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INFLUENCE OF LATERAL VEHICLE ACCELERATION ON DISTRIBUTION OF BRAKING FORCES AS CAUSES OF TRAFFIC ACCIDENTS

SUMMARY

Considering previous calculation of vehicle speeds negotiating curves, in this paper we are trying to define and determine lateral accelerations which occur when a vehicle negotiates a curve. We have also analysed how the speed of negotiating a curve influences the braking force, which is in many traffic accidents the cause of the accident, if the braking system is not adjusted to the speed (and vice versa).

1. INTRODUCTION

With regard to calculation of vehicle speeds negotiating the curve, and in order to determine the stability conditions of the driving axle, and defining of the relatively simple and for practice acceptable method of analytical determining of the speed at which the vehicle negotiated the curve, let us consider the model of a vehicle shown in Figure 1, for the limiting case of stationary vehicle movement along a horizontal curve. Therefore, in the limiting case of skidding, the rear driving axle will be loaded by the overall traction force needed for the stationary way of vehicle driving through a curve, which is by means of the differential gears practically equally distributed to the outer and inner wheel of the driving axle, and by the centrifugal force component belonging to the driving axle. This means that in the described case the outer wheel of the driving axle will be loaded by the overall force:

$$(G_3 \cdot \mu_3)^2 = \left(\frac{F_v}{2}\right)^2 + B_3^2 \qquad (1)$$

and the inner wheel of the same axle by the overall force:

$$(G_4 \cdot \mu_4)^2 = \left(\frac{F_v}{2}\right)^2 + B_4^2 \qquad (2)$$

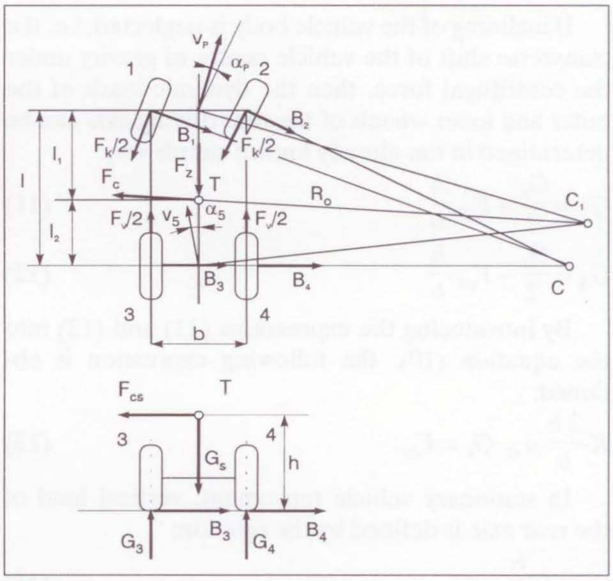

Figure 1

where the centrifugal force component, which in the limiting case can be transferred via the driving axle to the base, has to equal:

$$B_3 + B_4 = F_{cs} \qquad (3)$$

i.e. after squaring:

$$B_{33}^2 + 2 \cdot B_3 \cdot B_4 + B_4^2 = F_{cs}^2 \qquad (4)$$

By introducing expression (1) and (2) into the equation (4) it follows:

$$(G_3 \cdot \mu_3)^2 + (G_4 \cdot \mu_4)^2 - 2\left(\frac{F_v}{2}\right)^2 = F_{cs}^2 - 2 \cdot B_3 \cdot B_4 \qquad (5)$$

By analysing the results of studying the stability of movement of variously designed vehicles, the expression (5) can be transformed into the analytically more adequate form, if these substitutions are introduced

$$(G_3 \cdot \mu_3)^2 + (G_4 \cdot \mu_4)^2 = G_3^2 \cdot \mu_{st}^2 \cdot K^3 \qquad (6)$$

$$2\left(\frac{F_v}{2}\right)^2 = G_4^2 \cdot \mu_{st}^2 \cdot K^3 \tag{7}$$

$$2 \cdot B_3 \cdot B_4 = F_{cs}^2 \left(1 - \mu_{st} \cdot K^2\right) \tag{8}$$

where coefficient "K" is the function of the lateral vehicle acceleration, height of the centre of gravity, spacing between the wheels and value of the static friction between the pneumatics and the road, according to the expression:

$$K = \frac{b \cdot k_b}{2 \cdot h \cdot \mu_{st}} \tag{9}$$

If substitutions (6), (7), and (8) are introduced into the expression (9), and after having done other necessary operations, the analytically very suitable expression is obtained:

$$\mu_{st} \cdot K \cdot \left(G_3^2 - G_4^2\right) = F_{cs}^2 \tag{10}$$

If inclining of the vehicle body is neglected, i.e. the transverse shift of the vehicle centre of gravity under the centrifugal force, then the dynamic loads of the outer and inner wheels of the rear driving axle can be determined in the already known simple way:

$$G_3 = \frac{G_s}{2} + F_{cs} \cdot \frac{h}{b} \tag{11}$$

$$G_4 = \frac{G_s}{2} - F_{cs} \cdot \frac{h}{b} \tag{12}$$

By introducing the expressions (11) and (12) into the equation (10), the following expression is obtained:

$$K \cdot \frac{2 \cdot h}{b} \cdot \mu_{st} \cdot G_s = F_{cs} \tag{13}$$

In stationary vehicle movement, vertical load of the rear axle is defined by the relation:

$$G_s = G \frac{l_1}{l} \tag{14}$$

On the other hand, the value of the centrifugal force belonging to the front and rear vehicle axle, depends on a number of factors, i.e. on the position of the centre of weight, rigidity of the car body, rigidity of the suspension and pneumatics, etc. However, studies have shown that in practice one can consider with sufficient accuracy that the centrifugal force is distributed on the vehicle axles in the same way as the vehicle weight, so that the component of the centrifugal force, belonging to the rear driving axle, is defined by the following relation:

$$F_{cs} = F_c = \frac{l_1}{g} \frac{G}{l} \cdot \frac{v^2}{R_0} \cdot \frac{l_1}{l} \tag{15}$$

By substituting the expressions (14) and (15) into the equation (13), the final expression for the value of the limiting speed is obtained, i.e. the speeds of the vehicle negotiating the curve:

$$v = v_{gran} = \sqrt{K \cdot \frac{2 \cdot h}{b} \cdot \mu_{st} \cdot g \cdot R_0} \tag{16}$$

Statistical analysis of 497 types of vehicles has shown that the relation between the axle spacing and the wheels spacing in modern passenger cars, amounts to an average of:

$$-\frac{l}{b} = 1.84 \text{ with the standard deviation of } \pm 0.135 \tag{17}$$

whereas the relation between the vehicle centre of gravity height and axle spacing ranges as follows:

$$\frac{h}{l} = 0.19 - 0.22 \tag{18}$$

The average coefficient "K" value for the passenger vehicles can be determined by means of the empirical expression:

$$K = e^{\left(0.59 \cdot \mu_{st}^2 - 0.96 \cdot \mu_{st} + 0.24\right)} \tag{19}$$

However, this calculation can be even further simplified, with error less than 4% if the following is calculated for the passenger vehicles:

- in the range of static friction value $\mu_{st} > 0.6$, with coefficient K = 0.85,
- in the range of static friction value $0.2 < \mu_{st} \leq 0.6$, with coefficient K = 0.95,
- in the range of static friction value $\mu_{st} \leq 0.2$, with coefficient K = 1.15.

According to the suggested method, the limit speed of a vehicle negotiating a curve, for dry asphalt surface condition and radius of curvature $R_0 = 40\,m$, would amount to about:

$$v_{gran} = \sqrt{0.85 \cdot 2 \cdot 0.21 \cdot 1.84 \cdot 0.90 \cdot 40} = 15.2 \text{ m/s} = 55 \text{ km/h} \tag{20}$$

for dry asphalt surface condition and radius of curvature $R_0 = 80\,m$, it would amount to about:

$$v_{gran} = \sqrt{0.85 \cdot 2 \cdot 0.21 \cdot 1.84 \cdot 0.90 \cdot 9.81 \cdot 80} = 21.5 \text{ m/s} = 78 \text{ km/h} \tag{21}$$

and for the wet asphalt surface condition, and radius of curvature $R_0 = 80\,m$, it would amount to about:

$$v_{gran} = \sqrt{0.95 \cdot 2 \cdot 0.21 \cdot 1.84 \cdot 0.60 \cdot 9.81 \cdot 80} = 18.6 \text{ m/s} = 67 \text{ km/h} \tag{22}$$

For comparison, and for the same above mentioned conditions, the limit speeds of a vehicle negotiating a curve have been calculated, for the case of ideally considering the vehicle as a material point, as follows:

- for dry asphalt surface conditions and radius of curvature $R_0 = 40\,m$

$$v_{gran_o} = \sqrt{0.90 \cdot 9.81 \cdot 40} = 18.8 \text{ m/s} \quad (68 \text{ km/h}) \tag{23}$$

- for dry asphalt surface conditions and radius of curvature $R_0 = 80\,m$

$$v_{gran_o} = \sqrt{0.90 \cdot 9.81 \cdot 80} = 26.6 \text{ m/s} \quad (96 \text{ km/h}) \tag{24}$$

- and for wet asphalt surface condition and radius of curvature $R_0 = 80\,m$

$$v_{gran_o} = \sqrt{0.60 \cdot 9.81 \cdot 80} = 21.7 \text{ m/s} \quad (78 \text{ km/h}) \quad (25)$$

By comparing the results (23), (24), and (25), with the results (20), (21) and (22), it is obvious that the values of passing speeds through curves have been calculated based on the idealised observing of vehicle as a material point, greater compared to the actual condition by about 23.7% on dry asphalt surface, i.e. by about 16.7% on wet asphalt surface, which is undoubtedly a big difference.

2. DISCUSSION

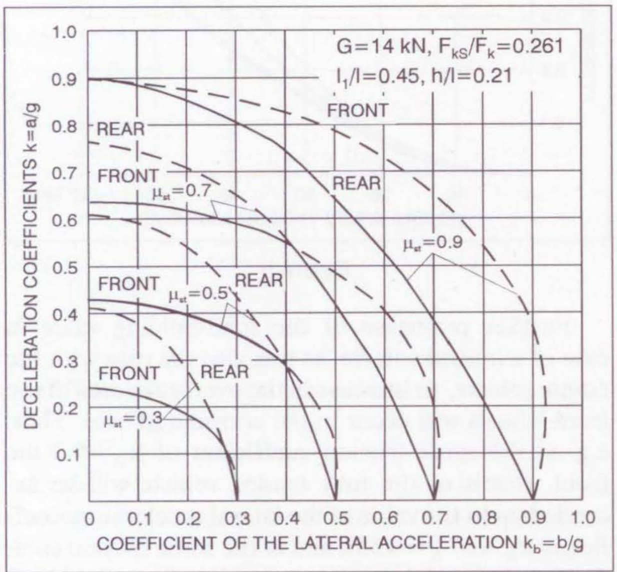

Figure 2 - Limits of road-holding features of the front and rear wheels in ready-for-movement condition with various road surface conditions for the installed braking system with linear characteristic

Figure (2) presents the front and rear wheels road-holding limits of a standard vehicle ready to start moving in various road surface conditions, i.e. at different maximum values of the road-holding coefficient. The installed braking system on the considered vehicle has a linear characteristic. Knowing, therefore, the road-holding coefficients for the front and rear wheels, we can also obtain the data on the maximal values of lateral acceleration and deceleration of the vehicle, and assess which wheels will get blocked while braking in the curve. This means that conditions may be assessed under which the vehicle will be brake-stable or brake-unstable. Considering Figure (2) it may be noticed immediately that the presented vehicle will exhibit blockage of the rear wheels already at the slightest occurrence of lateral acceleration, at the static friction of $\mu_{st}=0.9$. Thus, under conditions of intensive braking in curve, with static friction between the pneumatics and the surface of $\mu_{st}=0.9$, the considered vehicle will be brake-unstable at all values of lateral acceleration. However, at lower values of static friction coefficients, at which the installed braking system insured overbraking of the front wheels to a greater extent, and in the case of braking in curve, this braking system, as seen in Figure (2), will also result in overbraking of the front wheels, which means also brake-stable condition of the vehicle up to the certain values of the lateral vehicle acceleration. So, e.g. at the static friction coefficient of $\mu_{st}=0.7$, the vehicle will be brake-stable when braking in the curve, i.e. the front wheels of the considered vehicle will overbrake up to the value of the lateral acceleration, that is the coefficient of brake acceleration $k_b = b/g = 0.408$. At higher values of lateral acceleration, the installed braking system on the considered vehicle will result in overbraking of the rear wheels, that means in brake-unstable condition of the vehicle.

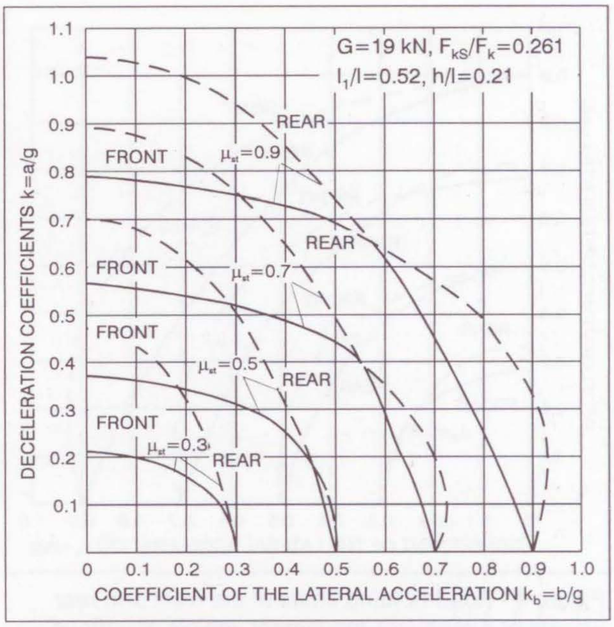

Figure 3 - Road-holding limits of the front and rear wheels of a fully loaded vehicle in different road surface conditions for the installed braking system with linear characteristic.

Also, as seen in Figure (2), further reduction of the friction coefficient value will result in increase of the overbraking area of the front wheels of the considered vehicle while braking in the curve. This means that by reduction of the friction coefficient value, the stability of braking in curve of the considered vehicle will increase. Thus e.g. at the static friction coefficient of $\mu_{st}=0.5$ the considered vehicle will be brake-stable, i.e. the front wheels will overbrake up to the value of lateral acceleration, that is the value of lateral acceleration coefficient value of $k_b = b/g = 0.4$. At the static friction coefficient of $\mu_{st}=0.3$, the front wheels of the considered vehicle will overbrake up to the value of the lateral acceleration coefficient of $k_b = 0.279$.

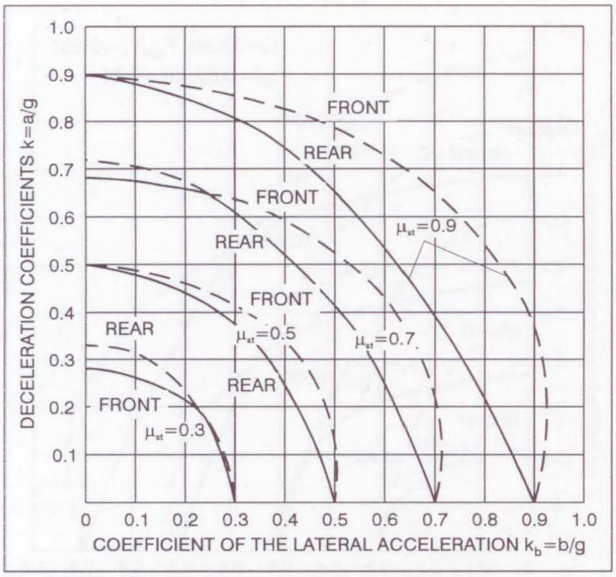

Figure 4 - Road-holding limits of the front and rear wheels of a ready-for-movement vehicle at various conditions of the road surface for the installed braking system with bi-linear characteristics

Figure 4 presents the relation between the maximal lateral acceleration and deceleration of the vehicle for the loaded vehicle. Thus, e.g. by comparing Figures (2) and (3) it follows that with the static friction coefficient of $\mu_{st}=0.9$, the empty vehicle, at all values of lateral acceleration will be brake-unstable, that is the rear wheels will overbrake. On the contrary, loaded (full) vehicle, as seen in Figure 4, will be brake-stable, that is the front wheels will overbrake up to the value of lateral acceleration value of the vehicle, $k_b = b/g = 0.578$.

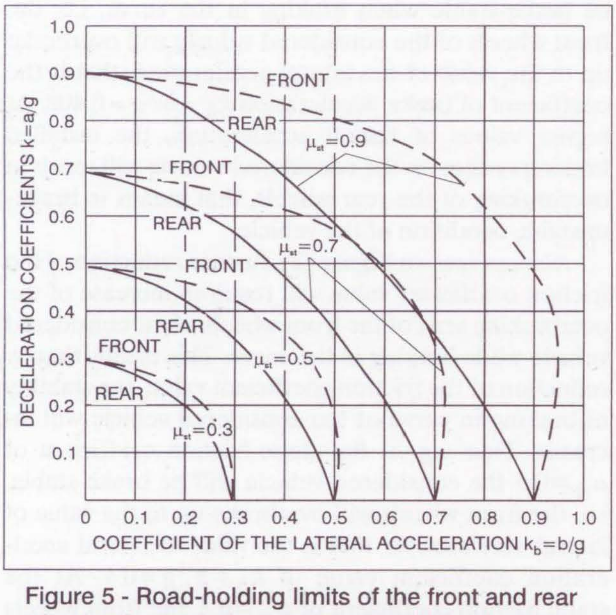

Figure 5 - Road-holding limits of the front and rear wheels of a ready-for-movement vehicle at various road surface conditions for the braking system with the ideal distribution of braking forces per vehicle axles.

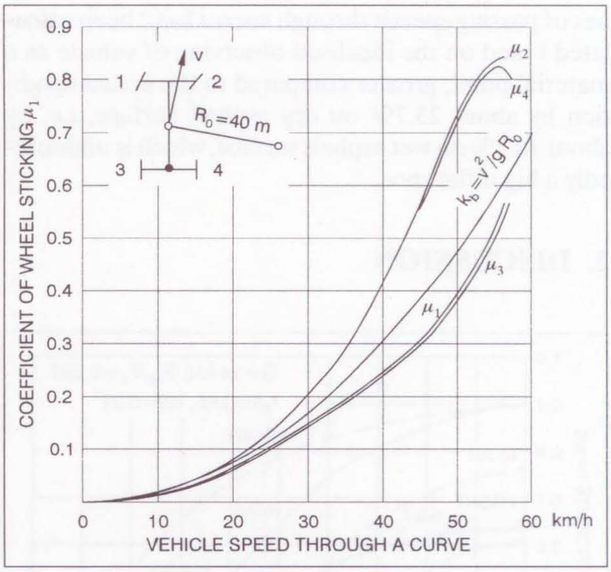

Figure 6

Further reduction of the road-holding value in case of a loaded vehicle, as was also the case with the empty vehicle, an increase in the overbrake area of the front wheels will occur in the considered case. Thus, e.g. at the static friction coefficient of $\mu_{st}=0.7$ the front wheels of the fully loaded vehicle will be exceeded up to the value of the lateral acceleration coefficient $k_b = b/g = 0.555$, and at the static friction coefficient of $\mu_{st}=0.5$ the front wheels will overbrake up to the value of lateral acceleration coefficient $k_b = 0.449$, and at the static friction coefficient of $\mu_{st}=0.3$ the front wheels of the fully loaded vehicle

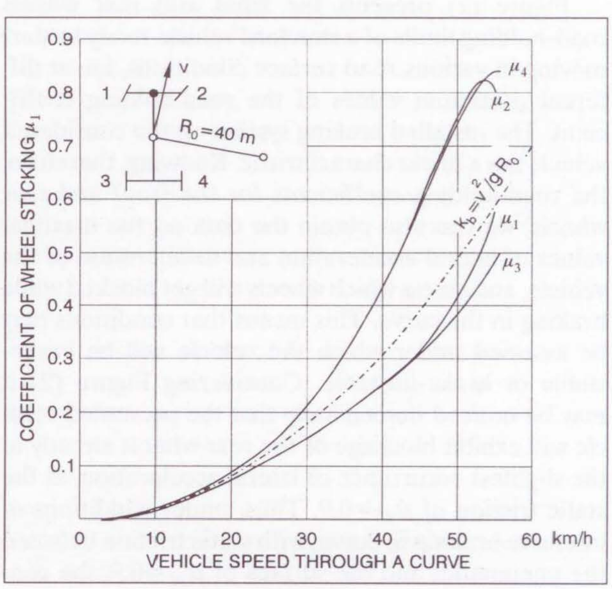

Figure 7

will overbrake up to the value of the lateral acceleration coefficient $k_b = 0.289$.

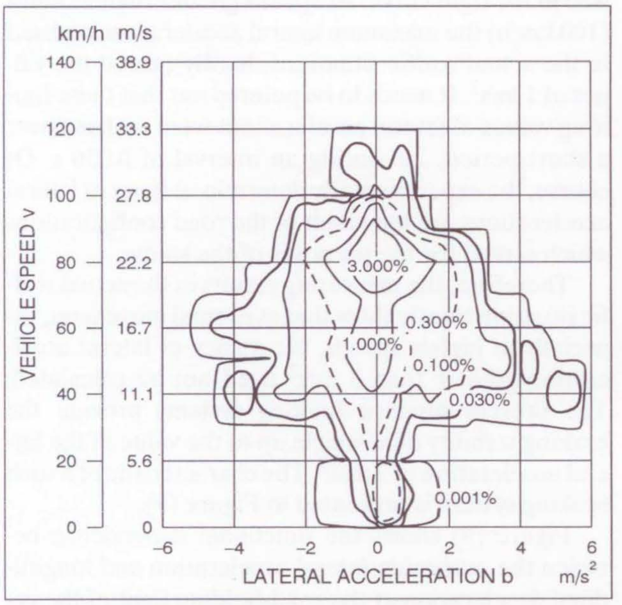

Figure 8 - The frequency of lateral accelerations in studying the vehicles in actual traffic situations

Therefore, it is obvious that the increased load of the vehicle benefits its brake-stability since this increase in the installed braking system with linear characteristic will result in an increase of the overbrake area of the front wheels.

This analysis has shown that the braking system with linear characteristic by its great store of braking stability shows satisfactory features even in the case of braking in the curve, regarding insuring the braking stability of the vehicle at the usual values of lateral acceleration, especially in case of decreased values of the road-holding coefficients, i.e. during braking in curve on a wet and slippery road surface.

Figure (4) shows the road-holding limits of the front and the rear wheels of an empty vehicle ready to start moving, in case of an installed braking system with a bilinear characteristic. By comparing the Figures (1) and (9), i.e. by comparing the road-holding limits of the front and the rear wheels in case of the installed braking system with a bilinear characteristic, it is obvious that the vehicle with the installed braking system with the bilinear characteristic, when braking in curve, will exhibit lower braking stability in relation to the case of installed braking system with the linear characteristic. Thus, e.g. at the static friction coefficient of $\mu_{st} = 0.7$, the braking coefficient with the linear characteristic provided in the considered case overbrake of the front wheels, that is, braking stability, up to the value of lateral acceleration coefficient of $k_b = 0.408$, while the braking system with the bilinear characteristic, as seen in Figure (4), will provide overbrake of the front wheels up to the value of the lateral

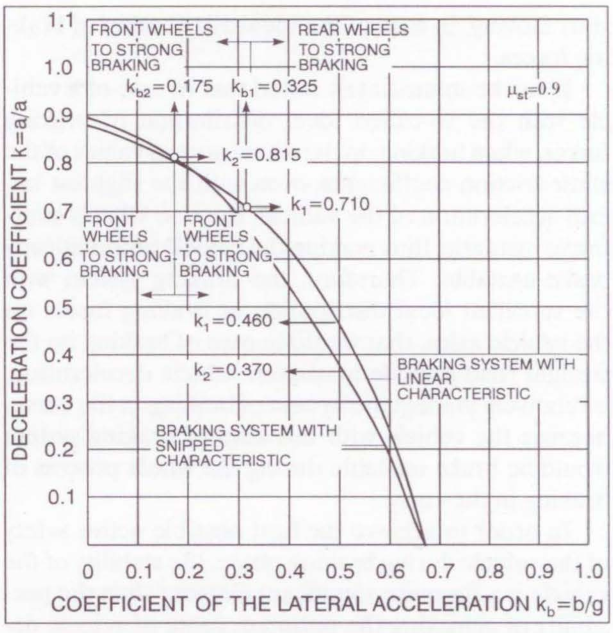

Figure 9 - Functional dependence between slowing down and lateral acceleration of the vehicle at the road-holding limit for the installed braking system with a linear characteristic and with a bilinear characteristic.

acceleration coefficient of as little as $k_b = 0.248$. Also, with the value of the static friction coefficient $\mu_{st} = 0.5$, the braking system with the linear characteristic provided the braking stability of the vehicle up to the value of the lateral acceleration coefficient $k_b = 0.4$, whereas in the case of the installed braking system with bilinear characteristic, the vehicle will be brake-unstable at the same value of static friction coefficient, in the whole area of lateral acceleration, i.e. the rear wheels will overbrake. Further, at the static friction coefficient value of $\mu_{st} = 0.3$, in case of the installed braking system with the linear characteristic, the considered vehicle was brake-stable up to the value of the lateral acceleration coefficient value $k_b = 0.279$, and in the case of the installed braking system with bilinear characteristic, the vehicle will be brake-stable up to the value of lateral acceleration coefficient $k_b = 0.237$.

The analysis of behaviour of vehicles when braking in curves considered the road-holding limits of the front and rear wheels in case of the installed braking system with a linear characteristic and for the case of the installed braking system with a bilinear characteristic, and it was concluded that the braking system with a linear characteristic provides higher braking stability than the braking system with the bilinear characteristic. The question is now, how a vehicle would react, regarding braking stability when braking in curve, which would have installed the braking system with ideal distribution of braking forces.

Figure (5) shows the road-holding limits of the front and the rear wheels of an empty vehicle ready to start moving, in case of the ideal distribution of braking forces.

It can be immediately noted that in case of a vehicle with the so-called ideal distribution of braking forces, when braking, in the whole area of values of the static friction coefficients, even with the slightest lateral acceleration of the vehicle, the rear wheels overbrake instantly, thus making the vehicle automatically brake-unstable. Therefore, the braking system with the so-called ideal distribution of braking forces on the vehicle axles, that would in case of braking on the straight road provide maximum vehicle deceleration, is relatively inadequate in case of braking in the curve, because the vehicle with this kind of braking system would be brake unstable during the whole process of braking in the curve.

In order to achieve the best possible active safety of the vehicle during braking phase, the stability of the vehicle is a far more significant element than the possibility of achieving the optimum value of vehicle deceleration. This means that the design of the braking system with the ideal distribution of the braking forces must necessarily take into consideration the simultaneous action of the longitudinal deceleration and lateral acceleration.

This leads to a question as to which values of lateral acceleration should be considered in actual traffic situations. The Daimler-Benz factory has done research of the lateral vehicle acceleration in actual traffic situations, and the results of the research are presented in Figure (8)

In the diagram in Figure (6) the left curves are presented by negative values (-), and the right curves by positive values (+). Also, every layer in Figure (6) represents a certain time frequency. Thus, e.g. since the trial drive took one hour i.e. 3600 seconds, it follows that the layer denoted by 0.003% represents the frequency of interval of 0.108 s, the layer denoted by 0.03% represents the frequency of interval of 1.08 s, etc. Since the frequencies of intervals which exceed one second are of special interest due to the possibility of driver's reaction, the layer denoted with 0.03%, which represents the frequency of intervals of 1.08 s, in Figure (6), is specially marked.

Considering, then, the empirically determined flows of lateral acceleration of vehicles in the actual traffic situations, as presented in Figure (6), we can note that maximum values of lateral vehicle acceleration are achieved in the area of low speeds. So, e.g. the maximum lateral acceleration of 5.3 m/s^2, was realised in the left curve at the speed of about 16.7 m/s (60 km/h), whereas in the right curve the maximum lateral acceleration of 5.8 m/s^2 was realised at the speed of about 11.1 m/s (40 km/h). The increase in speed the values of lateral acceleration decrease, so that at the speed of 27.8 m/s (100 km/h), the maximum lateral acceleration of 2.4 m/s^2 was realised in the left curve, and the maximum lateral acceleration of 1.8 m/s^2 was realised in the right curve. At speeds greater than 27.8 m/s (100 km/h) the maximum lateral acceleration, realised in the actual traffic situations, hardly exceed the values of 1 m/s^2. It needs to be pointed out that these limiting values of lateral accelerations were realised over a short period, i.e. during an interval of 0.036 s. Of course, the experimentally determined flows of lateral accelerations are the result of the road configurations which served for the purposes of the study.

Therefore, the measuring results in the actual traffic situation clearly show that at normal movement, especially at higher speeds, the values of lateral acceleration greater than 3 m/s^2 need not be calculated. The present modern braking systems provide the braking stability of a vehicle up to the value of the lateral acceleration of 3 m/s^2. The characteristic of a such braking system is presented in Figure (9).

Figure (9) shows the functional dependence between the maximum lateral acceleration and longitudinal deceleration at the road-holding limit of the vehicle with the installed one of today's standard braking systems with linear characteristic, as well as of the vehicle with the installed braking system with bilinear characteristic. It is obvious that the braking system with the linear characteristic will provide braking stability up to the value of the lateral acceleration coefficient $k_b = b / g = 0.325$, whereas the braking system with bilinear characteristic, regarding braking stability will be significantly worse, i.e. in the presented case in Figure (7) the transition of the vehicle from the brake-stable into the brake-unstable condition will follow in the considered braking system with bilinear characteristic already at the lateral acceleration coefficient of $k_b = 0.175$. Also, as seen in Figure (7), the braking system with linear characteristic and at relatively high values of lateral vehicle acceleration will provide higher values of deceleration in relation to the braking system with the bilinear characteristic. Thus, e.g. at the lateral acceleration coefficient $k_b = 0.5$, the braking system with linear characteristic will result in the braking coefficient value of $k = a / g = 0.46$, and the braking system with the bilinear characteristic with the braking coefficient value of $k = 0.37$. Therefore, this fact also benefits the braking stability of the vehicle in conditions of lateral acceleration, and for the case of installed braking system with linear characteristic, which is of special significance for an average driver in actual traffic situations. Of course, all the statements refer to the cases when the vehicle brakes under the influence of lateral acceleration. In case of braking while moving in a straight line, the braking system with the linear characteristic has a lower degree of utilisation than the braking system with bilinear characteristic. This means that during straight braking, in case of the vehicles with the installed braking system with the linear characteristic, the braking distance will be longer and the wear of the brakes and the pneumatics of the front wheels higher, than in the case of the installed braking system with bilinear characteristic. However, analyses of the influence of change of the friction coefficient of the brakes linings on the distribution of braking forces as consequence of the change in temperature in the brakes, influence of the moisture and dirt, as well as the above mentioned analyses of the braking problems under the influence of lateral acceleration, have shown that the braking system with linear characteristic is for the purposes of insuring brake-stability of the vehicle, regarding exploitation, safer than the braking system with bilinear characteristic. This conclusion is very important since in the evaluation of the active safety of a vehicle in actual traffic, as already mentioned, braking stability, i.e. stability of the vehicle during braking, plays a far more significant role than the slightly reduced value of vehicle deceleration, meaning a somewhat longer braking distance, and somewhat greater wear of the brake materials and pneumatics of the front wheels.

3. CONCLUSION

Clean analitical expresion would be in generally speaking and considering across incline of the road is

$$v_{gr} = \sqrt{\frac{g\,R_0\left(K\,\mu_{st}\,\frac{2h}{l}\cos\alpha \pm \sin\alpha\right)}{\cos \mp K\,\mu_{st}\,\frac{2h}{l}\sin\alpha}} \tag{26}$$

where the α is across incline of the road.

This leads to the conclusion that the analyses have shown that in practice often applied analytical calculation of speed values of vehicles negotiating a curve based on the idealised observation of vehicle as a material point, using Coulomb's friction, are completely inadequate, since in the movement of vehicle in conditions of lateral acceleration, the redistribution of dynamic loads on wheels occur, so that the inner wheels, i.e. wheels closer to the centre of rotation, regardless of the road surface condition and design type of the vehicle itself, reach their maximal road-holding values much earlier than the outer wheels. It is therefore necessary, when analytically determining the curve speed values, to take into consideration not only the value of the road-holding coefficient, as in the case of idealised observing of vehicle as a material point, but also of the position of the vehicle centre of gravity and wheels spacing. Therefore, a relatively simple and for the practice acceptable method of analytical determining of the speed value of the actual vehicle negotiating a curve has been suggested.

SAŽETAK

UTJECAJ BOČNOG UBRZANJA VOZILA NA RASPODJELU SILA KOČENJA KAO UZROK PROMETNIH NESREĆA

Uzevši u obzir prijašnja razmatranja gdje smo proračunavali brzine kojima možemo proći zavojem, u ovom radu obrađujemo i izračunavamo veličine bočnih ubrzanja koja se pojavljuju prolaskom vozila kroz zavoj. U svezi s time analizirali smo i kako brzina prolaska kroz zavoj utječe na silu kočenja , koja u velikom broju prometnih nesreća ukoliko nije kočioni sistem prilagođen brzini (ili obrnuto) predstavlja uzrok nastanka prometne nesreće.

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