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A CONTRIBUTION TO THE ANALYSIS OF DETERMINING THE POST-COLLISION VEHICLE SPEED

SUMMARY

About 50% of the analyses of actual road traffic accidents are the analyses of various collision conditions of vehicles, i.e. various collisions of two vehicles or crashes of vehicles into various types of solid barriers. Such analyses, however, require that the values of vehicle post-collision movement parameters of its kinematics are determined with as much accuracy as possible. Since methods commonly used in the present forensic expertise do not provide satisfactory results by not taking into account all the elements of the vehicle movement, this paper presents analytic considerations of the issue, thus determining relatively simple and for the practice acceptable analytical expressions which can be used to determine the vehicle movement parameters immediately following collision.

USED SYMBOLS

v_0 – speed of vehicle centre of gravity before collision

v_1 – speed of vehicle centre of gravity in the phase when the vehicle is rotating around centroidal axis with constant angular velocity, while the acceleration of vehicle centre of gravity is constant

v_2 – speed of vehicle centre of gravity in the phase when the vehicle centre of gravity has the constant speed, while the speed around centroidal axis is constant

v_3 –speed of vehicle centre of gravity in the phase when the vehicle centre of gravity have the constant speed, while angular speed and angular velocity around axis are not existing

v_p – speed of material point at half of distance between front wheels

v_s – speed of rear wheels centre of gravity

T – vehicle centre of gravity

v –generally shown – speed of vehicle centre of gravity wich can be changed with any other speed of vehicle centre of gravity

a –velocity

ε –momentary angular velocity

ω –momentary angular speed

t –time

ρ –angle of decline from primary vehicle diretion

s –path (distance)
(index on bottom marks is shawing the vehicle movement phase)

l_1 –distance of front axis of vehicle centre of gravity

l_2 –distance of rear axis of vehicle centre of gravity

K_ρ –coefficient bonding translation and rotation of vehicle

1. INTRODUCTION

An average of about 40% of all traffic accidents which occur every year on our roads, are collisions of two vehicles, and about 9% are collisions of vehicles into various types of solid barriers. Of course, a relatively big number of forensic expertise, therefore, refers precisely to the dynamic analysis of various types of vehicle collisions. However, although the problem of analytically performing the analysis of various collision conditions has already been relatively well known up to the present, still a part of the problem in these analyses has remained unknown, and it refers to the post-collision condition of the vehicle. In practical analyses, namely, the required condition of vehicle movement at the moment of collision is determined precisely on the basis of the results of the performed dynamic analysis of the vehicle movement condition after collision. This means that the vehicle movement is analysed over the period which starts at the moment of separation of vehicles, i.e. its separation from the barrier, until the moment of complete standstill of the vehicle following the accident. In this sense, the forensic specialist as an expert person performing the professional analysis of a certain traffic accident is provided with the documentation of the on-the-spot investigation, i.e. report on the investigation, drawing of the place of accident and survey, which provide the forensic specialist with qualitative and quantitative data on the final position of the vehicle, on the damage on the vehicle and trails found at the site of the accident.

This way the reconstruction of the vehicle kinematics following the collision seems simple and easy to solve, so that many forensic specialists in practical analyses of the actual traffic accidents and due to insufficient knowledge on the actual issues, simplify the problem almost to technical absurdity considering the vehicle as a material point. In reality, however, the problem is a much more difficult one, and it is for the moment practically impossible to solve it analytically completely exactly. The reason lies in the fact that uncontrolled movement of the vehicle following collision has a completely stochastic character and depends primarily on the value and way of acting of external resistant forces, which means that it depends on the current elastic characteristics of the pneumatics, value of air pressure in them, on the current properties of the contact surface between the pneumatics and the ground, on the current position of the drive wheels, on the design and the current technical condition of the support system of the vehicle, on the braking method i.e. deformity of certain wheels and the related part of the support system, etc. Therefore, determining the position of the direction and speed of the movement of the vehicle centre of weight immediately upon collision, is a physically and mathematically complex problem that can be solved only by approximate methods whose result deviates more or less from the actual situation. This is precisely the objective of this paper - an attempt to match the results of the analytical interpretation of the post-collision vehicle kinematics as much as possible with the reality.In this paper the taken values and positions of vehicles after collision are taken from numerous police reports.All phrases and equations are new considering all recichable sources and they represent true autors' contribution in solving the existing problem.

2. THE APPROACH TO THE PROBLEM

Generally speaking, because of the mainly eccentric character of a collision, immediately upon collision or crash into a solid barrier, the vehicle has translational speed of the movement of the centre of gravity (v_0) and angular velocity (ω_0) of the vehicle rotating around the vertical centroidal axis. In this way, the overall kinetic energy of the vehicle after the collision $(m \cdot v_0^2 / 2 + J \cdot \omega_0^2 / 2)$ has to be counteracted by external forces resisting the vehicle movement along the path (s) from the position of the vehicle at the moment of collision, until the final position after the accident,

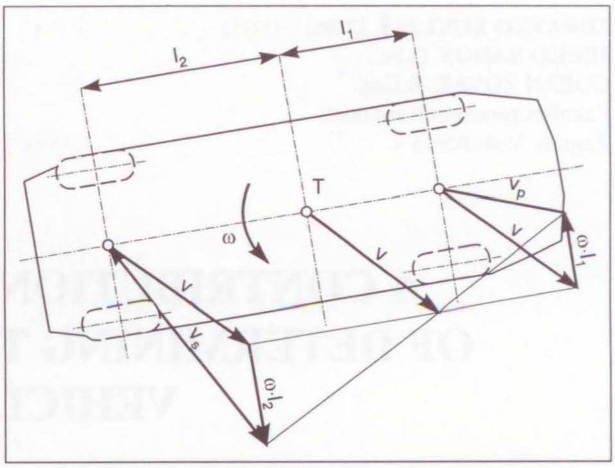

Figure 1 - Vehicle movement in second phase of horizontal turning

travelled by the vehicle over time (t), and during which the vehicle itself, in relation to its collision position, rotates about its vertical centroidal axis by an angle (ρ). Of course, in this way the absolute speed of the front and rear wheels, during the described post-collision kinematics, will constantly change their value and direction. At the time at which the vectors of absolute speeds of the front and rear wheels, as shown in Figure (1), have the same direction, and point in the direction of the movement of the vehicle centre of gravity, the vehicle movement kinematics can be considered as uniform slowed down translational movement of the centre of gravity with constant rotation of the vehicle around its vertical centroidal axis. On the other hand, however, at the time when the vectors of absolute speeds of the front and rear wheels, as illustrated in Figure (2), are opposite, the vehicle kinematics can be considered as uniform slowed down rotation of the vehicle around its vertical centroidal axis with constant translational speed of the centre of gravity itself. Therefore, the post-collision kinematics of the collided vehicle can be considered as a successive interchange of the two mentioned kinematic conditions.

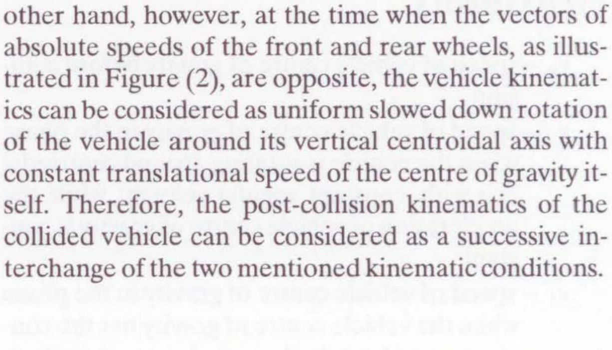

Figure 2 - Vehicle movement in third phase of horizontal turning

3. ANALYTICAL INTERPRETATION OF THE PROBLEM

The first phase (from $t = 0$ to $t = t_1$)

a=const.

ε=0

ω=const.

$$v_1 = v_o - a t_1 \tag{1}$$

$$s_1 = v_o t_1 - \frac{1}{2} a t_1^2 \tag{2}$$

$$\omega_1 = \omega_o \tag{3}$$

$$\rho_1 = \omega_o t_1 \tag{4}$$

The second phase (from $t = t_1$ to $t = t_2$)

a=0

v=const.

ε=const.

$$v_2 = v_1 \tag{5}$$

$$s_2 = s_1 + v_1 \left(t_2 - t_1\right) \tag{6}$$

$$\omega_2 = \omega_1 - \varepsilon_1 \left(t_2 - t_1\right) = 0 \tag{7}$$

$$\rho_2 = \rho_1 - \omega_1 \left(t_2 - t_1\right) - \frac{1}{2} \varepsilon \left(t_2 - t_1\right)^2 = \rho \tag{8}$$

The third phase (from $t = t_2$ to $t = t_3$)

a=const.

ε=0

ω=0

ρ=const.

$$v_3 = v_2 - a\left(t_3 - t_2\right) = 0 \tag{9}$$

$$s_3 = s_2 + v_2 \left(t_3 - t_2\right) - \frac{1}{2} a \left(t_3 - t_2\right)^2 = s \tag{10}$$

$$\rho_3 = \rho_2 = \rho \tag{11}$$

Using the presented analytical expressions, the required kinematic elements of the vehicle immediately following the collision may be determined. Using the expression (1),(5) i (9) follows the relation:

$$t_3 - t_2 = \frac{v_2}{a} = \frac{v_o}{a} - t_1 \tag{12}$$

which, if introduced into the expression (10), gives the result:

$$s = s_2 + \left(v_o - a t_1\right)\left(\frac{v_o}{a} - t_1\right) - \frac{1}{2} a \left(\frac{v_o}{a} - t_1\right)^2$$

which acquires the form:

$$s = s_2 + \frac{v_o^2}{2a} - v_o t_1 + \frac{1}{2} a t_1^2 \tag{13}$$

Furthermore, by using the expressions (3) and (7) the following relation is obtained:

$$t_2 - t_1 = \frac{\omega_0}{\varepsilon} \tag{14}$$

by means of which, as well as using the expressions (4) and (8), the result follows:

$$\rho = \omega_0 t_1 + \frac{\omega_0^2}{2\varepsilon} \tag{15}$$

which can be transformed into the form:

$$t_1 = \frac{\rho}{\omega_0} - \frac{\omega_0}{2\varepsilon} \tag{16}$$

Also, with the relation (14) and expression (6)the result follows:

$$s_2 = s_1 + v_1 \frac{\omega_0}{\varepsilon} \tag{17}$$

which, by using the expressions (1), (2) and (16) is transformed into the form:

$$\dot{s}_2 = \frac{v_0 \rho}{\omega_0} + \frac{v_0 \omega_0}{2\varepsilon} - \frac{a \rho^2}{2\omega_0^2} - \frac{a\rho}{2\varepsilon} + \frac{3a\omega_0^2}{8\varepsilon^2} \tag{18}$$

Using this expression, as well as using the expressions (13) and (16) the result follows:

$$s = \frac{v_0 \omega_0}{\varepsilon} - \frac{a\rho}{\varepsilon} + \frac{a\omega_0^2}{2\varepsilon^2} + \frac{v_0^2}{2a} \tag{19}$$

If, using the relation

$$s = \frac{v_0^2}{2a} \qquad v_0 = \omega_0 \frac{s}{\rho}$$

linearisation is done according to the time expression (19), it turns into the form:

$$\omega_0^2 \left(a\rho + 2\varepsilon s\right) = 2a\varepsilon\rho^2 \tag{20}$$

If substitution is introduced

$$K_\rho = \frac{a}{2\varepsilon} \tag{21}$$

which can be marked as the connection coefficient between translation and rotation of the vehicle, immediately, using the equation (20), the analytical expression for calculating the angular speed of the vehicle rotation following collision is obtained:

$$\omega_0 = \sqrt{\frac{a\rho^2}{\rho K_v + s}} \tag{22}$$

If distance (s) is eliminated from this expression, finally the analytical expression for calculating the values of translational speeds of the movement of centre of gravity immediately upon the collision is obtained.

$$v_0 = \frac{a\rho}{\omega_0} - \omega_0 K_v \tag{23}$$

4. CONCLUSION

The presented analysis shows clearly that the method often used previously in the forensic expert practice, of regarding the vehicle as behaviour of a material point, in the analysis of the post-collision kinematic condition of the vehicle, is a very rough and, regarding accuracy of results, unsatisfactory approximation of the actual condition. In other words, the performed analysis has undoubtedly shown that in determining the vehicle movement parameters during the post-collision phase of its kinematics, both the translation of the vehicle centre of gravity and the vehicle rotation about its own centre of gravity need to be considered. The paper suggests relatively simple, and therefore in practice acceptable, analytical expressions for determining the values of post-collision kinematic parameters of the vehicle, otherwise necessary in vector analysis which answer the question on vehicle movement parameters at the moment of collision, thus enabling also the reconstruction of events which caused the considered traffic accident.

SAŽETAK

PRILOG ANALIZI ANALITIČKOG ODREĐIVANJA VELIČINE POSTKOLIZIJSKE BRZINE KRETANJA VOZILA

Među analizama realnih prometnih nesreća, oko 5o % njih su analize raznih kolizijskih stanja vozila tj. Raznih međusobnih sudara vozila,ili pak udara vozila u razne vrste čvrstih zapreka.Takve analize međutim, zahtijevaju što točnije određivanje veličine parametara kretanja vozila u postkolizijskoj fazi njegove kinematike.Kako metode, koje se uglavnom koriste u današnjoj sudsko-ekspertnoj praksi,ne pružaju zadovoljavajuće rezultate, jer ne uvažavaju sve elemente kretanja vozila, a u ovom je radu obavljeno analitičko razmatranje iznijetog problema te su na taj način utvrđeni relativno jednostavni i za praksu prihvatljivi analitički izrazi s pomoću kojih se mogu odrediti veličine parametara kretanja vozila neposredno nakon njegove kolizije

REFERENCES

[1] **BURG, H., RAU, H.**,: *Handbuch der Verkehrsunfallrekonstruktion.*Verlag Information Ambs GmbH, Kippenheim, 1981.