MATIJA GLAD, M.Sc.
matija.glad@hrvatske-ceste.hr
Hrvatske ceste d.o.o.
Nikole Tesle 9/IX, HR-51000 Rijeka, Republic of Croatia
ZDENKO LANOVIĆ, B.Eng.
zdenko.lanovic@elipsa.hr
Elipsa - S.Z. d.o.o.
I. Vranovski ogranak 2, HR-10000 Zagreb,
Republic of Croatia
JASMINA PAŠAGIĆ, M.Sc.
jasmina.pasagic@fpz.hr
University of Zagreb,
Faculty of Transport and Traffic Sciences
Vukelićeva 4, HR-10000 Zagreb, Republic of Croatia

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MODEL FOR DETERMINING FIXED COSTS FOR THE WINTER SERVICE OPERATION

ABSTRACT

From the season 2005/06 a new dynamic model for the operation of the Winter Service in the Republic of Croatia will be used. The old model was based on three levels of readiness, and the roads were categorised primarily according to their administrative distribution. The new dynamic model has three levels of readiness, while the first level is further divided into two service levels. The road is classified to a certain readiness and service level according to the traffic, climate and economic conditions.

The new model splits the cost structure into fixed and variable costs. The investor wants to keep the fixed costs at a minimal level, which will guarantee proper readiness for quick intervention. The investor wants to ensure a technological infrastructure for quality cleaning of roads is created. The capital companies want larger fixed costs to ensure certain profit, and defined fixed costs enable them to asses the profitability of the Winter Service operation. Such structure forms the following relationship: in mild winters the capital companies "profit" and the investor "loses", and vice versa for cold winters. Mathematically, such relationship should be treated as a finite strategic two-player game.

This paper will show the model needed to forecast fixed costs in the new dynamic model for operation of Winter Service, through consideration of connection of linear programming and the matrix game theory, to study the problem in parallel, from the standpoint of both players.

KEY WORDS

winter service, dynamic model, game theory, fixed cost

1. INTRODUCTION

Modern society presumes a safe, fast and continuous traffic, regardless of weather conditions. In the Republic of Croatia, the changes to organisational structures for all factors in road maintenance, as well as many changes in legal regulations lead to the need for a different approach to planning and execution of maintenance. This especially applied to redefinition of standards for road maintenance, as well as to the need to adopt rules and technical conditions for winter service.

From the season 2005/06 onwards, a new dynamic model for winter service operation will be applied in the Republic of Croatia. The old model was based on three levels of priority, and the roads were classified primarily depending on their administrative division. The existing standards based on the average climatic conditions and are not adapted to dynamic traffic requirements and organisational changes to road management. The new dynamic model has three levels of priority, and the first of the priority classes is also split into two service levels. A road is classified into a priority level and a service class according to the traffic, climate and economic conditions. It is possible to forecast future periods when, by simulation of the conditions, the priority levels and service classes might change for particular roads.

The application of the new model into the operating documents was done by a team of experts from the company Hrvatske Ceste d. o. o. (Ltd.), and high probability exists that the project will be approved, and after the approval it will be implemented according to the quoted regulations, i. e. that new four-year contracts for maintenance of state, county and local roads will be signed.

The new model splits the cost structure to fixed and variable costs. Fixed costs should cover the establishing and maintaining of winter service readiness (technical preparation and maintaining working conditions, depreciation, insurance) and avoid complex calculations of passive and active readiness of workers and mechanisation. The variable costs represent effective work of winter service units to clear the snow and maintain the roads open. The investor wishes to reduce the fixed costs to a level which will guarantee quality of preparation and readiness for quick response, the aim being to establish a technical structure for satisfactory cleaning of the roads. The companies providing the service will want to keep the fixed costs as high as possible, to be sure of a certain profit level, defined fixed costs enable them to calculate the profitability of winter service operation. Such structure creates the following relation: for warmer winters the service companies will "profit", and the investor will "lose" and vice versa for colder winters. The possibility to divide the costs to fixed and variable encourages consideration of relations between the investors and service companies.

2. OLD AND NEW WINTER SERVICE MODELS

The winter service is a gathering of mutually connected elements with a multi-criteria goal "Maximal safety for drivers, with minimal maintenance costs". The existing models, and even the contracts between the investors and service companies, had not prescribed:

- minimal technical equipment for the service company:
  - machines, vehicles, permanent and temporary winter bases,
  - equipment (snow ploughs etc.)
- competencies and experience of legal and physical entities/persons,
- technological working procedures,
- the quality of materials to be spread on roads,
- method of accomplished work accounting,
- meteorological conditions (especially more recent meteorological data), and division of Croatia into climate zones, observing the specific climatic conditions.

For those reasons, constant conflicts and lack of satisfaction on both sides (i. e. investor and service company) were continuing all the time. This especially applied to readiness outside normal working hours, but also to waiting during the normal working hours, when there was no need for intervention. The new model introduces a set of variables by variation of which one can significantly change the functional operators: traffic-technological, technical, financial etc. The responsibilities of winter service are clearly defined, thus significantly reducing the scope for possible variations and manipulation of the variables.

However, regardless of differences and specifics of conditions, the proposed model for the winter service will clearly answer the two basic questions:

1. is the existing structure of the winter service optimal, and
2. can all the participants be satisfied (drivers, investor, service providers) with the services provided, for individual service levels, corresponding to the financial investments.

It was very difficult to answer these questions in the old model, since large errors were possible due to subjective decisions or assessments. Also, in the previous model, the quality of service was not mentioned at all. This is well covered in the new model, through the priority levels (shown by climatic zones) and through two service levels for the first priority level. The new model, compared to the existing one, shows four basic advantages:

1. no static levels of readiness,
2. the concession model is introduced, according to the optimal organisation, where both permanent (fixed) and variable costs are accounted for,
3. simulation is possible and changes to the priority levels and service levels are possible,
4. it is possible to plan costs of winter service, through simulations.

3. COST STRUCTURE

The current model for charging the cost of winter maintenance of public roads was not satisfactory for any participant. The new model for accounting and allocation of costs (fixed and variable) provides for the assumption that the fixed costs cover maintenance of readiness for winter service (technical preparation and maintaining the equipment in good order, depreciation and insurance). In this way the complicated charging for passive and active readiness of workers and machinery is avoided. The variable costs represent actual work of the winter service units to clear the roads from snow and keep them open.

3.1. Fixed costs

In the comprehensive system of winter service the following factors influence the fixed costs:

a) Climate zones, location and lengths of roads, by priority levels and service levels;

b) The number of vehicle units for roads inside and outside towns;

c) Participation of the vehicles in a transport unit;
d) Number of other transport units, equipment, machinery and road workers;
e) Levels of readiness by zones, with actual hours for transport units and road workers;
f) The levels of readiness by zones, with actual hours for other machines (graders, loading devices, rotary snow ploughs, and vehicles up to 2t);
g) Standardised number of interventions, quantity of materials spread, quantities of other materials, by climate zones;
h) Percentage of snow protectors by material types;
i) Percentage of depreciation and accounting days to calculate permanent costs;
j) Corrective coefficient for the hourly charges for workers, to calculate permanent costs;
k) Standardised number of vehicles and road workers engaged, depending on the duration of the levels of readiness.

The following Tables illustrate the complexity and importance of individual elements.

The analysis shows that the fixed costs represent about 33% of total costs of winter services. for that reason, special attention should be made to minimise

Table 1 - Lengths of state roads by climate zones (I. – V.):

No.	County	Total (km)	ZONE I	ZONE II	ZONE III	ZONE IV	ZONE V
1.	Međimurska county	75,510	75,510				
2.	Varaždinska county	205,154	205,154				
3.	Koprivničko - križevačka county	212,728	212,728				
4.	Bjelovarsko - bilogorska county	285,574	285,574				
5.	Virovitičko - podravska county	185,287	185,287				
6.	Zagrebačka county and Town Zagreb	304,578	304,578				
7.	Krapinsko - zagorska county	212,946	212,946				
8.	Požeško - slavonska county	228,867	228,867				
9.	Brodsko - posavska county	122,522	122,522				
10.	Osiječko - baranjska county	483,376	483,376				
11.	Vukovarsko - srijemska county	287,474	287,474				
12.	Karlovačka county	380,593	155,200	225,393			
13.	Sisačko - moslovačka county	413,223	254,311	158,912			
14.	Primorsko - goranska county	517,610		219,199		298,411	
15.	Ličko - senjska county	567,284			441,717	125,567	
16.	Zadarska county	601,139			97,500	324,629	179,010
17.	Šibensko - kninska county	353,065				104,863	248,202
18.	Splitsko - dalmatinska county	787,216				378,380	408,836
19.	Istarska county	310,609				310,609	
20.	Dubrovačko - neretvanska county	399,526				399,526	
	Total:	6.934,281	3.013,527	603,504	539,217	1.941,985	836,048

Table 2 - Location and lengths of roads by readiness levels and service levels in the first climate zone

1st climate zone			Length of the state roads by priority levels (km)				Total (km)
Number of readiness locations	Readiness location type	Road location	1st priority level		2nd priority level	3rd priority level	
			1st Service level	2nd Service level			
42	(S)	Roads outside towns	2.383,851		72,905	28,077	2.484,833
	(S)	Roads inside towns	469,627		46,667	12,400	528,694
Total:			2.853,478		119,572	40,477	3.013,527

Table 3a - Number of vehicle units for roads outside towns (1st level of readiness):

climate zones	1st level of readiness			
	1st priority level		2nd priority level	3rd priority level
	1st service level	2nd service level		
I.	60	70	80	90
II.	60	65	75	85
III.	55	55	65	75
IV.	120	120	120	125
V.	100	110	115	120

Table 3b - Number of vehicle units for roads inside town (1st level of readiness):

	1st level of readiness		
climate zones	1st priority level	2nd priority level	3rd priority level
I.	60	70	80
II.	50	65	70
III.	40	60	60
IV.	100	115	115
V.	90	110	110

Table 4 - Levels of readiness by zones, with effective hours for vehicles and road workers:

Climate zones	Duration of the readiness levels (days)				Ice on roads - effective (working hours by levels)				Cleaning the snow - effective (working hours by levels)				Total effective working hours by levels			
	1st	2nd	3rd	Total	1st	2nd	3rd	Total	st	2nd	3rd	Total	1st	2nd	3rd	G.Total
I.	50	45	25	120	0,50	1,10	1,30	2,90	1,00	2,60	3,50	7,10	1,50	3,70	4,80	10,00
II.	20	55	45	120	0,70	0,40	1,80	2,90	1,70	3,50	4,60	9,80	2,40	3,90	6,40	12,70
III.	20	53	47	120	0,30	0,40	2,10	2,80	1,70	2,10	4,00	7,80	2,00	2,50	6,10	10,60
IV.	100	14	6	120	0,60	1,00	2,00	3,60	0,30	0,50	0,80	1,60	0,90	1,50	2,80	5,20
V.	95	17	8	120	0,30	1,50	3,00	4,80	0,50	0,60	0,90	2,00	0,80	2,10	3,90	6,80

Table 5 - Number of interventions, quantity of spread and quantity of other materials, by climate zones:

Climate zones	NaCl		$CaCl_2$		Spread		Other matereials				
	No of intervent.	t/km	No of intervent.	t/km	No of intervent.	m3 /km	Edge poles (pcs/km)	Snow barriers (m/km)	Signs (pcs/km)	Addi-tional plates (pcs/km)	Grambit stuff (t/km)
I.	75	0,08	75	0,0009	38	0,12	20	5	1	0,5	0,2
II.	95	0,10	95	0,0007	48	0,21	14	3	1	0,5	0,2
III.	87	0,11	87	0,0008	44	0,18	14	3	1	0,5	0,2
IV.	20	0,05	20	0,0020	10	0,15			0,1	0,05	0,2
V.	45	0,06	45	0,0040	23	0,18	6		0,5	0,25	0,2

Table 6 - Standard number of engaged vehicles and road workers depending on duration of levels of readiness

No	Level of readiness	Lorry (pcs)	Unimog (pcs)	Grader (pcs)	Loader (pcs)	Rotary snow plough (pcs)	Workers (No)
1.	I.	16,53	4,13		8,75		20,67
2.	II. i III.	47,78	11,94	3,52	12,25	1,17	59,72
Total:		64,31	16,08	3,52	21,00	1,17	80,39

them. The approach must be founded on the partnership relation between the investor and service companies, since they share a common goal - in winter conditions enable good quality and safe traffic. Their business relation is one of conflicts: the service company wants to make as much profit as possible, while the investor wants to pay as little as possible.

4. RELATION OF INVESTOR AND SERVICE COMPANIES AS A STRATEGIC GAME

Determination of fixed costs in the dynamic model of winter service represents a "conflict" of two players - the investor and the service company. The sum of the game is zero, since the service company's profit is equal to the sum which the investor pays. Also, the players have a finite number of behaviour strategies for the 'conflict': mild winter with frequent snowfall, cold winter with little snow, etc. The specifics of the problem intuitively show that the players will have different numbers of strategies. For example, it is in the service company's interest to have several strategies and to negotiate more money through a number of similar scenarios, while the investor will prefer smaller number of clearly defined strategies regarding fast and good assessment of operating costs.

According to that, the business relation between the investor and the service company represents a matrix strategy game with two players and the sum of zero, since:

- the players have a finite number of possible strategies, and
- the loss to one player is equal to profit of the other player.

It will rarely occur that a saddle point will exist, i. e. that a strategy will exist in which one of the players (the services company) will win as much as possible and the other (investor) lose as little as possible. A game without saddle is more likely, i. e. the players will have mixed strategies. This corresponds to the real life situation, since the meteorological forecasts are not yet precise enough for longer time periods, and at the beginning of the winter both players determine their business strategies for various scenarios.

The described relation can be represented in the matrix as follows:

Player A – service company

Player B – investor (1)

M – game matrix

		Player B					
	Strategy	B_1	...	B_j	...	B_n	$\min_j(a_{ij})$
Player A	A_1	a_{11}	...	a_{1j0}	...	a_{1n}	a_1
	...	...	...	...	...	...	...
	A_i	a_{i1}	...	a_{ij}	...	a_{in}	a_i
	...	...	...	...	...	...	...
	A_m	a_{m1}	...	a_{mj}	...	a_{mn}	a_m
	$\max_i(a_{ij})$	b_1	...	b_j	...	b_n	

(2)

If player A selects strategy A_i, depending on the strategy chosen by player B, in the worst possible case he will get:

$$\min_j(a_{ij}) = a_i \tag{3}$$

These minimal profits are listed in the last column of the matrix (2). For player A the most favourable strategy is the one enabling him the largest profit:

$$\max_i(a_i) = \max_i \min_j(a_{ij}) = a_i \tag{4}$$

Player B in the worst possible case pays, if he selects strategy B_j, the following amount:

$$\max_i(a_{ij}) = b_j \tag{5}$$

The largest payment by player B is shown in the last row of the matrix (2), and the best strategy corresponds to the lowest of these payments:

$$\min_j(b_j) = \min_j \max_i(a_{ij}) = b_j \tag{6}$$

If player A with the best strategy gets at least the amount (4), and player B pays at the most the amount (6) the following unequal equation applies:

$$\max_i \min_j(a_{ij}) \leq \min_j \max_i(a_{ij}) \tag{7}$$

If in the expression (7) the equal sign applies, the element a_{ij} is called *the saddle point* and its value represents the *value of the matrix game*. For the games without the saddle points in the expression (7) the strict un-equality applies. In reality, the players will decide for the strategies:

1. player always decides for one strategy (the newer one makes the decision for some other one) – *the clean strategy*,
2. player decides on various alternatives; for each of them with a certain probability (the sum of these probabilities being equal to one) – *mixed strategy*.

The probabilities of choosing a specific strategy for individual players are marked as:

$$A: x = \{x_1, \dots, x_m\},\ 0 \leq x_i \leq 1 \tag{8.1}$$

$$x_1 + \dots + x_m = 1 \tag{8.2}$$

$$B: y = \{y_1, \dots, y_n\},\ 0 \leq y_i \leq 1 \tag{8.3}$$

$$y_1 + \dots + y_n = 1 \tag{8.4}$$

Since this as far as the winter service is concerned, the investor pays to the service company, i. e. player *A* (the services company) always wins, and player *B* (investor) always loses, the mean value of the player *A* wins is monitored – the mathematical expectation to win of the player *A*:

$$E(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}x_i y_j = x^T My \quad (9)$$

If x_0 and y_0 represent the optimal strategies, the following expression applies (*von Neumann's* theorem):

$$E(x_0,y)\geq E(x_0,y_0)\geq E(x,y_0) \quad (10)$$

The solution or the value of the game is represented by the *strategy game saddle* and is calculated to insert into (9) the optimal strategies for both players.

The advantage of matrix strategic games is the possibility to solve them indirectly via the linear programming techniques. The optimal strategy for player *A* is:

$$x=\{x_1,\ldots,x_m\} \quad (11)$$

it has the characteristics that player *A* gets the value w as the minimum, regardless of which strategy is chosen by player *B*. If player *B* plays the clean strategies:

$$\begin{aligned} y_1 &= \{1,0,\ldots,0\} \\ y_2 &= \{0,1,\ldots,0\} \\ &\vdots \\ y_n &= \{0,0,\ldots,1\} \end{aligned} \quad (12)$$

and player *A* the optimal strategy (11), the following applies:

$$\begin{aligned} E(x,y_1) &= a_{11}x_1+\ldots+a_{m1}x_m \geq w \\ E(x,y_2) &= a_{12}x_1+\ldots+a_{m2}x_m \geq w \\ &\vdots \\ E(x,y_n) &= a_{1n}x_1+\ldots+a_{mn}x_m \geq w \end{aligned} \quad (13)$$

All components in (11), optimal strategy of player *A*, are non-negative and fulfil the equation (8.2). If all non-equality expressions in (13) are divided by w and if one introduces the value:

$$u_i=\frac{x_i}{w} \quad (14)$$

the condition of non-negativity is preserved (u_i?0), and the equation (8.2) is transformed to:

$$u_1+\ldots+u_m=\frac{1}{w} \quad (15)$$

Player **A** (service company) wants to make the game value w as large as possible, i. e., to reduce its reciprocal value *1/w* and the following linear programming problem is formulated:

$$\begin{aligned} &\min(u_1+\ldots+u_m) \\ &a_{11}u_1+\ldots+a_{m1}u_m \geq 1 \\ &\vdots \\ &a_{1n}u_1+\ldots+a_{mn}u_m \geq 1 \\ &u_1\geq 0,\ldots,u_m\geq 0 \end{aligned} \quad (16)$$

By solving this linear program the value of the game w is calculated according to (15), and according to (14) the components of the optimal strategy for player *A* are obtained.

Calculation for optimal strategy for player **B** (investor) represents a dual linear program (16) and has the form of:

$$\begin{aligned} &\max(v_1+\ldots+v_n) \\ &a_{11}v_1+\ldots+a_{1n}v_n \leq 1 \\ &\vdots \\ &a_{m1}v_1+\ldots+a_{mn}v_n \leq 1 \\ &v_1\geq 0,\ldots,v_n\geq 0 \end{aligned} \quad (17)$$

Similar as for the player *A*, the calculation of program (17) produces the elements for the optimal strategy for player *B*:

$$v_j=\frac{y_j}{w} \quad (18)$$

$$v_1+\ldots+v_n=\frac{1}{w} \quad (19)$$

Hence, the programs (16) and (17) show how, when available financial values are known, one can calculate the fixed costs for the compound structure of the winter service, i. e. how one can, from the known strategies (standpoints) of individual players, as defined in matrix (2), obtain exact and desirable results: satisfied investor who knows he paid as much as was needed for establishing of good quality winter service and the service company that knows it will achieve a required level of profit.

5. CONCLUSION

The new dynamic model for winter service respects the climate differences and the demanded traffic volumes. The new model defines the priority levels and service levels according to the following criteria:

- traffic (road type, traffic flow and structure),
- climate (altitude, snow, ice, snowdrifts),
- economic (vehicles /1000 inhabitants, population/km² of the county).

The clear structure of the new model enables definition of the cost structure. Since the fixed costs contribute to approximately 33% of the total, significant savings can be obtained by their rationalisation, since it is known that for the investor Hrvatske ceste d. o. o. the total annual costs for winter services in Republic of Croatia amount to approximately 70.7 million kuna.

This paper has shown that by application of the game theory the fixed costs can be assessed realistically, and form the basis for establishing of fair partnership relation between the investor and service companies.

Mr. sc. **MATIJA GLAD**
matija.glad@hrvatske-ceste.hr
Hrvatske ceste d.o.o.
Nikole Tesle 9/IX, 51000 Rijeka, Republika Hrvatska
ZDENKO LANOVIĆ, dipl. ing.
zdenko.lanovic@elipsa.hr
Elipsa - S.Z. d.o.o.
I. Vranovski ogranak 2, 10000 Zagreb, Republika Hrvatska
Mr. sc. **JASMINA PAŠAGIĆ**
jasmina.pasagic@fpz.hr
Sveučilište u Zagrebu, Fakultet prometnih znanosti
Vukelićeva 4, 10000 Zagreb, Republika Hrvatska

SAŽETAK

MODEL ODREĐIVANJA FIKSNIH TROŠKOVA U RADU ZIMSKE SLUŽBE

Od sezone 2005/06. primijenit će se novi dinamički model rada zimske službe u Republici Hrvatskoj. Stari model zasnovan je na tri razine prednosti, a ceste su uvrštene u pojedini razred primarno temeljem njihove administrativne podjele. Novi dinamički model ima tri razine prednosti, a prva razina prednosti dva razreda služnosti. Cesta se svrstava u pojedinu razinu prednosti i razred služnosti prema: prometnim, klimatskim i gospodarskim uvjetima.

Novi model dijeli strukturu troškova na fiksne i varijabilne. Investitor želi fiksne troškove svesti do razine koji jamče kvalitetnu pripremu i spremnost za brzu intervenciju; želi osigurati stvaranje tehnološke strukture za adekvatno čišćenje cesta. Društva kapitala kroz veće fiksne troškove žele ostvariti sigurnost određenog profita; definirani fiksni troškovi omogućuju im procjenu profitabilnosti u radu zimske službe. Ovakva struktura stvara sljedeći odnos: u slabijim zimama društva kapitala „profitiraju" i investitor „gubi", a obrnuto je za jakih zima. Matematički se ovakav odnos promatra kao konačna strateška igra dva igrača.

U radu će se prikazati model procjene fiksnih troškova u novom dinamičkom modelu rada zimske službe kroz razmatranje sveze između linearnog programiranja i teorije matričnih igara te proučiti problematika paralelno s gledišta oba igrača..

KLJUČNE RIJEČI

zimska služba, dinamički model, teorija igara, fiksni troškovi

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