JURAJ SAUCHA, D. Sc.
Fakultet strojarstva i brodogradnje
I. Lučića 5, Zagreb
JERKO RADOŠ, D. Sc.
Fakultet prometnih znanosti
Zagreb, Vukelićeva 4
DIANA MILČIĆ, M. Sc.
Fakultet strojarstva i brodogradnje
I. Lučića 5, Zagreb

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SIMPLIFIED APPROACH TO DETERMINATION OF CRITICAL AXIAL LOAD OF MARINE LINE SHAFTING

ABSTRACT

The critical axial load of the marine line shafting is obtained by using simplified mechanical models of two span bars with various supporting modes, loaded with constant compressive force. The values of critical forces plotted as functions of the ratios of span lengths, for several ratios of span flexural rigidities, are shown in diagrams.

KEYWORDS

critical load, marine line shafting

1. INTRODUCTION

The propulsive force, produced by the work of the propeller, is transmitted to the marine line shafting and received by the thrust bearing. Thus, from mechanical point of view, the marine line shafting can be considered a long compressively loaded bar. This calls for the checking of elastic stability in the design of marine line shafting.

This paper deals with determination of critical axial load of the marine line shafting through considering three simplified mechanical models of two span bars with various supporting modes, loaded with constant axial compressive force. The spans have different lengths and flexural rigidities.

In solving the problem, the governing fourth-order differential equation is used. The constants, determined by using the boundary conditions, are expressed in terms of the bending moments at the supports. Formulating the conditions of continuity of the elastic curve of the buckled bar produces a system of homogenous linear equations with these moments as unknowns. The condition of non-trivial solution to the system demanding that its determinant equals zero yields the equation from which the critical force is obtained.

2. THE EQUATION OF THE ELASTIC CURVE FOR THE BUCKLED BAR

The equilibrium equation of a buckled bar of constant flexural rigidity EI, loaded with constant compressive force F, takes the form

$$[EIw'']'' + Fw'' = 0 \qquad (1)$$

This equation can be used as the governing equation describing the behaviour of a span of the multispan bar between the supports L and R. The span under observation has been isolated from the bar by cutting hinges into the bar over the supports and applying moments M_L and M_R to replace removed constraints with adjacent spans (Figure 1)

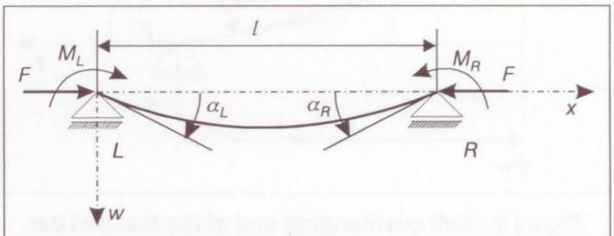

Figure 1 - Isolated span of the buckled bar

The general solution to the equation (1) is

$$w = C_1 \cos kx + C_2 \sin kx + C_3 x + C_4 \qquad (2)$$

where C_1 to C_4 are constants, and

$$k = \sqrt{\frac{F}{EI}} \qquad (3)$$

From the boundary conditions

$$\text{at } x = 0,\ w = 0; \qquad \text{at } x = l,\ w = 0 \qquad (4)$$

and in the range of small displacements being valid

$$\text{at } x = 0,\ E\,I\,w'' = -M_L; \qquad \text{at } x = l,\ E\,I\,w'' = -M_R \qquad (5)$$

the constants are determined as follows:

$$C_1 = -C_4 = \frac{M_L}{k^2 EI}$$

$$C_2 = \frac{1}{k^2 EI}\left(\frac{M_R}{\sin kl} - \frac{M_L}{\operatorname{tg} kl}\right) \tag{6}$$

$$C_3 = \frac{M_L - M_R}{k^2 EIl}$$

Substituting (6) in (2) and determining $w' = \frac{dw}{dx}$, when $x = 0$ yields the slope of the elastic curve at the left support

$$w'_L = \frac{M_L l}{EI} f_1(u) + \frac{M_R l}{EI} f_2(u) \tag{7}$$

and when $x = l$ yields the slope of the elastic curve at the right support

$$w'_R = \frac{M_L l}{EI} f_2(u) + \frac{M_R l}{EI} f_1(u) \tag{8}$$

where

$$u = kl \tag{9}$$

$$f_1(u) = \frac{1}{u}\left(\frac{1}{u} - \frac{1}{\operatorname{tg} u}\right) \tag{10}$$

$$f_2(u) = \frac{1}{u}\left(\frac{1}{\sin u} - \frac{1}{u}\right) \tag{11}$$

The equation (1) and its solution (2) can be applied in determining the slope of the elastic curve of the buckled overhanging ends of the bar. A buckled left overhang is shown in Figure 2.

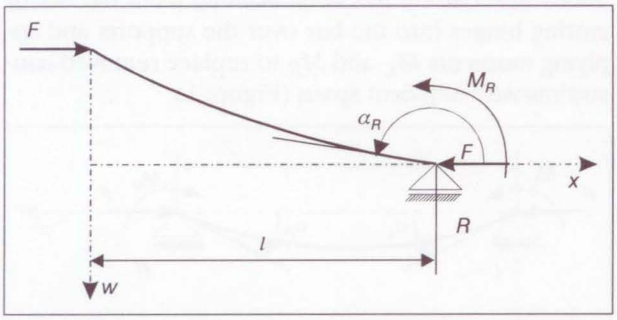

Figure 2 - Left overhanging end of the buckled bar

The boundary conditions according to Figure 2 are:

$$\begin{array}{ll} \text{at } x = l, w = 0; & \text{at } x = l, M = M_R \\ \text{at } x = 0, M = 0; & \text{at } x = 0, F_T = 0 \end{array} \tag{12}$$

Here the bending moment is

$$M = -EIw'' \tag{13}$$

and transverse force F_T can be obtained from the equations (Figure 3)

$$F_T = F_Q \cos\alpha + F_N \sin\alpha \tag{14}$$

$$F_L = -F_Q \sin\alpha + F_N \cos\alpha \tag{15}$$

With $\sin\alpha \approx \operatorname{tg}\alpha = w'$, $\cos\alpha \approx 1$, $w' \cdot w' \approx 0$ and $F_Q = \frac{dM}{dx} = M'$ it follows from (14) and (15)

$$F_T \approx F_Q + F_L w' = M' - Fw' = -EIw''' - Fw' \tag{16}$$

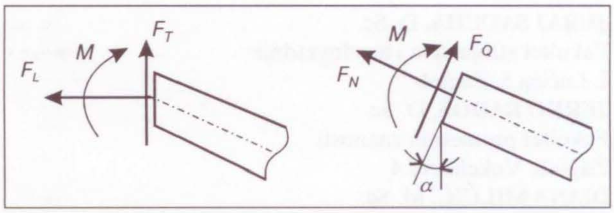

Figure 3 - Transverse force F_T, longitudinal force F_L, shearing force F_Q and normal force F_N

From the boundary conditions (12) expressed by (2), respecting (13) and (16), the constants in (2) are determined as follows:

$$C_1 = C_3 = C_4 = 0, C_2 = \frac{M_R}{k^2 EI}\frac{1}{\sin kl} \tag{17}$$

If (17) is inserted in (2) and $w' = \frac{dw}{dx}$ is determined, the slope of the elastic curve at support is obtained for $x = l$:

$$w_R = \frac{M_R l}{EI} f_3(u) \tag{18}$$

where $u = k\,l$ and

$$f_3(u) = \frac{1}{u}\frac{1}{\operatorname{tg} u} \tag{19}$$

3. THE CONTINUITY CONDITIONS

The condition of continuity of the elastic curve requires the equality of the slopes of elastic curves of the adjacent spans l_{i-1} and l_i at the support i (Figure 4)

$$w'_{R,i-1} = w'_{L,i}$$

which expressed by (8) and (7) yields

$$M_{i-1}\frac{l_{i-1}}{I_{i-1}} f_2(u_{i-1}) + M_i\left[\frac{l_{i-1}}{I_{i-1}} f_1(u_{i-1}) + \frac{l_i}{I_i} f_1(u_i)\right] + \\ + M_{i+1}\frac{l_i}{I_i} f_2(u_i) = 0 \tag{20}$$

where

$$u_{i-1} = k_{i-1} l_{i-1} = \frac{F}{E}\frac{l_{i-1}}{I_{i-1}}; \; u_i = k_i l_i = \frac{F}{E}\frac{l_i}{I_i} \tag{21}$$

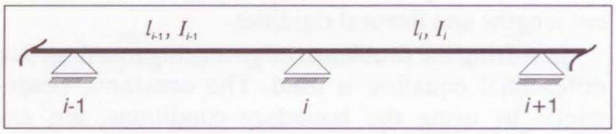

Figure 4 - Adjacent spans l_{i-1} and l_i

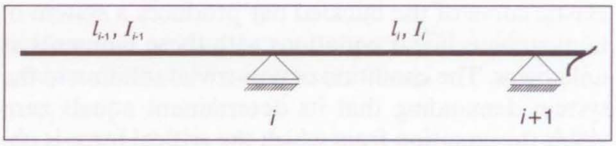

Figure 5 - Left overhanging end of the bar

The condition of continuity of the elastic curve at the support i of the left overhanging end of the bar (Figure 5)

$$w'_{R,i-1} = w'_{L,i}$$

expressed by (18) and (7) yields

$$M_i\left[-\frac{l_{i-1}}{I_{i-1}} f_3(u_{i-1}) + \frac{l_i}{I_i} f_1(u_i)\right] +$$

$$+M_{i+1}\frac{l_i}{I_i} f_2(u_i) = 0 \tag{22}$$

Finally, the fixed end of the bar has to be considered. From the boundary condition (Figure 6)

$$w'_{R,i} = 0$$

expressed by (8) it follows

$$M_i \frac{l_i}{I_i} f_2(u_i) + M_{i+1}\frac{l_i}{I_i} f_1(u_i) = 0 \tag{23}$$

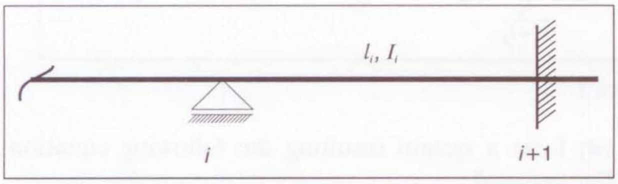

Figure 6 - Fixed right end of the bar

If the bar is fixed at the left end (Figure 7), from the boundary condition

$$w'_{L,i} = 0$$

expressed by (7) it follows

$$M_i \frac{l_i}{I_i} f_1(u_i) + M_{i+1}\frac{l_i}{I_i} f_2(u_i) = 0 \tag{24}$$

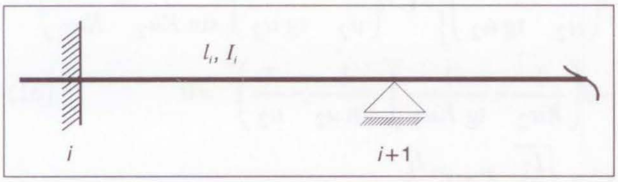

Figure 7. Fixed left end of the bar

4. DETERMINATION OF THE CRITICAL FORCES

For a two span bar with fixed right end and overhanging, supported or fixed left end, equation (20) can be set up for intermediate support, equation (23) for fixed right end and equation (22) for overhanging and equation (24) for fixed left end.

Setting up these equations yields a system of homogeneous linear equations with moments M_i, M_{i+1} and M_{i-1} (if it exists) as unknowns. The coefficients in the equations are functions of u_{i-1} and u_i. The condition of non-trivial solution to the system demanding that its determinant equals zero produces the equation $Det\ (u_{i-1}, u_i) = 0$. If u_{i-1} is expressed by reference variable u_i respecting (9) and (3)

$$u_{i-1} = u_i \frac{l_{i-1}}{l_i}\sqrt{\frac{I_i}{I_{i-1}}} \tag{25}$$

equation $Det\ (u_i) = 0$ is obtained. The critical force follows from the smallest root u_{io} of this equation by (3)

$$F_{cr} = u_{io}^2 \frac{EI_i}{l_i^2} \tag{26}$$

The described procedure of determining the critical forces will be applied on the three of the possible mechanical models of the marine line shafting, as follows.

a) Two span bar with fixed right end and overhanging left end

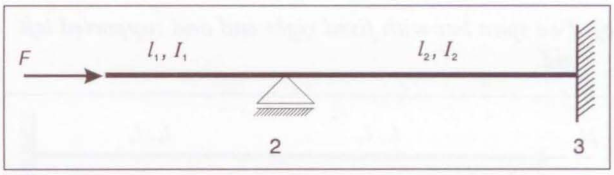

Figure 8 - Two span bars with fixed right end and overhanging left end

Equation (22) for support 2 and equation (23) for fixed end 3 (Figure 8) form a system from which results the following equation $Det\ (u_2) = 0$

$$\left(\frac{1}{u_2} - \frac{1}{\operatorname{tg} u_2}\right)\left[\left(\frac{1}{u_2} - \frac{1}{\operatorname{tg} u_2}\right) - C\frac{1}{\operatorname{tg} Ku_2}\right] -$$

$$-\left(\frac{1}{\sin u_2} - \frac{1}{u_2}\right)^2 = 0 \tag{27}$$

$$C = \sqrt{\frac{I_2}{I_1}},\ K = C\frac{l_1}{l_2}$$

From the smallest root u_{2o} of equation (27) follows

$$F_{cr} = u_{2o}^2 \frac{EI_2}{l_2^2} \tag{28}$$

Diagram 1 presents F_{cr} dependent on $\frac{l_2}{(l_1 + l_2)}$ and $\sqrt{\frac{I_2}{I_1}}$. The value of F_{cr} can be computed from the plotted quantity $\frac{F_{cr}(l_1 + l_2)^2}{EI_2}$ as follows:

$$F_{cr} = \left[\frac{F_{cr}(l_1 + l_2)^2}{EI_2}\right]\frac{EI_2}{(l_1 + l_2)^2} \tag{29}$$

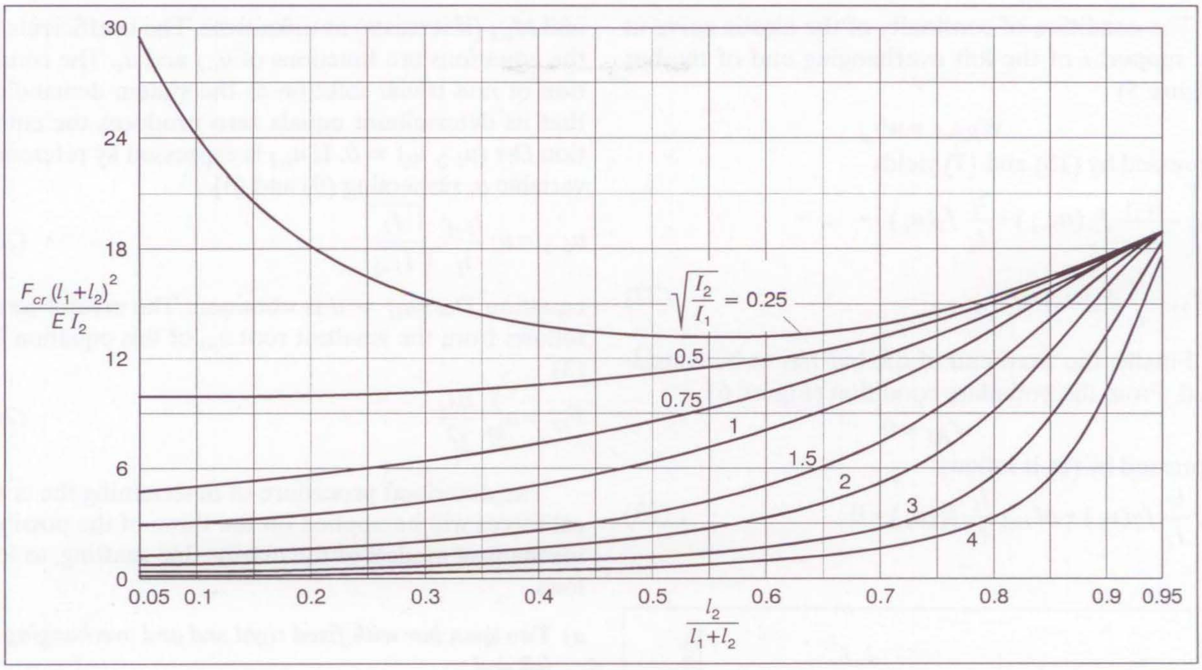

Diagram 1

b) Two span bar with fixed right end and supported left end

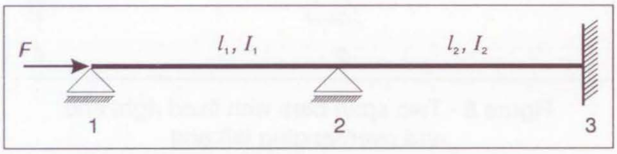

Figure 9 - Two span bar with fixed right end and supported left end

Equation (20) for support 2 and equation (23) for fixed end 3 (Figure 9), respecting $M_1 = 0$, form a system resulting the following equation $Det\ (u_2) = 0$:

$$\left(\frac{1}{u_2}-\frac{1}{\text{tg}\, u_2}\right)\left[C\left(\frac{1}{Ku_2}-\frac{1}{\text{tg}\, Ku_2}\right)-\left(\frac{1}{u_2}-\frac{1}{\text{tg}\, u_2}\right)\right]-\left(\frac{1}{\sin u_2}-\frac{1}{u_2}\right)^2=0 \quad (30)$$

$$C=\sqrt{\frac{I_2}{I_1}},\ K=C\frac{l_1}{l_2}$$

From the smallest root u_{2o} of equation (30) F_{cr} follows by (28). The quantity $\frac{F_{cr}(l_1+l_2)^2}{EI_2}$ is plotted as function of $\frac{l_2}{(l_1+l_2)}$ and $\sqrt{\frac{I_2}{I_1}}$ in diagram 2. The value of F_{cr} can be computed from this quantity by (29).

c) Two span bar with fixed ends

Equation (20) for support 2, equation (23) for fixed end 3 and equation (24) for fixed end 1 (Figure 10) form a system resulting the following equation $Det\ (u_2) = 0$:

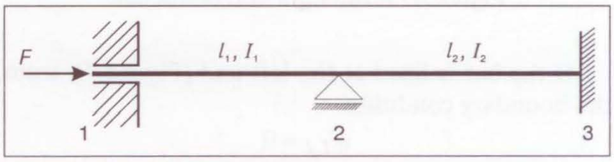

Figure 10 - Two span bars with fixed end

$$C\left(\frac{1}{u_2}-\frac{1}{\text{tg}\, u_2}\right)\left(\frac{1}{Ku_2}-\frac{1}{\text{tg}\, Ku_2}\right)\left[C\left(\frac{1}{Ku_2}-\frac{1}{\text{tg}\, Ku_2}\right)+\left(\frac{1}{u_2}-\frac{1}{\text{tg}\, u_2}\right)\right]-C^2\left(\frac{1}{u_2}-\frac{1}{\text{tg}\, u_2}\right)\left(\frac{1}{\sin Ku_2}-\frac{1}{Ku_2}\right)^2-C\left(\frac{1}{Ku_2}-\frac{1}{\text{tg}\, Ku_2}\right)\left(\frac{1}{\sin u_2}-\frac{1}{u_2}\right)^2=0 \quad (31)$$

$$C=\sqrt{\frac{I_2}{I_1}},\ K=C\frac{l_1}{l_2}$$

From the smallest root u_{2o} of this equation F_{cr} follows by (28). It can be computed from the quantity $\frac{F_{cr}(l_1+l_2)^2}{EI_2}$ plotted in diagram 3 as function of $\frac{l_2}{(l_1+l_2)}$ and $\sqrt{\frac{I_2}{I_1}}$ by (29).

5. CONCLUSION

The analysis of the influence of the ratios of span lengths and span flexural rigidities upon the critical

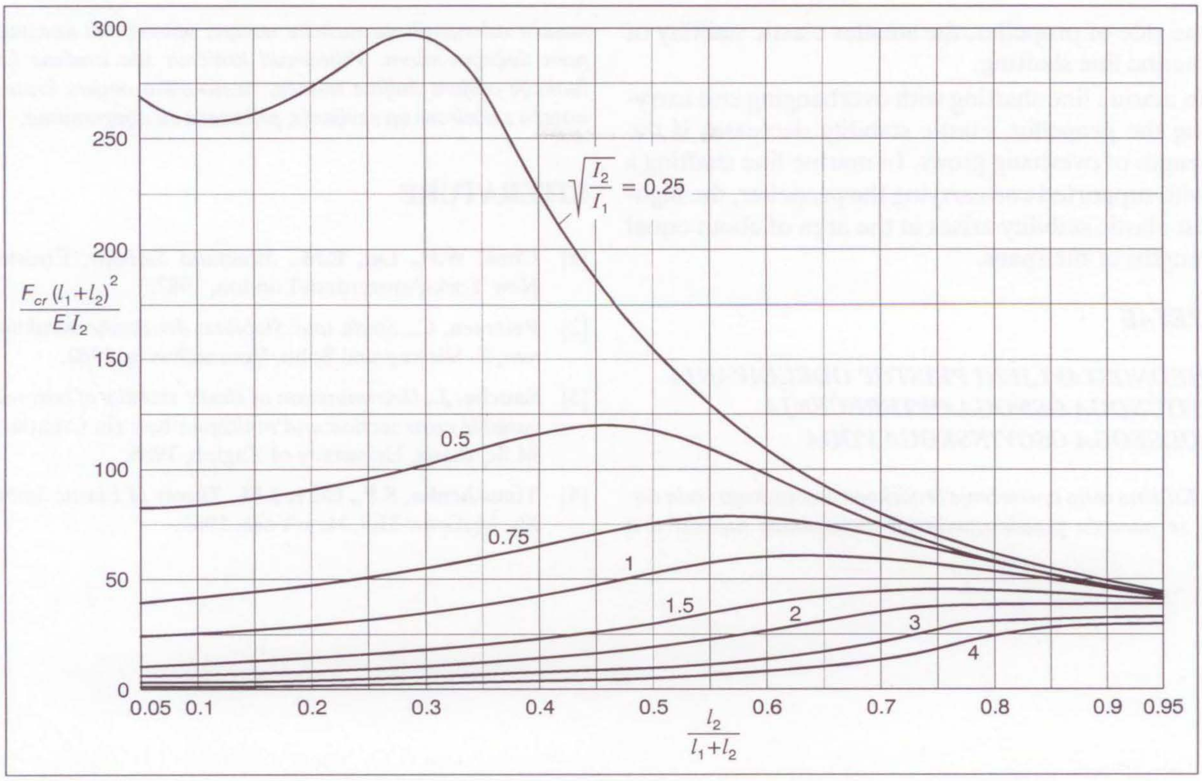

Diagram 2

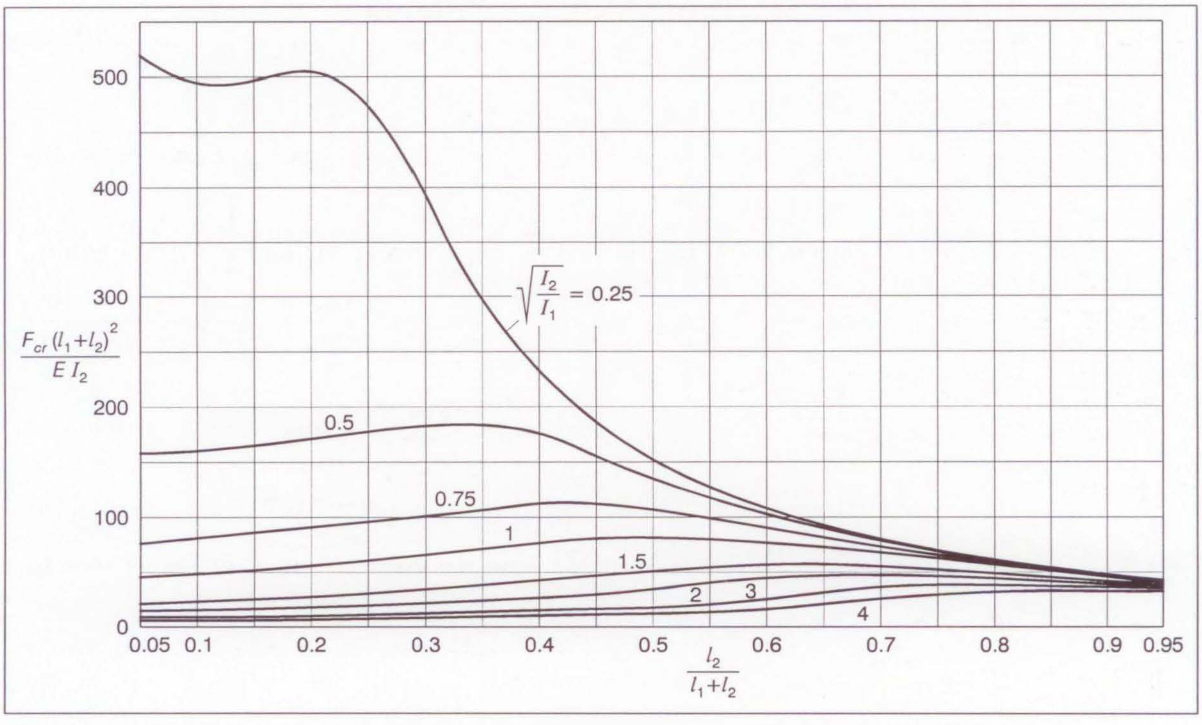

Diagram 3

force, which can be observed in diagrams 1, 2 and 3, results the following conclusions:

- The elastic stability of marine line shafting with supported end carrying the propeller is multiply larger than the one of marine line shafting with overhanging end.
- The bigger flexural rigidity of the span on the side of thrust bearing, with regard to the one of the span on

the side of propeller, the smaller elastic stability of marine line shafting.

- In marine line shafting with overhanging end carrying the propeller, elastic stability decreases if the length of overhang grows. In marine line shafting s with supported end carrying the propeller, the highest elastic stability arises in the area of about equal lengths of the spans.

SAŽETAK

POJEDNOSTAVLJENI PRISTUP ODREĐIVANJA KRITIČNOGA OSNOGA OPTEREĆENJA BRODSKOGA OSOVINSKOGA VODA

Kritično osno opterećenje brodskoga osovinskoga voda dobije se pomoću pojednostavljenih mehaničkih modela dva nosača oslonjenih na različite načine, opterećenih konstantnom tlačnom silom. Vrijednosti kritičnih sila izražene kao funkcije omjera duljine nosača, za nekoliko omjera krutosti nosača s obzirom na savijanje, prikazane su dijagramima.

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