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ALTERNATIVE POSSIBILITIES FOR DEFINING THE LENGTH OF THE SEPARATE LEFT-TURN LANE AT THE EXISTING LEVEL INTERSECTION

ABSTRACT

On the market today, there are various computer programs for simulating traffic flows at level intersection, which are all based on mathematics not seen by the end user. In this way the user only supplies data without being aware of how the program works and neither of the mathematical background. The results obtained are critically judged under this influence, resulting thus in subjective decisions.

Therefore, the article presents simple alternative mathematical possibilities for the requirements and the length of a separate left-turn lane at intersection.

KEY WORDS

left turn lane, level intersection

1. INTRODUCTION

In a three arm intersection (Figure 1) the separate left-turn lane has a capacity of n vehicles and the length of L meters respectively. In most cases [1,2,3,4] the focus of attention is on the probability that the length of the separate left-turn lane will be sufficient for all vehicles which in the N series of vehicles arrive at the intersection to stop in the separate left- (intersection) or right-turn lane (gasoline stations, bus stations, highway exits, level intersection with or without traffic signals, roundabouts with by-pass, etc.)

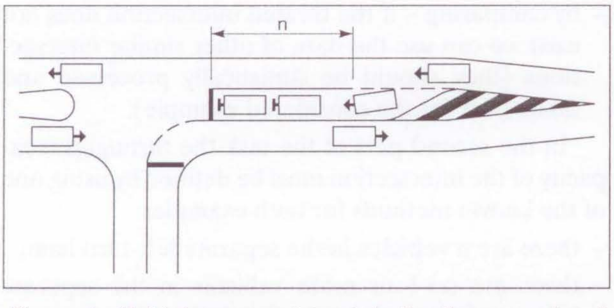

Figure 1 - Vehicles are arriving into the three arm level intersection with a separate left-turning lane

2. METHODOLOGY

The task should be worked out in two steps:

In the first step we define how more vehicles than n vehicles are likely to come to the intersection in order to turn left. The problem is treated statically. Vehicles that turn left have to give priority to the vehicles from the opposite direction (Figure 2) which are driving straight through and we ignore the fact that the first vehicle can drive off before the n^{th} vehicle, which intends to turn left, reaches the intersection.

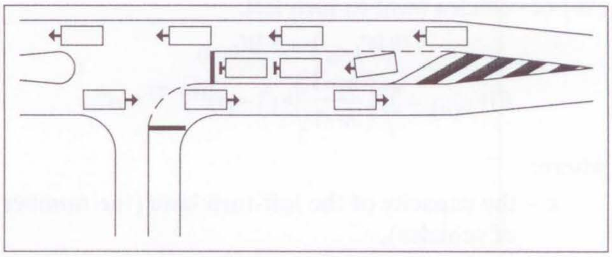

Figure 2 - Vehicles that turn left have to give priority to the vehicles from the opposite direction which are driving straight

In the second step of the task we must find out the throughput capacity of the intersection by using one of the known methods for both possible examples: when there are exactly n or fewer vehicles waiting in the intersection or when there are more than n vehicles (Figure 3).

The first part of the task my be worked out in two ways, by using:

- negative binomial, or
- Poisson distribution.

2.1. Negative binomial distribution

The negative binomial distribution is used when we want to find out in which test a k^{th} negative result occurs. In this case the task should be set in the following way:

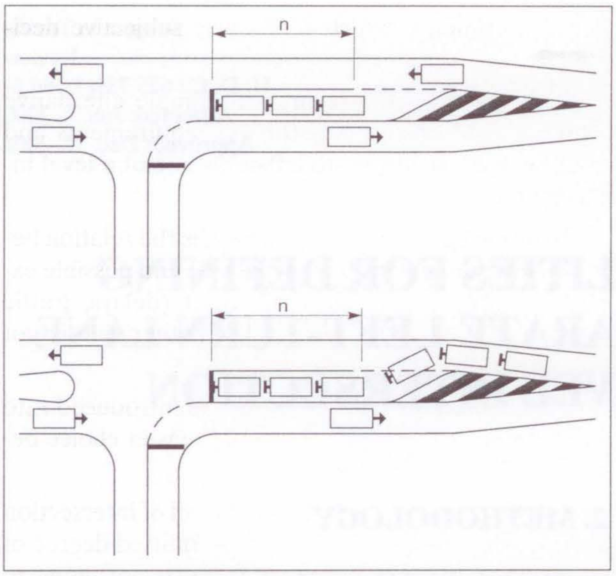

Figure 3 - Possible examples: a) there are exactly n or fewer vehicles waiting in the separate left-turning lane or b) there are more than n vehicles

In the intersection the separate left-turn lane has the capacity of k vehicles. What is the probability that the lane will not be sufficient for all those vehicles to stop which want to turn left in the series of n vehicles when arriving at the intersection? On the average p (%) of vehicles want to turn left:

$$P(W_{k \leq n}) = FW_{k(n)}$$

$$FW_{k(n)} = \sum_i \binom{w-1}{n-1} * (1-\rho)^{(w-n)} * p^n$$

where:

k – the capacity of the left-turn lane (the number of vehicles),

n – the number of vehicles in the series that arrives into the intersection,

p % – the percentage of the vehicles that turn left.

2.2. Poisson distribution

This problem can be worked out also in another way – by means of the Poisson distribution. The Poisson model is successfully used in finding faults in materials, in demanding repair services or where there is a certain input flow but arrivals are at random. The Poisson distribution describes also very well the flow of vehicles arriving into the intersection at particular, known intervals.

The course of events can be treated according to the Poisson distribution when the following items are satisfied:

a) probability depends on the duration of the time interval and the number of arrivals and not on the beginning of the interval (which is in most cases already being satisfied);

b) $p_n(t)$ does not depend on the number of arrivals before the beginning of the time interval – a flow without any consequences (already satisfied in the intersection);

c) we can always choose such an interval in which two or more vehicles have not arrived yet:

$$\lim_{t \to \infty} \frac{\sum_{i=2}^{n} p_i(t)}{t} = 0$$

Supposing that the time interval is t = 1, then the Poisson distribution gets the form of:

$$p_n = \frac{\lambda^n}{n!} * e^{-\lambda}$$

Deviation from a stationary often occurs in disturbed flows of arrivals meaning that the arrival probability depends also on the moment of the beginning of the observation ($p_n = p_n(n, t, r)$), where r stands for the beginning of the time interval t. In this case:

$$p_n(r,t) = \frac{\lambda(r,t)^n}{n!} * e^{-\lambda(r,t)}$$

which represents the Poisson rule of distribution of the probability for the non-stationary flow of arrivals.

When solving a concrete example we must first find out whether the course of events really refer to the Poisson distribution: Where the average value and the standard deviation of a stochastic process are close enough then the process can be analysed as the Poisson process.

3. OTHER POSSIBILITIES

Concrete examples that occur every day can be also worked out in another way:

- by t-distribution – with time intervals chosen at random, with a known number of arrived vehicles in a particular time interval and with a supposed condition for introducing a longer (if any) left-turn lane;
- by the simulation method – in cases when arrivals are not known and when the problem cannot be transferred into a theoretical model respectively;
- by comparing – if the treated intersection does not exist we can use the data of other similar intersections (they should be statistically processed and taken over for the considered example).

In the second part of the task the throughput capacity of the intersection must be defined by using one of the known methods for both examples:

- there are n vehicles in the separate left-turn lane,
- there are n+1 or more vehicles in the separate left-turn lane or for the probability of both examples respectively.

4. AN EXAMPLE OF A SOLUTION

On the three-arm intersection (Figure 4) the separate left-turn lane has a capacity of three vehicles. What is the probability that the separate lane will not be long enough for all those vehicles to stop which want to turn left in the series of six vehicles? On the average 30 % of vehicles want to turn left.

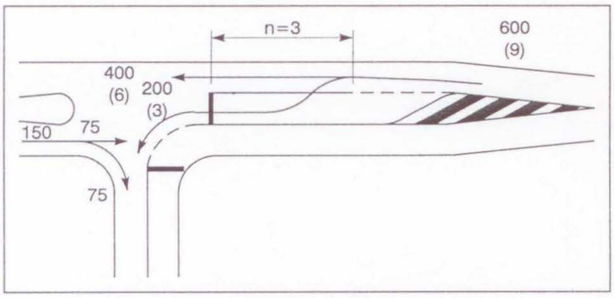

Figure 4 - Concrete example

The problem is first solved by means of the negative binomial distribution, which is used when we want to find out in which experiment k-times an advantageous event will happen:

Number of experiments until the fourth advantageous event (vehicle in the left) is $4 \leq W_4$

$$P(w_4 \leq 6) = FW_4$$

$$FW_4 = \sum_{W=4}^{6} \binom{w-1}{4-1} * (1-0.3)^{(w-4)} * 0.3^4$$

$$P(W_4 \leq 6) = 0.07$$

The problem can be worked out in another way by means of the Poisson distribution. In this case, the procedure is the following:

$$P(Y) = 6$$

$$P(z) = Fz(3) = e^{(-\lambda)} * \frac{6^z}{z!}$$

$$P(z) = Fz(3) = 0.152$$

If three vehicles arrive to the intersection, the length of the left-turn lane is sufficient.

$$P(A) = P(z4) = 1 - P(z) = 1 - Fz(3)$$

$$Y = 6, p = 0.3 - p * Y = 1.8$$

$$P(A) = 1 - 0.859 = 0.141$$

5. CONCLUSION

In order to decide whether a reconstruction on a particular intersection before being overloaded is adequate it is reasonable to carry out certain simulations of traffic flows.

On the market nowadays there are various computer programs for simulating traffic flows, which are all based on mathematics which cannot be seen by the end user. The results obtained are critically judged under this influence which may cause subjective decisions.

The article therefore presents simple alternative mathematical possibilities for the requirements and the length of the separate left-turn lane of a level intersection.

In our valid road design guidelines the relation between invested means on the one side and possible expenses owing to inadequate project (delays, traffic jams, traffic accidents, etc.) on the other side is not considered.

New ideas and models should be introduced into this field so that designers will have easier choice between different variants.

It is the client (who defines the level of intersection service) who also determines the permitted degree of probability at which the intersection is not going to work. It must be clear that the degree of permitted risk is being decreased with the importance of the intersection e.g. on highways 5%, on main roads 15%, on regional roads 30%, on local roads 50%. It is not known whether a degree of risk has been suggested or recommended anywhere in the world (apparently with the exception of the USA).

POVZETEK

Na trgu je danes več različnih računalniških programov za simulacijo prometnih tokov v nivojskih križiščih, ki temeljijo na matematičnem ozadju, ki pa je načeloma neznano končnemu uporabniku programa. Uporabnik večinoma vnaša vhodne podatke brez, da bi poznal model in matematične postopke, ki privedejo do končnih rezultatov. Končnih rezultatov tako ni možno ovrednotiti, kar pa lahko privede do subjektivnih odločitev.

V prispevku so prikazane enostavne alternativne matematične možnosti za določitev potrebe po uvedbi ločenega pasu za leve zavijalce in dolžine le-tega.

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