exits of traffic flows

equal number of entries and exits (nodes or sources and sinks) of traffic flows.

However, further research should establish mathematical models that describe intersecting of two-way traffic flows with different number of incoming and outgoing nodes. Moreover, the number of intersecting points needs to be found as well as the intensity of intersecting traffic flows in an ideal intersecting model.

The next phase of research should be directed to developing programming tools that would enable simple implementation of the presented theory of traffic flow organisation, i.e. the recording of routines and algorithm as additional program support to AutoCAD program.

Equipping of vehicles by GPS satellite receiver for global positioning, the inertia system and GSM device for mobile communications would mean creating a dynamic database on vehicles in the traffic network. This would provide better information to motorists, and the dispatcher centre could control and possibly, if need arises, redirect traffic in real time. Every vehicle, namely, has a concrete target, and its route would be optimised by means of a central computer. Thus, unnecessary self-intersecting and intersecting of traffic flows would be avoided. It would also mean an increase in the throughput capacity of the existing traffic networks.

SAŽETAK

PRILOZI TEORIJI ORGANIZIRANOSTI PROMETNIH TOKOVA

Svaka prometna mreža može se reorganizirati, tj. mogu se promijeniti smjerovi prometnih tokova i voditi ih drugim pravcima. Matematičkim izračunavanjem broja točaka presijecanja, ulijevanja i odlijevanja prometa moguće je utvrditi trenutačnu količinu sukobljavanja prometnih tokova u mreži. Promjenom usmjerivanja prometnih tokova želi se dobiti manji

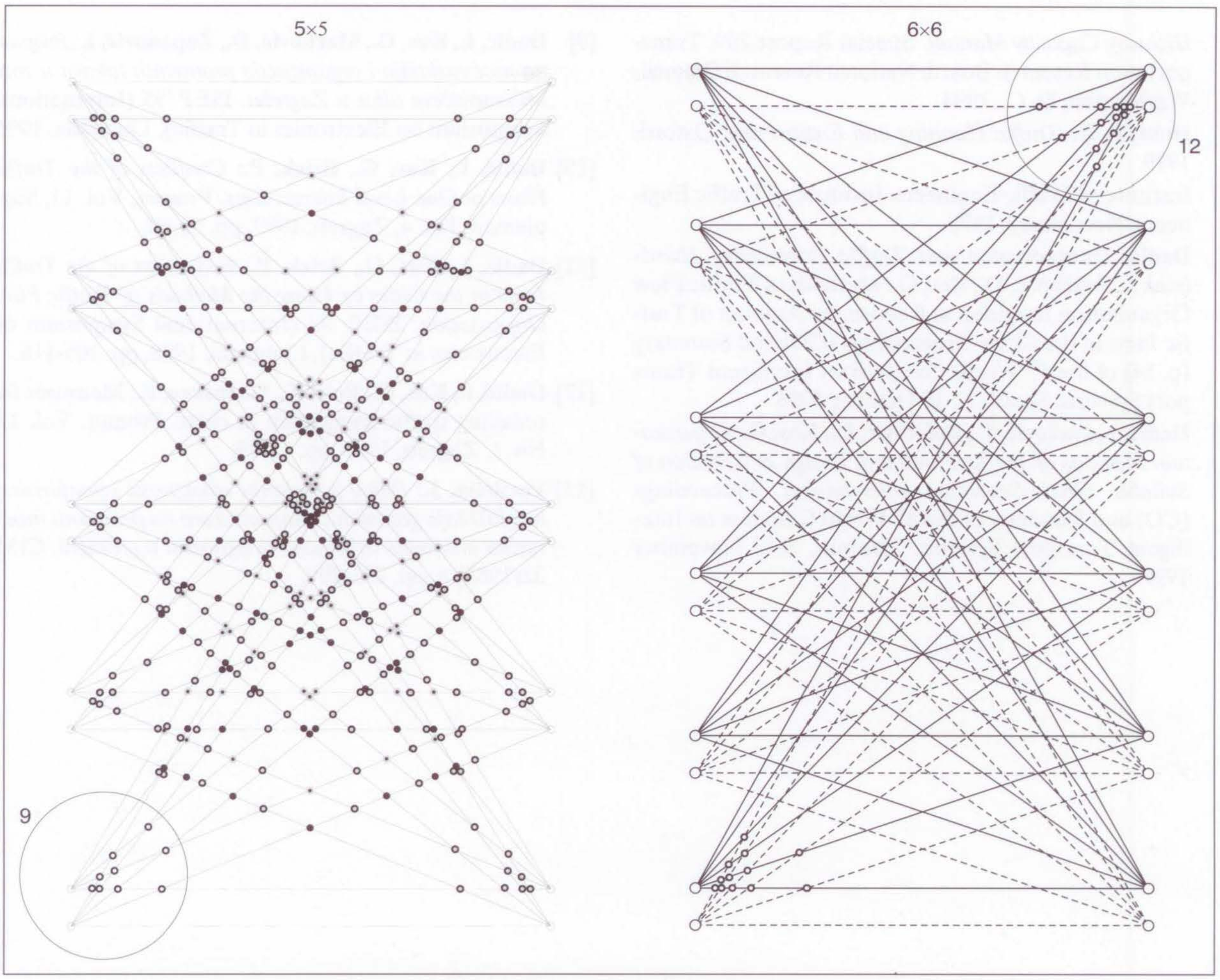

Figure 9 - Intersecting of traffic flows for the same number of entries and exits in two-way flow for the cases of 5x5 and 6x6 entries and exits

intenzitet presijecanja tokova, što ujedno ukazuje na kvalitetnije organiziranje prometnih tokova. U radu je prikazan model presijecanja prometnih tokova na izoliranoj dionici ceste i napravljeni su matematički obrasci za izračunavanje broja kolizionih točaka za isti i različit broj ulaza i izlaza (čvorova). Problem je dalje razvijen na slučaj traženja broja točaka presijecanja prometnih tokova s dvosmjernim prometnim tokovima. Pronađen je matematički obrazac za isti broj ulaza i izlaza (čvorova ili izvora i ponora) prometnih tokova.

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2. Dadić, I.: Prilog izučavanju organizacije prometnih tokova u gradovima. Doctoral dissertation, Belgrade, 1988, pp. 90-96.
3. The expression "EJA" (Cro: Ekvivalent Jednog Automobila) is used for reducing different vehicles (cargo vehicles of various categories, buses, commercial vehicles, motorcycles and bicycles) to a passenger car unit (i.e. Single Car Equivalent).
4. Cf. Dadić, I., Kos, G., Brlek, P.: Metode organiziranosti prometnih tokova za smanjenje zagušenja prometa na dionicama prometnica između raskrižja. ISEP'99 (International Symposium on Electronics in Traffic), Ljubljana, 1999, pp. 39-46
5. According to Figures 8 and 9 it may be observed that the conflicting points tend to be arranged along the node boundary (included in the circle). Their number increases and the sequence is determined by the graphical method. Since the string is a component of the mathematical formula, it is called the marginal intersecting coefficient among traffic flows.

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