ZORAN RADMILOVIĆ, D.Sc.
E-mail: z.radmilovic@sf.bg.ac.yu
University of Belgrade,
Faculty of Transport and Traffic Engineering
Vojvode Stepe 305, 11000 Belgrade, Serbia&Montenegro
SAŠA JOVANOVIĆ, B.Eng.
E-mail: sjovanovic73@yahoo.co.uk
Technical University of Catalonia
Department of Nautical Science and Engineering
Pla. de Palau 18, 08003 Barcelona, Kingdom of Spain

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BERTH OCCUPANCY AT CONTAINER TERMINALS: COMPARISON OF ANALYTICAL AND EMPIRICAL RESULTS

ABSTRACT

The different kinds of container ships with variable number of containers arrive to ports, each container requiring single service.

In this paper, an analytical approach is developed with the help of bulk queueing system, to analyze and to plan the berth occupancy depending on the number of containers in on board and on shore and on the average waiting time/average service time ratios. The appropriate numerical results and graphs are presented for direct determination of the berth occupancies for different number of containers.

The arrivals of container ships at container terminal are usually a stochastic process. The number of berths required will depend on the berth occupancy. In order to determine the number of berths required, we have to know the distribution of ship arrivals and the distribution of ship service times including peak factors or seasonal variations. In this paper, the relationship between berth occupancy and container ship turnaround time at container terminal is based on bulk-arrivals and single service queueing models. We have assumed that the inter-arrival times and service times follow appropriate probability distributions with determined limitations. However, given results can be used with a high degree of confidence for first approximate solutions and as the control of berth occupancy or arrival of ship to berth.

KEYWORDS

berth occupancy, container ship, container terminal, bulk queueing, ship turnaround time

1. INTRODUCTION

Container terminal planning and management may be a very complex task. These activities must make the most of the available local resources to meet the required level of productivity, while attempting to reach a balance between the needs of the container port authorities, port operators, stevedoring companies and container shipping lines. The goal for all container port development should be the possibility of working on a 24/7/365 basis.

The container ship turnaround time in port includes the following processes and operations:

- arrival at port or estimated time of arrival,
- start of waiting for free berth, if no berths are available,
- free berth available,
- proceeding to berth
- arrival at berth, including berthing,
- start of loading/unloading operations,
- end of loading/unloading operations,
- unberthing and leaving of berth,
- leaving the port, including corresponding manoeuvres.

These processes cannot ideally follow one after the other, nor can they be distributed in constant time intervals. The factors affecting these processes are, for example, meteorological conditions, water levels at berths, number of containers, ships' capacity in TEU, etc. On the other hand, large container ships are very expensive to run, making any delays very costly and thus reducing economic benefits that would otherwise result from running a large container ship. Due to these reasons, such ships demand an optimal turnaround time of approximately 24 hrs top.

In general case, the average total port time of container ship or, port time of containers in defined group size per ship equals:

$$T_{cs} = T_w + T_b + T_{ber} + T_{unber} \quad (1)$$

where:

T_w – average waiting time of container ship or container group per ship for free berth

T_b – average service time of container ship or container group per ship

T_{ber} – average berthing time (T_{ber} = 1,0 hour per ship, according to international practice),

T_{unber} – average unberthing time (T_{unber} = 1,0 hour per ship)

In this paper, we shall compare the empirical and analytical results for the berth occupancy in container terminals. The empirical results were obtained from many container ports, according to Thoresen, 2003, whilst analytical results were derived from queueing theory. Therefore, the relationship between berth occupancy and container ship turnaround time at container terminal is based on bulk arrival and single service queueing models. We assumed that the inter-arrival times and service times follow appropriate probability distributions with defined limitations. In order to calculate the turnaround time of container ship it is essential to know if the ships arrive randomly and operational and control measurements such as average waiting time / average service time ratio depending on the berth occupancy. By means of this ratio we can analytically determine the turnaround time of container ship in port at a reasonable level of berth occupancy.

2. BERTH OCCUPANCY AT EXISTING CONTAINER TERMINALS IN THE WORLD

The berth occupancy ratio in percents due to working time including peak factor / week (Thoresen, 2003) is defined as:

$$B_{or} = \frac{T_{wtc} \times 100}{B_n \times \dfrac{W_d + W_h}{S_{cs}}} \tag{2}$$

or

$$B_{or} = \frac{G_{sts} \times 100}{B_n \times W_d \times W_h} \tag{3}$$

Where the following are the parameters for determining the berth occupancy ratio / week:

B_{or} – berth occupancy ratio in percentage,

T_{wtc} – total working time per container ship from berthing to unberthing in hours,

B_n – number of berths,

W_d – working days / week,

W_h – working hours / day,

G_{sts} – total ship to shore container gantry cranes working hours / week,

S_{cs} – number of container ships berthing / week.

The working time per container ship in hours can be calculated as

$$T_{wtc} = \frac{S_{bcs}}{C_n \times G_{bh} \times L_{sc} \times W_{ct}} \tag{4}$$

where:

S_{bcs} – number of container boxes handled by one container ship,

C_n – total number of ship to shore cranes working on each container ship,

G_{bh} – number of container boxes handled / container crane / hour,

L_{sc} – working time due to commencing and completing operations, due to basic output time, usually varying between 0.8 – 0.95

W_{ct} – working crane time due to ship total berthing time, usually varying between 0.7 – 0.9

The total ship to shore container cranes number is derived from:

$$G_{sts} = S_{cs} \times T_{wtc} \tag{5}$$

Where the total number of container ships needed to berth per week, including peak factor is

$$S_{cs} = \frac{C_{box}}{S_{bcs}} \tag{6}$$

Where the following are the parameters for determining the capacity per week:

C_{box} – number of container boxes handled / week,

S_{bcs} – number of container boxes handled by one container ship.

The total berth capacity or number of container boxes handled / week:

$$C_{box} = \frac{C_{TEU} \times P}{W_w \times R_{bt}} \tag{7}$$

where:

C_{TEU} – container movements / year,

P – peak factor per week, usually varying between 1.1 – 1.3

W_w – number of working weeks / year, advisable to use 50 weeks / year,

R_{bt} – ratio between number of boxes (total number of 20' and 40' containers) and number of TEU containers, usually varying between 1.4 – 1.7

Based on Equations (1) – (7) and on data obtained from container ports, the berth occupancies for container cargo berth operation are presented in Table 1.

These figures will depend on the port administration control of the arrival of the ship to the berth. High berth occupancy factor can seem attractive because it yields the highest berth utilization, but it is usual to assume a ratio between the average waiting time and average service time not higher than 5 – 20%.

Table 1 – Berth occupancy

Number of berths	Berth occupancy factor in percentage		
	Low	Average	High
1	25	35	45
2	40	45	50
3	45	50	55
4	55	60	65
5	60	65	70
6 or more	65	70	75

(Source: Thoresen, 2003)

3. ANALYTICAL DETERMINATION OF BERTH OCCUPANCY AT CONTAINER TERMINALS

In our study, we consider the total port time of container ships dependence on the number of containers in groups requiring single service at berths. For these reasons, the container ship – berth link is considered as the queueing system with bulk arrivals and single service for containers. Defining this queueing system we assumed the following:

- the discussed queueing system is the system with infinite waiting capacity where sources of arrival pattern are not integral parts of a container ship – berth link,
- service channels are berths with the similar or identical and independent cargo handling capacities,
- ship arrivals with containers are Poisson distributed random arrivals; containers arrive usually in bulk, for example, 10, 20, 30, 40, 50, 100, 300, 500 or 1000 TEU in container ship at container terminal or berth,
- all containers on board or at the berth wait until served at the port,
- service times (loading/unloading time of containers) are Erlang distributed with $k = 1$
- container group size is a random variable,
- queue discipline is first come – first served (FCFS, FIFO) by container group and random within the group.

In extended Kendall's queueing notation these systems are covered by $M^X/M/c\ (\infty)$ symbol, where M stands for Poisson inter-arrival times distribution and exponential service times distribution, X stands for random variable of the number of containers in group, c stands for number of berths, and ? stands for unlimited waiting area capacity.

According to theoretical results (Kabak, 1970, Chaudhry and Templeton, 1983, Radmilović 1992 and 1994, and Jovanović, 2003), the basic objective of the application of multichannel bulk queueing systems is determination of average waiting time / average service time ratio for container ships in the port and the turnaround time depending on the berth occupancy, number of berths, number of containers in group and other ship – berth link operating performance measures.

The average waiting time / average service time ratio can be generally defined (Radmilović, 1992) as:

$$\gamma = f(c,\rho) = \frac{T_w}{T_b} = \frac{L}{c\rho} - 1 \tag{8}$$

where:

T_w – average waiting time

T_b – average service time

ρ – berth occupancy

L – average number of container ships or groups of containers in port

Using mathematical derivations (Chaudhry and Templeton, 1983 and Radmilović, 1994), the average waiting time / average service time ratio in $M^X/M/c\ (\infty)$ queueing system when the number of containers in groups is constant, is obtained from the following equation:

$$\gamma = \frac{1+\bar{a}}{2c(1-\rho)} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c^2\rho(1-\rho)} - 1 \tag{9}$$

where:

$\bar{a}$ – average number of containers in the group,

P_n – steady-state probability that n containers are in port at time t.

In this case, the berth occupancy is defined as:

$$\rho = \frac{\lambda\bar{a}}{c\mu} \tag{10}$$

where:

λ – average arrival rate of container ships or groups of containers;

μ – average service rate of containers

If the number of containers in group is constant, the variance of random variable of container group X is $\sigma_a^2 = 0$, and then the average waiting time / average service time ratios and berth occupancy according to Eqs. (9) and (10) for the number of berths, for instance, $c = 1$, 2 and 3 are:

One berth, $c = 1$, $M^X/M/1\ (\infty)$ delayed system:

$$\gamma = \frac{1+\bar{a}}{2(1-\rho)} - 1,\ \rho = \frac{\lambda\bar{a}}{\mu} \tag{11}$$

Two berths, $c = 2$, $M^X/M/2\ (\infty)$ delayed system:

$$\gamma = \frac{1+\bar{a}}{4(1-\rho)} + \frac{1}{2(\bar{a}+\rho)} - 1,\ \rho = \frac{\lambda\bar{a}}{2\mu} \tag{12}$$

Three berths, $c = 2$, $M^X/M/3\ (\infty)$ delayed system:

$$\gamma = \frac{1+\bar{a}}{6(1-\rho)} + \frac{2}{3}\,\frac{3(\bar{a}+\rho)-a_1\bar{a}}{3\rho^2+\rho\bar{a}(5-a_1)+2\bar{a}^2} - 1 \qquad (13)$$

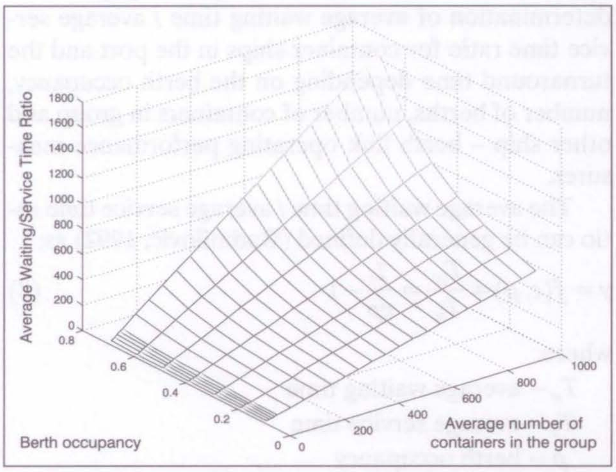

Figure 1 - Average waiting time / average service time ratios for $M^X/M/1\ (\infty)$ delayed system

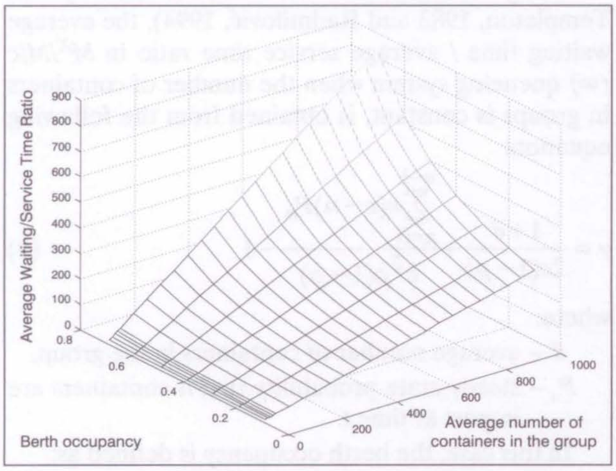

Figure 2 - Average waiting time / average service time ratios for $M^X/M/2\ (\infty)$ delayed system

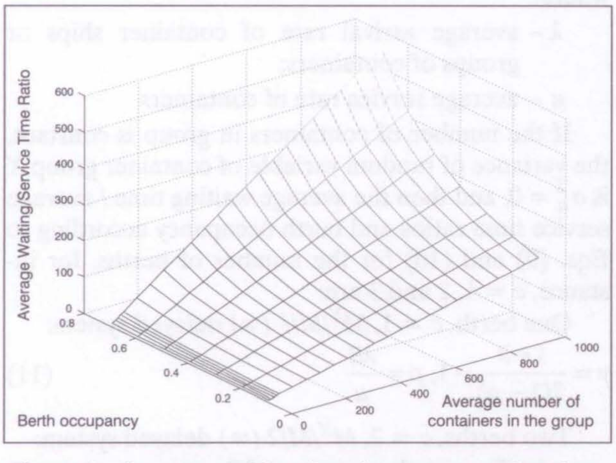

Figure 3 - Average waiting time / average service time ratios for $M^X/M/3\ (\infty)$ delayed system

where a_1 is a probability of one container or ship being present in the port. In our case a_1 will be equal to zero.

In Figures 1, 2 and 3, we present the numerical results of the average waiting time / average service time ratios obtained from $M^{X=\bar{a}}\ /\ M\ /\ c(\infty)$ delayed system and Eqs. (11), (12) and (13), depending on the number of berths, c = 1, 2 and 3, on the number of containers in groups $\bar{a} = 10, 20, 30, 40, 50, 100, 300, 500$ and 1000 and on the berth occupancy $\rho = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ and 0.7 (expected values for berth occupancies in container terminals).

4. COMPARISON OF ANALYTICAL AND EMPIRICAL RESULTS

The crane capacity per hour for handling containers can vary between 10 and the extreme 70 containers, with the average capacity of about 25 containers/h per crane. As for guidance only, the following can be used (Thoresen, 2003):

a) Rail-mounted harbour cranes can handle from ship to shore about 15 containers/h;
b) Mobile container cranes capacity is about 15 – 25 containers/h;
c) Shore-to-ship gantry cranes are capable of handling about 30 – 40 containers/h, whilst ship-to-shore gantry cranes with a secondary trolley system handle about 40 – 70 containers/h.

Using the Eqs. (1) through (13) and the data from Table 1 and Figures 1, 2 and 3, we can perform different comparisons between empirical and analytical values of berth occupancy, turnaround time of container ship or container group per ship, total working time per container ship, number of berths, etc.

It is known that the average service time for $M^{X=\bar{a}}\ /\ M\ /\ c(\infty)$ delayed system is

$$T_b = \frac{1}{\mu} \qquad (14)$$

or in this case for $\mu = 10, 20, 30, 40, 50, 60$ and 70 containers per hour, we obtain

$$T_b = 1/10 = 0{,}1;\ 1/20 = 0{,}05;\ 1/30 = 0{,}0333;$$
$$1/40 = 0{,}025;\ 1/50 = 0{,}020;\ 1/60 = 0{,}0166;$$
$$1/70 = 0{,}0143$$

For example, if mean number of containers in group is $\bar{a} = 100$, the number of berths is $c = 2$ and the empirical average berth occupancy $\rho = 0{,}45$ we yield from Eq. (12) the average waiting time / average service time ratio as:

$$\gamma = \frac{1+100}{4(1-0.45)} + \frac{1}{2(100+0.45)} - 1 = 44.9141$$

From Eq. (8) we have for $\mu = 40$ containers / hour

$$T_w = \gamma T_b = 44.9141 \cdot 0.025 = 1.1228\,\mathrm{h}$$

The total turnaround time of container ship in port or the container group per ship from Eq. (1) equals to

$$T_{cs} = 1.1228 + 0.025 + 1.0 + 1.0 = 3.1478\,\mathrm{h}$$

Under assumption of the possibility of working on 24/7/365 basis, the working time per container ship for loading and unloading in hours and number of container ships berthing per week $S_{cs} = 2$, from Eqs. (2) and (4) is:

$$T_{wtc} = B_{or}\left(B_n \frac{W_d \times W_h}{S_{cs}}\right) = 0.45\left(2\frac{7 \times 24}{2}\right) = 75.6\,\mathrm{h}$$

Total ship to shore container gantry cranes working hours per week including peak factor Eq. (5) is:

$$G_{sts} = 2 \times 75.6 = 151.2\,\mathrm{h}\,/\,\mathrm{week}$$

The number of containers boxes handled per week, Eq. (6) and Eq. (7) is:

$$C_{box} = 2 \times 100 = 200\ \text{containers / week}$$

If we presume the peak factor per week $P = 1{,}5$, number of working weeks per year $W_w = 50$ and ratio between number of boxes to number of TEU containers $R_{bt} = 1{,}2$ we yield

$$C_{TEU} = \frac{200 \cdot 50 \cdot 1.5}{1.2} = 12.500$$

container movements / year

For each empirical berth occupancy we can perform the analysis of main operating performance measures in feeder or hub container terminals under different conditions in relation with theoretical results for average waiting time / average service time ratio, for example in Figures 1, 2 and 3. Computational results can easily be extended to other queueing models and cases.

5. CONCLUSIONS

Conclusions obtained from this study can be summarized as follows:

1. The berth occupancy and other main operating measures in the container ship – berth link are examined as empirical and analytical values.
2. Analytical results were obtained from $M^{X=\bar{a}} / M / c(\infty)$ bulk arrivals and single service queueing system used for the operational analysis in container ship – berth link.
3. For different combinations of input values, one can easily see adequate solutions for turnaround time of container ship or total time of container groups per ship in container ship – berth link within the container terminal.
4. Estimation of ship (or container group per ship) turnaround time during the considered time period depends on the acceptable average waiting time / average service time ratio. This parameter is obtained as the function of berth occupancy, number of berths, number of containers, waiting and service times, and berthing and unberthing times.
5. At lower berth occupancies, $\rho < 0{,}5$, the average waiting time / average service time ratio increases slowly whilst at higher berth occupancies, $\rho \geq 0{,}5$ and greater number of containers in group, the average waiting time / average service time ratio increases more rapidly.

Finally, the applied methodology is rather simple in the estimation of the existing state and planning requirements for the container ship – berth link on container terminals. Furthermore, it provides better management and control as well as decision-making processes related to container ship and their standing times in port and port utilization.

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