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Tehnološki marketing - Technological marketing  
Prethodno priopćenje - Preliminary communication  
U.D.C. 656:659:519.7  
Primljeno - Accepted: 23.02.1996.  
Prihvaćeno - Approved: 28.05.1996.

## TECHNOLOGICAL MARKETING MIX DESIGNING AS A MATHEMATICAL DECISION-MAKING PROBLEM

### DIZAJNIRANJE MIXA TEHNOLOŠKOG MARKETINGA KAO MATEMATIČKI PROBLEM ODLUČIVANJA

Dizajniranje mixa tehnološkog marketinga (TM-mix) u različitim prometnim, transportnim i telekomunikacijskim sustavima, ima bazična svojstva "problema odlučivanja" (DMP). U radu su razmatrane matematičke konstrukcije i procedure uporabljive u razvijanju TM alternativa i izboru najbolje kombinacije (kompozicije) TM-mixa. Prezentiran je algoritam za rješavanje "Općeg problema izbora". Ekspertni sustavi i druga analitička podrška odlučivanja imaju temeljnu teorijsku podršku u naznačenim konceptima, procedurama i algoritmima.

### 1. INTRODUCTION

The concept of technological marketing-mix (TM-mix) is the core concept in the theory and practice of technological (HI-Tech) marketing. In designing TM-mix for different traffic transport and telecommunication systems (services) we use conceptual scheme "7P" with instruments:

- $P_1$  - Product (Service),
- $P_2$  - Price (Tariffs),
- $P_3$  - Place (Distributions),
- $P_4$  - Promotion,
- $P_5$  - People (in Services Marketing Process),
- $P_6$  - Physical Evidence of Service,
- $P_7$  - Process.

Concept of TM suggests that TM-mix is an effective composition of instruments ( $P_1, \dots, P_7$ ) for product/service (i), selling to the user-type (j) in area (k) at time (t); with notation:

$$\text{TM-mix} = (P_1, P_2, P_3, P_4, P_5, P_6, P_7)_{i,j,k,t} \quad (1)$$

Technological marketing-mix may be selected from a great number of possibilities with qualitative and quantitative attributes and complex relations among them. De-

velopment of TM alternatives must respect different strategies for various product/technology/demand cycle stages, product (service) portfolio matrix, technology alternatives etc.

Designing TM alternatives and choice of the effective combination of TM instruments (MT-mix composition) is a very complex problem that has the basic features of a "decision-making problem". We assume that the theory of choice and decision-making gives concepts, procedures and algorithms which may be used in the development of MT alternatives and the choice of TM-mix, taking into account many properties simultaneously. Rational choice under risk or uncertainty is based on expected utility concept.

Theorems in the (normative) theory of choice and decision-making<sup>1</sup> show that it is always possible to specify separate issues or aspects in a description of objects, and to set links among them so that a choice can be described by the composition of several partial functions (problems). Basic mathematical constructs needed to describe decision-making and choice problem included: binary relation (comparison), concept of optimality, choice function, coordinate relations, hierarchical relations, and specific constructions like utility functions. Optimal control problem with aspect (criteria) has rational solution with application of  $\lambda$ -convolutions. This enables us to use the well developed methods and algorithms for "one-criterion problems" to get particular and sometimes general solutions. [2], [3]

### 2. THEORETICAL BACKGROUND AND DEFINITIONS

The theory of decision-making and rational engineering deals with mathematical models of decision-making, concepts and algorithms, which are relevant for different practical situations and problem. "Optimal" decision (effective and efficient) allows goals to be reached with minimal expenditure of resources. In the simplest cases the decision maker solves the problem directly, without

any special procedure or algorithm. However, mathematical models and methods are often required to help the decision maker find effective and efficient decisions for complex problems.

"Classical" methods in searching for optimal solutions are studied in mathematical and operational researches. A solution is mathematical object for which a given function has an extreme, often made with respect to one aspect or criterion.

*Statement 1:* In "decision-making problems" (DMP) alternatives and the decision must be evaluated from different points of view, which may involve different dimensions, different factors (technological, marketing, economics,...) or different functions; models for DMP must respect several aspects or criteria, with optimality principle which is not fixed.

Previous statement comprises the basic features of a decision for which we can use the following formal definitions.

*Definition 1:* A decision-making problem is a pair (A, OP) where A is a set of alternatives or variants, and OP is an optimality principle. The solution to (A, OP) is the set  $A_{OP} \subseteq A$ , selected by the optimality principle OP.

The general optimization problem does not necessarily assume the maximization of any numeric function.

The information about the set A and optimality principles OP, classify decision-making problems. In "general DMP" both A and OP may be unknown and information required to find the solution  $A_{OP}$  is extracted during the solution process.

*Definition 2:* A "choice problem" is a special case of general DMP where set of alternatives is known.

*Definition 3:* A "general optimization problem" is a special case of a general DMP where set of alternatives A and optimality OP are known.

The process of solving the problem (A, OP) can be divided into two main phases: generating of alternatives and then solving the choice problem. When generating the set A, the feasibility of each alternative must be considered according to specific constraints of the problem under consideration.

The problem of generating A may be treated as the specific choice problem (A<sub>U</sub>, OP<sub>1</sub>), where A<sub>U</sub> is the universal set of all imaginable alternatives and OP<sub>1</sub> is an optimality principle for the feasibility of the alternatives. The feasibility conditions are defined by the technical, technological, economical or other constraints.

A formal representation in choice theory is based on binary relations (R), coordinate relations on Euclidean space  $E^m$  (m-dimensional), hierarchical relations, construct of decomposition and expected utility concept. All these concepts and construct are interlinked which allows us to describe the results of arbitrary choice from a finite set<sup>4</sup>. Axiomatic foundations imply asymmetric, transitive and acyclic relations.

Concepts of particular and mathematical choice problems, procedures and algorithms for decision making, methods for solving estimation problems and optimizing utility function - are the main contributions for decision-making implementation. Decision making under risk and under uncertainty applies the operator "expected utility" defined on a set of outcomes.

Practical modeling and description of alternatives are impossible without methods for solving the following problems:

- constructing possible and feasible alternatives,
- forming sets of aspects which are essential for estimating,
- constructing a criterial space,
- ordering alternatives in terms of aspects,
- finding the mapping of A into criterial space  $E^m$ .

*Definition 4:* Estimation problem is the operation of assigning of a vector from m-dimensional space  $E^m$  to a system.

We can represent an estimation problem as a decision-making problem (A, OP). It is defined by the choice function:

$$C_{OP}(X) = \begin{cases} a & \text{if } a \in X \subseteq A \\ 0 & \text{if } a \notin X \subseteq A \end{cases} \quad (2)$$

where a is a system's estimate and a solution of (A, OP).

Estimation problems arise at different stages of a decision-making process. They may be solved directly by the decision maker or with help of consultants, decision analysts or experts with special knowledge and experience.

### 3. DESIGNING ALTERNATIVES AND TM-MIX

Developing technological marketing alternatives (instruments) is suggested by the concept of TM-mix and TM methodology (methodic). [1], [4]

Analysis of the concrete situation and TM strategy are the input for TM-mix planning work (program). The TM-mix concerns the "seven P-s" for traffic, transport and telecommunication services (products/service, price, place, promotion, people, physical evidence, process). The term "mix" has a recipe connotation, implying that these instruments are adjusted depending on the market that is being targeted with adequate technology.

Figure 1 shows how strategic TM planning works according to hierarchical representation.

*Statement 2:* Developing TM alternatives as a decision-making problem, has the following steps:

- (1) Creating an initial set (list) of alternatives (ISA),
- (2) Refining the list of initial alternatives removing any that are nonfeasible,
- (3) Preparing a final set for the evaluation.



Figure 1

Creating and formulating the initial set of alternatives can be assisted by:

- the use of creativity-enhancing/problem-solving approaches, like group dynamic sessions, "brainstorming", expert procedures, etc.,
- the review of major strategy concepts (product/technology/demand life cycle, SWOT, etc.),
- DSS (Decision Support System) tools.

Without a priori information about the properties of alternatives, a universal set of all imaginable alternatives  $A_u$  is used. With such ISA, the choice problem (phase) will be complex and may necessarily have a solution in all cases. To avoid such situations, some of the possible alternatives  $A$  must be selected from  $A_u$ . We can assume:

$$A = C_{OP1}(A_u) \quad (3)$$

where  $C_{OP1}$  are the choice functions identifying the alternatives to the set of the possible ones. In selecting nonfeasible alternatives, the relevant questions are:

- Are the alternatives presented feasible and practical? (Limited resources, legal restrictions, technological restrictions)
- Are the alternatives reasonable given the situation presented?
- Are they consistent with the goals and objectives of the company?
- Is the list of alternatives collectively exhaustive?

Whatever the structure of TM problem, criteria are needed to evaluate the alternatives ( $\rightarrow$ rank them), ac-

ording to how well they satisfy the decision-maker's objectives. Because most TM decisions involve more than one objective, we must usually deal with multiple criteria TM decision problems. Criteria must be defined in a proportional context, relative to the specific objectives being sought.

*Definition 5:* An optimal control (management) problem using multiple criteria is represented by a triple  $(U, \phi, R)$ , where  $U$  is the set of controls,  $\phi$  is the mapping of  $U$  into the space of gains  $E^m$ , and  $R$  is a binary relation of  $E^m$  by which gains can be compared.

The "optimal control problem with multiple criteria" may be formulated as: find all or some  $u^* \in U$  such that:

$$\phi(u^*) \in C_{OP}(\phi) \quad (4)$$

where a vector  $\phi(u^*) \in E^m$  is interpreted as the gain delivered by a control  $U$ ; and the domain of the optimality principle is the set of gains  $\phi = \phi(U) \subseteq E^m$ .

*Statement 3:* A problem  $(U, \phi, R)$  with multiple criteria can be reduced to a one-criterion problem by using the concept of separability and  $\lambda$ -separability; the problem has a solution if  $\phi(U)$  is closed, bounded set in  $E^m$  and  $R$  is a  $\lambda$ -separable relation.

A proof for the statement is presented in reference literature [3]. The application of  $\lambda$ -convolution is the basic approach towards the solution of optimal control problems with multiple criteria. This enables us to use developed methods and algorithms for one-criterion problems to get particular, or sometimes general solutions to a problem  $(U, \phi, R)$ .

In many dynamic TM problems with multiple criteria, a control  $u \in U$  is a finite or infinite collection  $u = \{u_0, u_1, u_2, \dots\}$  of TM actions. These control (management) problems may be reduced to a sequence of simpler problems each associated with the particular action  $u_0, u_1, u_2, \dots$ . In all cases the crucial point is that  $\varphi(u)$  is the sum (the integral) of gains given by particular controls and relation  $R$  which is transfer invariant.

*Statement 4:* The optimality control  $u^* \in U$  may be expressed by binary relation  $R$  on  $E^m$  as follows:

$u^*$  is  $R$  - optimal if:

$$\varphi(u) \bar{R} \varphi(u^*) \quad (\bar{R} - \text{the dual relation to } R)$$

is true for all  $u \in U$ , where:

$$\varphi(u) = (\varphi_1(u), \dots, \varphi_m(u)).$$

A majority relation and a Pareto relation<sup>3</sup> are normally chosen for  $R$ .

#### 4. CHOICE PROBLEM AND ALGORITHM FOR SOLVING GENERAL CHOICE PROBLEM

In the choice phase, the TM decision maker has a set of feasible alternatives and a set of evaluation criteria. Evaluating of the final list of alternatives involves measuring; trading off, or scoring them in terms of the specified criteria. Several complexities are included in problems:

- there are multiple criteria and multiple alternatives,
- there may be a fairly large number of criteria and sub-criteria,
- all criteria are not equally important to the decision makers,
- some of the criteria may be quantitative while others may be qualitative.

Advanced decision-analysis tools (decision-support system) have the above characteristics. The methodology of technological marketing for managerial (TM) analysis includes highly flexible and versatile decision-analysis tools like "Analytic Hierarchy Process" which incorporate both objective and subjective factors in evaluating alternatives and arriving at a decision. Other multi-criteria decision-making methodologies (such as goal programming and other) cannot handle subjective considerations.

The main components of the solving choice problem are:

- (1) Using quantitative analysis for objective criteria (mathematical models for decision support, data analysis, optimization models, heuristic, simulations...);
- (2) Using qualitative analysis with subjective criteria (expert judgment, expert support systems...);

- (3) Merging quantitative and qualitative analysis (multi-criteria decision analysis framework, such as AHP);
- (4) Performing a synthesis of the AHP model (the result is a single measure of "score" of the overall utility);
- (5) Conducting a sensitivity analysis of the AHP model (sensitive to possible changes in the weights of the criteria);
- (6) Checking the analytical result against your intuition (decision is a good one if you will be able to explain and justify it).

In the next paper we will present a review of Analytic Hierarchy Process as a practical model (tool) for Complex TM decisions. In this chapter we analyze the algorithm for solving general choice problem including complex constructions like compositions.

A solution to choice problem is obtained by reducing it to partial and simple problems using the choice function decomposition. The decompositions<sup>2</sup> of choice function will be treated as its equivalent representation in terms of a set of other choice functions whose composition gives the initial choice function. A choice function is the most general concept; the next level downwards is the binary relation and the coordinate relation. All these constructs must be interlinked, which allows us to describe the results of arbitrary choices from a finite set in terms of any of them. Concept of (general) decomposition makes these links explicit.

*Statement 5:* Choice function  $C^*$  is normal if it is generated by binary transitive and acyclic relation  $R \subseteq A^2$ . In this case we have:

$$C^R(X) = X^R$$

where  $X^R$  is the set of elements dominated in terms of  $R$ . The function  $C^R(X)$  is called a preference with dual relation "blocking".

We assume that  $R_1, \dots, R_k$  are binary relations on  $A$ , and that  $\psi(\gamma_1, \dots, \gamma_k)$  is a Boolean function. We define a choice function  $C = C(R_1, \dots, R_k, \psi)$  on  $A$  as:

$$x \in C(X) \Leftrightarrow \psi(\gamma_1, \dots, \gamma_k) = 1 \tag{5}$$

$$\text{where } \gamma_i = \begin{cases} 1 & \text{if } x \in C^{R_i}(X) \\ 0 & \text{if } x \notin C^{R_i}(X) \end{cases}$$

Choice function  $C = C(R_1, \dots, R_k; \psi)$  defined by formula (5) is said to be a  $\psi$ -composition of the normal choice functions generated by the binary relations  $R_1, \dots, R_k$ . In [3] there is a proof for the theorem that for a given arbitrary choice function  $C$  on  $A$ , binary relations  $R_1, \dots, R_k$  exist and a Boolean function  $\psi$  such that:

$$C = C(R_1, \dots, R_k; \psi) \tag{6}$$

Thus, an arbitrary choice function, irrespective of its complexity or to which classes it belongs, may always be decomposed (expanded) into functions generated by binary relations. Constructs applicable to any choice functions will be called general decomposition, while

those only applicable to specific choice functions will be called partial or special decomposition. [3]

When  $B^*=0$  and  $M^*=M(A)$ , information about the function  $C$  is absent. When  $B^*=B(A)$  the function  $C$  is completely described. We can say that the description of the function  $C$  is given by the pair  $(M^*, B^*)$ .

**Definition 6:** Mathematical choice problem is the triple  $(A, M^*, B^*)$ , where  $A$  is the set of alternatives,  $C \in M^* \subseteq M(A)$ , and  $C(X)$  is given for all  $X \in B^* \subseteq B(A)$ .

A solution to  $(A, M^*, B^*)$  is the set  $A^* = C(A)$ . A problem is solvable if  $A^*$  is uniquely defined with the set  $M^*$  and values of  $C$  for all  $X \in B^*$ . If  $A \in B^*$ , the problem is solvable with solution  $C(A)$ .

In relation to the general choice problem  $(A, OP)$ , we shall call  $(A, OP)$  a simple problem if its solution  $A_{OP}$  can be found immediately and directly without a special algorithm. In the case when the problem is not a simple one we need to be able to reduce it to a mathematical choice problem. We must find a set  $M^*$  containing  $C_{OP}$  and  $B^*$  which is the set of all  $X \subseteq A$  for which  $(X, OP)$  is a simple one with the same optimality principle. If the resultant mathematical problem is solvable, then its solution is the solution to the initial problem  $(A, OP)$ .

When a complex problem is dealt with, a decomposition is necessary.

**Definition 7:** We define  $(A, OP_i)$  as a particular choice problem, where  $OP_i$  is the optimality principle for making a choice from  $A$  using the  $i$ -th aspect.

We formulate the particular problem  $(A, OP_i)$  and  $\psi$ -composition of the  $C_{OP_i}$  ( $i=1, n$ ) functions. A solution of the initial problem  $(AT_{OP})$  is the set  $A_{OP} = C(A)$ , where  $C = C(C_{OP_1}, \dots, C_{OP_n}; \psi)$ . This construction discovers the basic aspects affecting the choice and how they are related to each other. [3]

**Algorithm 1:** Algorithm for Solving a General Choice Problem

- (1) Examine whether the problem  $(A, OP)$  is a simple one. If so, then stop. Otherwise go to Step 2.
- (2) Formulate the mathematical choice problem  $(A, M^*, B^*)$ , corresponding to the initial problem.
- (3) Verify whether  $(A, M^*, B^*)$  is solvable. If so, then stop. Otherwise go to Step 4.
- (4) Formulate the particular problems  $(A, M^*, B^*)$ , ( $i=1, n$ ).
- (5) Set  $i=0$ .
- (6) Set  $i=i+1$ .
- (7) Verify whether  $(A, OP_i)$  is a simple one. If so, then find  $A_i = C_{OP_i}(A)$  and go to Step 12, otherwise go to Step 8.
- (8) Formulate the mathematical choice problem  $(A, M_i^*, B_i^*)$  corresponding to the particular choice problem  $(A, OP_i)$ .
- (9) Verify whether  $(A, M_i^*, B_i^*)$  is solvable. If so, then find  $A_i = C_{OP_i}(A)$  and go to Step 12. Otherwise go to Step 10.

(10) Find  $\bar{M}_i^* \subseteq M_i^*$  and  $\bar{B}_i^* \subseteq B_i^*$  such that at least one inclusion is rigorous.

(11) Set  $M_i^* = \bar{M}_i^*$ , and  $B_i^* = \bar{B}_i^*$  and go to Step 9.

(12) If  $i \neq n$ , then go to Step 6, otherwise go to Step 13.

(13) Define the  $\psi$ -composition of the choice function  $C_{OP_i}$  from the particular problems ( $i=1, n$ ).

(14) Find  $A_{OP}$  from

$$x \in A_{OP} \Leftrightarrow \psi(\gamma_1, \dots, \gamma_n) = 1$$

$$\text{where } \gamma_i = \begin{cases} 1 & \text{if } x \in A_{OP_i} \\ 0 & \text{otherwise} \end{cases}$$

Solving a choice problem in TM by Algorithm 1, one of the following possibilities occurs:

- the initial problem  $(AT_{OP})$  is simple,
- the mathematical choice problem  $(A, M^*, B^*)$  is solvable,
- the mathematical choice problem  $(A, M^*, B^*)$  is not solvable.

For the simple TM problems, Steps 1-3 are sufficient. Complex TM problems need special procedures for solving mathematical choice problems (Steps 3 and 9) and a procedure for constructing a  $\psi$ -composition of the  $C_{OP_i}$  choice functions (Step 13). If the functions  $C_{OP_1}, \dots, C_{OP_n}$  correspond to the normal components of  $C_{OP}$ , then the mathematical choice problems  $(A, M^*, B^*)$  at Step 8 are solvable and there is no necessity to find  $\bar{M}_i^*$  and  $\bar{B}_i^*$ .

In the same cases the solution  $A_{OP}$  may be empty. This means that none of the variants in  $A$  are satisfactory to the decision maker.

The general algorithm is open to many modifications depending on the TM problem under consideration. We can formulate choice problems with utility function where aspects may be transformed to criteria. In many traffic problems, the elements of  $A$  are random quantities and problem has probability properties.<sup>4</sup>

## CONCLUSION

Designing technological marketing-mix is a very complex problem with the basic feature of "decision-making problem". The theory of choice and decision-making gives concepts, mathematical models and procedures with operational significance<sup>5</sup>. Application of the various forms of rational decision-making and decision support tools may improve TM decision, if the situations are correctly specified and the procedure correctly applied.

Usefulness of decision engineering rested primarily not on the kind of direct empirical confirmation but on the possibility to produce TM decisions that are less irrational than the unsystematized action of an intelligent decision maker.

Modern decision support (DSS) and Expert Systems can use mathematical models, concepts and algorithms developed in theory of decision-making and decision engineering.

**SUMMARY**

Designing technological marketing mix (TM-MIX) in different traffic, transport and telecommunication systems has a basic feature of a "decision-making problem" (DMP). The paper discusses mathematical constructs and procedures applicable in developing TM alternatives and in the choice of the best combination (composition) of TM-mix. Algorithm for solving a "General Choice Problem" is presented. Expert Choice Software and other decision-support tools have substantial theoretical support in the denoted concepts, procedures and algorithms.

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**ENDNOTES**

1. Normative (or prescriptive) decision theory is based on specific axiom systems like any other mathematical system. Descriptive decision theory is highly empirical and concerned with how and why people think and act the way they do.
2. Decomposition (expansion) of ordinary functions of real or complex variables into a power or a Fourier's series are well known for the engineers. In our context we can develop similar constructions to represent choice functions and for solving complex TM decision-making problems.
3. With Pareto relation  $P$  Pareto's set on  $A \subseteq E^m$  is the set  $A^P = \{x \in A : (\forall y \in A)[yPx]\}$
4. These problems will be analyzed in other papers.
5. Apart from the presented concepts, theory of choice and decision-making considers several rationality concepts: bounded rationality, alternative rationality, contextual rationality, game rationality, adaptive rationality.

**CONCLUSION**

Designing technological marketing mix is a very complex problem with the basic feature of "decision-making problem". The theory of choice and decision-making gives concepts, mathematical models and procedures with operational significance. Application of the various forms of rational decision-making and decision support tools may improve TM decision if the denoted are correctly specified and the procedure correctly applied.

Usefulness of decision engineering tested primarily not on the kind of direct empirical confirmation but on the possibility to produce TM decisions that are less irrational than the unassisted action of an intelligent decision maker.

Algorithm 1: Algorithm for Solving a General Choice Problem

(1) Examine whether the problem (A, OP) is a simple one. If so, then stop. Otherwise go to step 2.

(2) Formulate the mathematical choice problem (A, M\*, B\*) corresponding to the initial problem.

(3) Verify whether (A, M\*, B\*) is solvable. If so, then stop. Otherwise go to step 4.

(4) Formulate the particular problems (A, M\*, B\*) (i=1, n).

(5) Set i=0.

(6) Set i=i+1.

(7) Verify whether (A, OP) is a simple one. If so, then find  $A^P = C_{OP}(A)$  and go to step 12, otherwise go to step 8.

(8) Formulate the mathematical choice problem (A, M\*, B\*) corresponding to the particular choice problem (A, OP).

(9) Verify whether (A, M\*, B\*) is solvable. If so, then find  $A^P = C_{OP}(A)$  and go to step 12. Otherwise go to step 10.