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Traffic Planning
Original Scientiffic Paper
U.D.C. 656:629.433
Accepted: Sept. 24, 1998
Approved: Nov. 5, 1998

THE EFFICIENCY LINKS ON A GRAPH IN TRANSPORTATION NETWORK

SUMMARY

The access to a single vertex of the non-directional transportation network graph can be defined in various ways, but if we want to classify the vertices and at the same time avoid ambiguity, then each of them has to be assigned a certain number. The paper uses the method for determining the access to a single vertex based on the description of the transportation network using the symmetrical matrix of incidence. By knowing the matrix of incidence it is possible to determine the eigenvalues i.e. eigenvectors, and their comparison determines the values for single vertices. Apart from describing the method, the concrete example of the tram network of the city of Zagreb has been analysed. Based on the obtained analysis results, it is possible to determine the efficiency of connecting single vertices in the considered transportation network, i.e. to propose its improvements.

1. INTRODUCTION

In considering a certain transportation network, it is possible to ask which vertices in this network are easier and which less easy to access. The problem is contained in the fact that the access can be defined in several ways, depending on the space in which the distance has been defined. In order to avoid ambiguity, i.e. uncertainty, the methods should be used which will define access numerically, requiring a mathematical description of the network.

This paper aims to determine the access to single vertices in a topological sense. The distance between certain vertices is defined as the shortest chain consisting of arches which need to be passed in order to arrive from one vertex to another. The starting vertex is the realisation of the network using the non-directional graph consisting of vertices and arches, to which the incidence matrix of vertices is assigned.

2. CLASSICAL METHOD OF DETERMINING CONNECTIVITY

Regarding the ambiguity in determining the accessibility of a single transportation network vertex, each needs to be assigned a quantitative value in the description. Let graph G be the non-directional transportation network graph and let this graph have the matrix A which is called the matrix of incidence. The matrix contains zeroes and ones, where one means the connection between the two neighbouring vertices, and zero means the absence of connection of the two neighbouring vertices.

The classical definition of the single vertex accessibility introduced according to Garrison (1960) is:

$$T = A + A^2 + A^3 + \ldots + A^m \qquad (1)$$

Then the access to a certain vertex D_1 can be determined as:

$$D_l = \sum_{j=1}^{n} t_{lj} \qquad l = 1,2,\ldots,n. \qquad (2)$$

where n is the number of columns, i.e. rows of the matrix A, and t_{lj} represents the elements of the matrix T.

According to the given expression (1) it is possible to classify the vertices in the interval limited by the minimal and maximal value of the parameter D, where:

$$D_{\min} = \min D_l \qquad D_{\max} = \max D_l \qquad (3)$$

For better clarity and comparison of single vertices, parameter Φ_1 is introduced, which represents the relative access to each of them.

$$\Phi_l = \frac{D_l - D_{\min}}{D_{\max} - D_{\min}} \cdot 100 \qquad (4)$$

By analysing the parameter Φ_1 it is possible to make the following conclusions regarding the reason why such access i.e. way of determining the accessibility of a certain vertex is not right:

1. the question is what value of factor m to select, the selection of various values results also in various values of accessibility to a single vertex, which leads to ambiguity
2. with the increase of factor m, the structure of matrix A^m differs significantly from the matrix A, i.e. the numbers appear in the matrix greater than one thus making the value of the parameter Φ_1 increasingly dependent on the redundant connections rather than the shortest connections between two vertices.

The solution is in the selection of the factor ω from the interval [0,1], which reduces the influence of redundant connections so that:

$$T=\omega A+\omega^2 A^2+\omega^3 A^3+\ldots+\omega^m A^m \tag{5}$$

However, this causes a problem as well regarding the value of ω, since again the selection of different values leads to ambiguity in determining the value of Φ_1.

If, however, we choose the value for ω from the interval $[1/2\lambda_1, 1/\lambda_1]$, where $1/\lambda_1$ is the highest eigenvalue of the matrix A, then by solving the system of linear equations of the form (6), we can determine the components of vector t which is composed of the sum of row elements of the matrix T.

$$\left(\frac{1}{\omega} I-A'\right)t=s \tag{6}$$

where s is the bar vector whose elements represent the sums of columns of the matrix A.

If the value of the parameter ω changes, towards the upper limit of the interval $1/\lambda_1$ it can be noted that the relations between the components of the vector $\boldsymbol{t}$ have a tendency towards fixed values. This can be explained by the fact that vector **s** is the approximation of the eigenvector $v^{(1)}$, which equals the eigenvalue $1/\lambda_1$. According to the expression (6), it follows that the vector **t** is a still better approximation of the eigenvector $v^{(1)}$, and better insofar as the values ω and $1/\lambda_1$ are closer.

It follows that the eigenvalue $1/\lambda_1$ and the eigenvector $v^{(1)}$ play the key role in determining the accessibility, speaking topologically.

3. DETERMINING CONNECTIVITY BY USING EIGENVECTORS OF THE MATRIX OF INCIDENCE

Let A be the matrix of incidence of a non-directional graph G of a given transportation network. If matrix A is real and symmetric, then it contains only the real eigenvalues λ_1 so that it follows:

$$\lambda_1 \succ \lambda_2 \succ \ldots \succ \lambda_n \tag{7}$$

where n is the number of rows i.e. columns of the matrix A.

For each eigenvalue there is a corresponding eigenvector $v^{(1)}$, I = 1, 2, ... n, i.e. the following relation holds:

$$A v^{(i)}=\lambda_i v^{(i)} \tag{8}$$

If the transportation network is considered, then the matrix A is primitive as well i.e. there is a number L such that $A^{-1}>0$.

If we denote the matrix A in the following way:

$$A=\left(a_{.1}, a_{.2}, \ldots, a_{.n}\right) \tag{9}$$

where the elements a.i are the columns of the matrix, then, since the eigenvectors of the matrix A are linearly independent, every column of the matrix can be represented as:

$$a_{.i}=\sum_{j=1}^{n} c_j^{(i)} v^{(j)} \tag{10}$$

where $c_j^{(i)}$ are the coefficients of the given linear combination.

By the use of the given notation it follows that the matrix $A^2=\left(A a_{.1}, \ldots, A a_{.n}\right)$ and the following holds:

$$A a_{.i}=\sum_{j=1}^{n} c_j^{(i)} A v^{(j)}=\sum_{j=1}^{n} c_j^{(i)} \lambda_j v^{(j)} \tag{11}$$

$$A^m a_{.i}=\sum_{j=1}^{n} c_j^{(i)} A^m v^{(j)}=\sum_{j=1}^{n} c_j^{(i)} \lambda_j^m v^{(j)}=$$

$$=c_1^{(i)} \lambda^m\left\{v^{(1)}+\sum_{j=1}^{n} \frac{c_j^{(i)}}{c_1^{(i)}}\left(\frac{\lambda_j}{\lambda_1}\right)^m v^{(j)}\right\} \tag{12}$$

Since $j>1$, it follows that:

$$\lim\left(\frac{\lambda_j}{\lambda_1}\right)^m=0 \tag{13}$$

and the expression (12) can be written as:

$$A^m a_{.i} \cong c_1^{(i)} \lambda_1^m v^{(1)} \tag{14}$$

By analysing the expression (14) it follows that all the columns of the matrix A^{m+1} differ from $v^{(1)}$ according to the scalar values which are assumed to be other than zero. The speed at which the columns of the matrix A^m approach the eigenvector $v^{(1)}$ depends on the dominance of the eigenvalue λ_1 over others.

Let $S_l^{(m)}$ be the sum of the l-th row of the matrix A^m, it follows:

$$S_l^{(m)} \cong \sum_{i=1}^{n} \lambda_l^{m-1} c_l^{(i)} v_l^{(1)}=\lambda_1^{m-1} v_l^1 \sum_{i=1}^{n} c_1^{(i)} \tag{15}$$

If we want to determine the accessibility of a single vertex, it is necessary, according to the expression (1) to define the sum of the following form:

$$\sigma_l^{(m)}=\sum_{i=1}^{m} S_l^{(m)} \qquad l=1,\ldots,n. \tag{16}$$

i.e. according to the expression (4) the value of accessibility of a single vertex is:

$$D_l^{(m)}=\frac{\sigma_l^{(m)}-\sigma_k^{(m)}}{\sigma_j^{(m)}-\sigma_k^{(m)}}.=\frac{\sum_{i=1}^{m} S_l^{(i)}-\sum_{i=1}^{m} S_k^{(i)}}{\sum_{i=1}^{m} S_j^{(i)}-\sum_{i=1}^{m} S_k^{(i)}} \cong$$

$$\cong \frac{v_l^{(1)}-v_k^{(1)}}{v_j^{(1)}-v_k^{(1)}} * 100 \tag{17}$$

where indices $\boldsymbol{j}$ and $\boldsymbol{k}$ denote the maximum and the minimum value of the eigenvector $v^{(1)}$.

$$v_j^{(1)} = \max v_l^1 \quad v_k^{(1)} = \max v_l^1 \tag{18}$$

The analysis of the results obtained according to the expression (17) shows that the least accessible vertex has the value zero, which would mean that it is isolated from all the others which is not true in the transportation network.

In order to avoid this, the accessibility to a certain vertex is defined in the following way:

$$\Psi_l = v_l^{(1)} * 100 \tag{19}$$

where $v^{(1)}$ is the eigenvector with the following properties:

1. all the components are positive,
2. the vector norm is defined in the following way, $\left\|v^{(1)}\right\| = \max_{1 \le i \le n} v_i^{(1)} = 1$
3. v_l^1 is the l-th component which defines the value of accessibility to the l-th vertex of the given transportation network graph.

The analysis and the comparison of the method based on determining the eigenvector $v^{(1)}$, and other methods for determining the accessibility of a certain transportation network vertex, the following conclusions may be made:

1. instead of calculating the partial sums $$A + A^2 + .. + A^m,$$ only the eigenvector $v^{(1)}$ needs to be determined, which is much simpler,
2. the method does not assign the value of zero to the least accessible vertex, because it is not isolated from the others,
3. there is an independence on the parameter m, which in other methods leads to ambiguity in determining the approach due to the selection of its different values,
4. independence of the accessibility value to certain vertices regardless of the fact whether they have a loop or not.

4. APPLICATION OF THE METHOD ON THE TRAM NETWORK IN THE CITY OF ZAGREB

The growth of the city population results in an increase of traffic, and this leads to the increase in the urban public transport. The specially dense traffic in the busy city streets is certain to cause congestion. As known, motor vehicles cause the greatest environmental pollution. In the densely populated urban areas, the widespread use of motor vehicles, the unfavourable engine operating conditions, and the insufficient air circulation are the primary pollutants of the urban environment. In order to alleviate the environmental pollution, the traffic of motor vehicles has been increasingly banned from the city centres, with the traffic being taken over by the urban public transport.

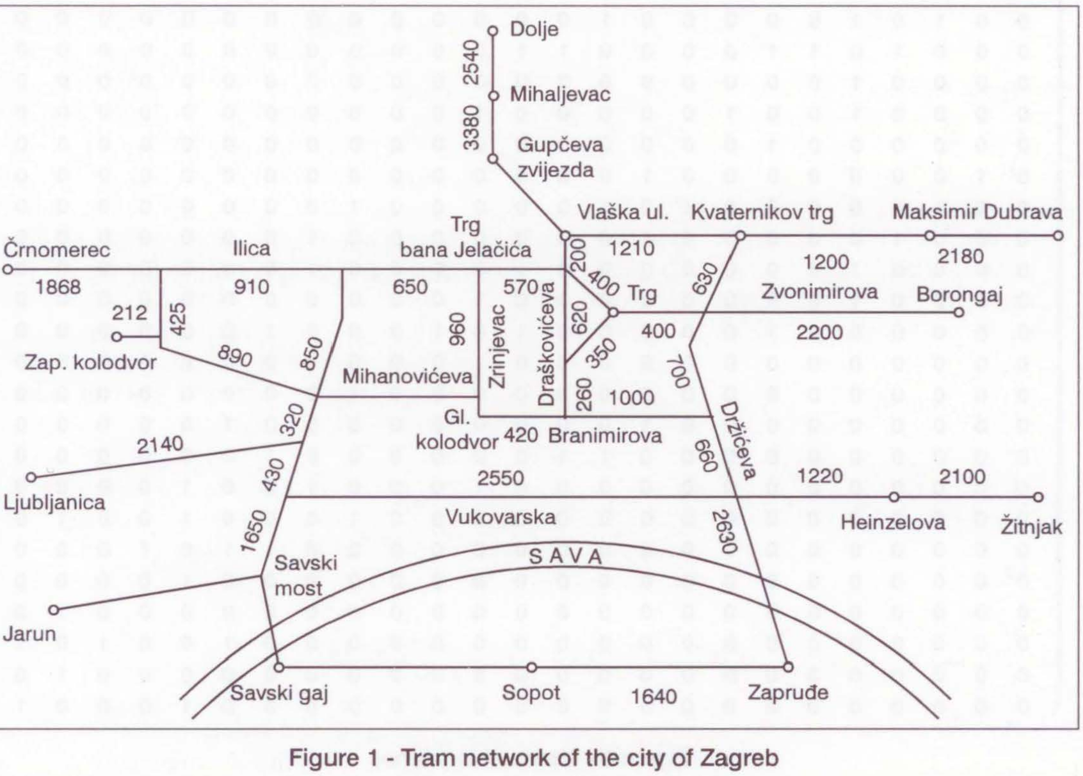

Figure 1 - Tram network of the city of Zagreb

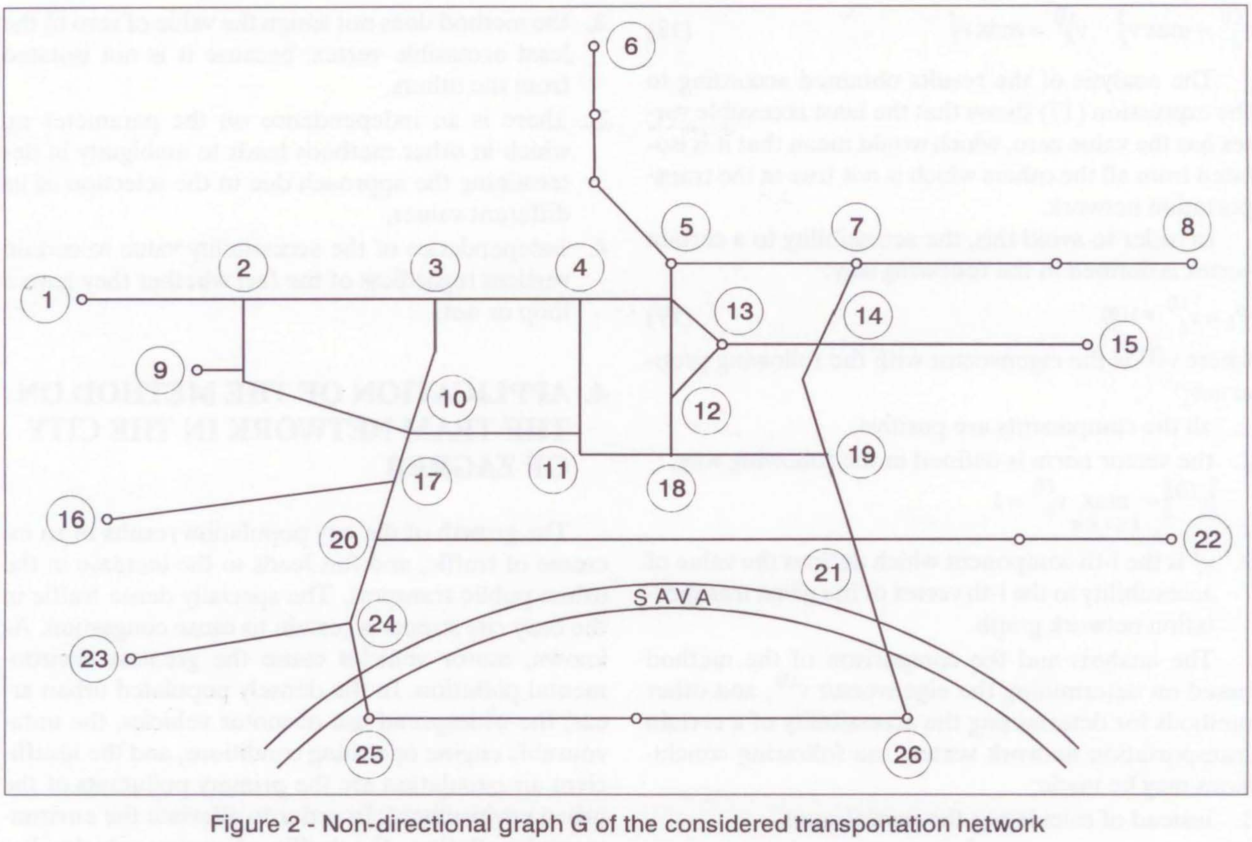

Figure 2 - Non-directional graph G of the considered transportation network

$$A = \begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0
\end{pmatrix}$$

Figure 3 - Matrix of incidence

The development of cities cannot be thought of without a general urban planning, and this includes the traffic infrastructure planning. Often such plans indicate the need to re-structure the existing urban units. The tram traffic network in the existing urban centres, in case of a plan change, has to undergo significant changes. In order to carry out this change of the tram network in an optimal way, it is necessary to make the necessary plans. Therefore, the current state needs to be analysed, and this means that the existing network and the connectivity of the characteristic vertices in this network need to be monitored.

Based on the obtained results, a model is made which allows simulation of the possible solutions, and the application of mathematical methods provides several solutions out of which the optimal one is selected.

The considered method of eigenvectors has been applied in the analysis of the actual transportation network, using the example of the tram network of the city of Zagreb, shown in Figure 1. The given transportation network has been assigned the non-directional graph G, presented in Figure 2 and the related matrix of incidence A - Figure 3. Graph G of the considered transportation network has 26 vertices, and it is a 26x26 size matrix of incidence.

For the given matrix of incidence, the eigenvector $\nu^{(1)}$ which corresponds to the maximum eigenvalue λ_1 can be determined. By knowing the eigenvector, also the accessibility values of a certain vertex can be determined.

The programming packet Mathematica 3.0 has been used for data processing i.e. for determining the values of the eigenvectors and eigenvalues.

Table 1.

Ord. No.	The name of the vertex	Relation (17)	Relation (19)
1.	Črnomerec	4.947	7.627
2.	Ulica Republike Austrije	19.201	21.479
3.	Frankopanska ulica	41.076	42.738
4.	Trg bana J. Jelačića	59.354	60.500
5.	Vlaška ulica	100.000	100.000
6.	Dolje	29.317	31.310
7.	Kvaternikov trg	57.472	58.671
8.	Dubrava	17.189	19.524
9.	Zapadni kolodvor	19.973	22.229
10.	Vodnikova	40.179	41.866
11.	Glavni kolodvor	52.208	53.556
12.	Draškovićeva	74.310	75.035
13.	Trg hrvatskih velikana	76.191	76.863
14.	Zvonimirova	66.870	67.804
15.	Borongaj	19.006	21.290
16.	Ljubljanica	3.439	6.162
17.	Savska	18.442	20.742
18.	Branimirova	55.369	56.628
19.	Branimirova-Držićeva	47.235	48.723
20.	Vukovarska-Savska	15.590	17.970
21.	Držićeva	26.678	28.745
22.	Žitnjak	6.347	8.988
23.	Jarun	0.000	2.819
24.	Savski most	6.587	9.221
25.	Savski gaj	3.777	6.490
26.	Zapruđe	8.524	11.103

Table 2.

Ord. No.	The name of the vertex	Relation (17)	Relation (19)
1.	Črnomerec	1.578	7.631
2.	Ulica Republike Austrije	16.401	21.543
3.	Frankopanska ulica	39.024	42.774
4.	Trg bana J.Jelačića	57.996	60.580
5.	Vlaška ulica	100.000	100.000
6.	Dolje	26.824	31.325
7.	Kvaternikov trg	56.009	58.715
8.	Dubrava	14.247	19.521
9.	Zapadni kolodvor	17.170	22.264
10.	Vodnikova	38.254	42.052
11.	Glavni kolodvor	50.572	53.612
12.	Draškovićeva	73.440	75.074
13.	Trg hrvatskih velikana	75.398	76.911
14.	Zvonimirova	65.745	67.852
15.	Borongaj	16.160	21.317
16.	Ljubljanica	2.192	8.208
17.	Savska	15.795	20.974
18.	Branimirova	53.879	56.716
19.	Branimirova-Držićeva	45.570	48.918
20.	Vukovarska-Savska	13.370	18.698
21.	Držićeva	24.493	29.137
22.	Žitnjak	3.176	9.131
23.	Jarun	0.000	6.150
24.	Savski most	4.719	10.579
25.	Savski gaj	0.992	7.081
26.	Zapruđe	5.614	11.419

The current state of the given tram network of the city of Zagreb, i.e. the accessibility results of certain vertices are presented in Table 1. The values have been determined by applying the relations (17) and (19).

As an example of the possible simulations on the model, the vertices 8, 15, 16, 22 and 23 have been selected in order to use the newly obtained data on the vertices connectivity to optimise the network.

The first version of model simulation is the connecting of the vertices 16 and 23 (Ljubljanica - Jarun), the results of which are presented in Table 2. The next model simulation version is the connecting of the vertices 8 and 22 (Dubrava - Žitnjak), with results given in Table 3 and finally, the version connecting the vertices 8, 15, and 22 (Dubrava - Borongaj - Žitnjak), with results presented in Table 4.

The comparison of the obtained results according to Tables 1-4 enables the classification of accessibility of certain vertices on the transportation network into three basic groups:

1. the vertices whose accessibility has been reduced,
2. the vertices whose accessibility has not changed significantly,
3. the vertices whose accessibility has been increased.

As the criterion for selecting the optimal solution topologically, the sum of the accessibility for all the vertices for certain versions of simulation has been selected, thus trying to maximise the connectivity.

The results are presented in Table 5. The table shows that the version 4 provides the optimal solution for the given simulation, which can then be taken into consideration in redesigning the tram network in Zagreb.

Table 3.

Ord. No.	The name of the vertex	Relation (17)	Relation (19)
1.	Črnomerec	1.006	7.399
2.	Ulica Republike Austrije	15.573	21.026
3.	Frankopanska ulica	37.893	41.904
4.	Trg bana J.Jelačića	57.115	59.885
5.	Vlaška ulica	100.000	100.000
6.	Dolje	26.409	31.162
7.	Kvaternikov trg	58.153	60.856
8.	Dubrava	21.408	26.484
9.	Zapadni kolodvor	16.315	21.720
10.	Vodnikova	37.378	41.423
11.	Glavni kolodvor	49.613	52.867
12.	Draškovićeva	72.951	74.698
13.	Trg hrvatskih velikana	75.170	76.774
14.	Zvonimirova	66.900	69.038
15.	Borongaj	16.114	21.532
16.	Ljubljanica	2.047	8.373
17.	Savska	15.808	21.246
18.	Branimirova	53.819	56.802
19.	Branimirova-Držićeva	47.015	50.437
20.	Vukovarska-Savska	14.953	20.446
21.	Držićeva	29.481	34.036
22.	Žitnjak	13.091	18.704
23.	Jarun	0.000	6.458
24.	Savski most	5.398	11.508
25.	Savski gaj	1.559	7.917
26.	Zapruđe	7.074	13.076

Table 4.

Ord. No.	The name of the vertex	Relation (17)	Relation (19)
1.	Črnomerec	0.000	5.677
2.	Ulica Republike Austrije	12.228	17.211
3.	Frankopanska ulica	31.082	34.995
4.	Trg bana J.Jelačića	52.627	55.317
5.	Vlaška ulica	100.000	100.000
6.	Dolje	26.470	30.645
7.	Kvaternikov trg	74.778	76.210
8.	Dubrava	58.023	60.406
9.	Zapadni kolodvor	12.293	17.273
10.	Vodnikova	31.754	35.629
11.	Glavni kolodvor	43.593	46.796
12.	Draškovićeva	71.948	73.541
13.	Trg hrvatskih velikana	79.445	80.612
14.	Zvonimirova	87.172	87.901
15.	Borongaj	59.962	62.235
16.	Ljubljanica	2.336	7.881
17.	Savska	14.695	19.538
18.	Branimirova	53.289	55.941
19.	Branimirova-Držićeva	57.150	59.583
20.	Vukovarska-Savska	20.443	24.960
21.	Držićeva	46.005	49.071
22.	Žitnjak	50.453	53.266
23.	Jarun	0.913	6.539
24.	Savski most	7.751	12.988
25.	Savski gaj	4.269	9.704
26.	Zapruđe	13.203	18.131

Table 5.

Ord. No.	The name of the vertex	(1)	(2)	(3)	(4)
1.	Črnomerec	7.627	7.631	7.399	5.667
2.	Ulica Republike Austrije	21.479	21.543	21.026	17.211
3.	Frankopanska ulica	42.738	42.774	41.904	34.995
4.	Trg bana J.Jelačića	60.500	60.580	59.885	55.317
5.	Vlaška ulica	100.000	100.000	100.000	100.000
6.	Dolje	31.310	31.325	31.162	30.645
7.	Kvaternikov trg	58.671	58.715	60.856	76.210
8.	Dubrava	19.524	19.521	26.484	60.406
9.	Zapadni kolodvor	22.229	22.264	21.720	17.273
10.	Vodnikova	41.866	42.052	41.423	35.629
11.	Glavni kolodvor	53.556	53.612	52.867	46.796
12.	Draškovićeva	75.035	75.074	74.698	73.541
13.	Trg hrvatskih velikana	76.863	76.911	76.774	80.612
14.	Zvonimirova	67.804	67.852	69.038	87.901
15.	Borongaj	21.290	21.317	21.532	62.235
16.	Ljubljanica	6.162	8.208	8.373	7.881
17.	Savska	20.742	20.974	21.246	19.538
18.	Branimirova	56.628	56.716	56.802	55.941
19.	Branimirova-Držićeva	48.723	48.918	50.437	59.583
20.	Vukovarska-Savska	17.970	18.698	20.446	24.960
21.	Držićeva	28.745	29.137	34.036	49.071
22.	Žitnjak	8.988	9.131	18.704	53.266
23.	Jarun	2.819	6.150	6.458	6.539
24.	Savski most	9.221	10.579	11.508	12.988
25.	Savski gaj	6.490	7.081	7.917	9.704
26.	Zapruđe	11.103	11.419	13.076	18.131
	TOTAL:	918.083	928.182	955.771	1102.050

5. CONCLUSION

In the cities, in which this was possible, the tram traffic developed as the main component of the urban public transport, especially in the centres. The organisation of the public transport, its realisation and keeping the time-table affect directly the increase in the traffic efficiency and reduce the transportation costs.

Very often the decision-maker has to make the decision of which two vertices should be connected within the transportation network in order to increase its overall connectivity. The existing tram network is analysed for the city of Zagreb, and the quantitative indicators of the efficiency of connecting the selected vertices are given, which can be used by the decision-makers as a basis for selecting the vertices that need to be connected in the next period in order to maximise the overall connectivity. According to Table 5 the best connectivity will be obtained if Dubrava is connected over Borongaj to Žitnjak.

The same mathematical procedure can be used for the analysis of the efficiency of connecting the cities by roads or railway lines. The method of eigenvectors is more efficient for such and similar real problems.

SAŽETAK

EFIKASNOST POVEZANOSTI NA GRAFU TRANSPORTNE MREŽE

Pristup pojedinoj točki neusmjerenog grafa transportne mreže može se definirati na razne načine, međutim ukoliko želimo izvršiti klasifikaciju točaka i pritom izbjeći višeznačnost

tada je potrebno svakoj od njih pridružiti određen broj. U radu je korištena metoda za određivanje pristupa pojedinoj točki koja se temelji na opisu transportne mreže pomoću simetrične matrice incidencije. Poznavanjem matrice incidencije moguće je odrediti svojstvene vrijednosti odnosno svojstvene vektore. čijom usporedbom se određuju vrijednosti za pojedine točke. Uz opis metode, izvršena je analiza na konkretnom primjeru tramvajske mreže grada Zagreba. Na temelju dobivenih rezultata analize moguće je odrediti efikasnost povezivanja pojedinih točaka navedene transportne mreže, odnosno predložiti poboljšanja iste.

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