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HYBRID TIME-SERIES FORECASTING MODELS FOR TRAFFIC FLOW PREDICTION

ABSTRACT

Traffic flow forecast is critical in today's transportation system since it is necessary to construct a traffic plan in order to determine a travel route. The goal of this research is to use time-series forecasting models to estimate future traffic in order to reduce traffic congestion on roadways. Minimising prediction error is the most difficult task in traffic prediction. In order to anticipate future traffic flow, the system also requires real-time data from vehicles and roadways. A hybrid autoregressive integrated moving average with multilayer perceptron (ARIMA-MLP) model and a hybrid autoregressive integrated moving average with recurrent neural network (ARIMA-RNN) model are proposed in this paper to address these difficulties. The transportation data are used from the UK Highways dataset. The time-series data are preprocessed using a random walk model. The forecasting models autoregressive integrated moving average (ARIMA), recurrent neural network (RNN), and multilayer perceptron (MLP) are trained and tested. In the proposed hybrid ARIMA-MLP and ARIMA-RNN models, the residuals from the ARIMA model are used to train the MLP and RNN models. Then the efficacy of the hybrid system is assessed using the metrics MAE, MSE, RMSE and R2 (peak hour forecast-0.936763, non-peak hour forecast-0.87638 on ARIMA-MLP model and peak hour forecast-0.9416466, non-peak hour forecast-0.931917 on ARIMA-RNN model).

KEYWORDS

time-series analysis; traffic flow forecasting; random walk model; residual analysis; ARIMA-MLP; ARIMA-RNN.

1. INTRODUCTION

In developing technology, real-time traffic flow prediction is a crucial problem in intelligent transportation systems because traffic congestion problems need to be managed [1, 2]. Especially in the peak hours, analysing the time-series traffic data and understanding the traffic characteristics is crucial in the transportation process [3]. Additionally, the traffic flow behaviour has been analysed using monitoring devices that help controlling traffic congestions. The traffic monitoring system records the time-series activities in peak time and non-peak time to estimate the traffic congestion [4]. The continuous monitoring of traffic data helps in detecting the traffic congestion. Apart from these issues, the traffic flow data collected from the source will have an irregular pattern most of the time, which creates difficulties in building the model [5]. The forecasting process has to overcome these issues. The forecasting techniques analyse the historical traffic data and predict the future data. Several forecasting techniques such as autoregression (AR), moving average (MA), autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA), vector autoregression (VAR), simple exponential smoothing (SES), etc. are normally being used to successfully analyse the traffic flow time-series data [6–8]. The residual analysis consists of several assumptions while performing the statistical test on the traffic data forecasting process [9]. Therefore, the time-series dataset is successfully handled by forecasting technique which uses the day, hour, week, and monthly data to predict the future data. As discussed, the error or residual value reduces the entire performance of the forecasting system. Different machine learning and deep learning models are utilised in many researches to reduce the forecasting errors and maximise the precision value [10–12]. These techniques examine the computational relevance and correlation that exists in the time-series data, which helps the models to forecast the future time-series data pattern from the past analysis. This leads to an increase in the overall forecasting accuracy. The machine learning techniques undergo the following steps, such as: data gathering, exploration, data preparation, forecast modelling, and performance evaluation in order to effectively analyse the time-series information according to the defined criteria, and the future variables are forecasted successfully. Even though these methods work effectively, the time-series forecasting process has few challenges, such as lack of data, lack of domain knowledge and low accuracy results, leading to computational complexity. Therefore, in this work, the various forecasting models are combined to build the hybrid models in order to enhance the prediction accuracy. The hybrid model is a mixture of linear and nonlinear models. The residual from the linear model is given to nonlinear model to learn its nonlinear behaviour. The development of hybrid autoregressive integrated moving average with multilayer perceptron (ARIMA-MLP) and hybrid autoregressive integrated moving average with recurrent neural network (ARIMA-RNN) models examines the traffic flow time-series data to predict the future traffic flow. The main advantage of integrating the linear and nonlinear model is to enhance cooperative selection of inherent features in the time-series data. The hybrid models are implemented and are evaluated with the performance metrics.

The remaining work is organised as follows: Section 2 examines the different researcher opinions regarding the time-series forecasting process. Section 3 discusses the traditional forecasting models such as autoregressive integrated moving average (ARIMA), multilayer perceptron (MLP) and recurrent neural networks (RNN) models. Section 4 examines the hybrid time-series forecasting models. Section 5 discusses about the results obtained from the traditional models and hybrid models. Finally, Section 6 discusses the conclusion of the work.

2. RELATED WORK

This section elaborates on the different researcher opinions, working processes, thoughts and ideas on the traffic flow forecasting process. Alghamdi et al. [13] used the autoregressive integrated moving average (ARIMA) model to implement the traffic congestion forecasting system. Long-term traffic forecasting is not normally recommended due to the unpredictable nature of road traffic circumstances. As a result, this system examined the non-Gaussian traffic data using a short-term, time-series technique to predict traffic congestion and identify abnormal traffic status. Short-term traffic estimates on highways are created based on current and historical traffic data and range from a few seconds to a few hours in the future. The system successfully preprocessed the traffic data and noted the traffic congestion and traffic flow with a 95% confidence level. Pan et al. [14] implemented the time-series forecasting system by applying hybrid extreme learning machine models. This process examined both linear and nonlinear patterns in creating the time-series forecasting model. The hybrid forecasting model efficiency was evaluated with linear autoregression neural network technique.

Ma et al. [15] developed the time-series-based traffic forecasting model using a hybrid machine learning approach. During this process, the multilayer perceptron derived the scale-co-movement patterns from the traffic flow. In addition to this, location-related traffic features were derived using an autoregressive integrated moving average (ARIMA) approach. The extracted features were further examined using a multidimensional support vector machine approach that predicted the traffic flow with maximum forecasting accuracy.

Wu et al. [16] proposed a hybrid deep learning approach to develop a traffic flow prediction system. The hybrid approach analysed the spatial and temporal characteristics of the traffic flow and resolved the involved issues. During this process, daily or weekly traffic flow characteristics were analysed, and the spatial features were extracted with convolution networks. The recurrent network was utilised to derive the temporal features used to predict the traffic flow with maximum accuracy. Boukerche et al. [17] efficiently analysed the hybrid machine learning models while predicting vehicular traffic flow. This system utilised the graph convolution neural networks, gated recurrent unit and the deep aggregation structures for analysing the traffic flow pattern by resolving the real-time online prediction task. The developed system used the refinement-learning concept in understanding the online prediction strategy. The efficiency of the system was determined using the vehicular cloud structure.

Kumar et al. [18] developed a short-term traffic flow forecasting system using seasonal autoregressive integrated moving average (SARIMA) approach. This system utilised the three-lane Chennai arterial roadway traffic information. The collected data were analysed using partial autocorrelation function that derived the traffic patterns. The model parameters were further studied with the maximum likelihood value presented in R. The derived features were also analysed using the ARIMA model that predicted the traffic flow with 4 to 10 of mean absolute percentage error. Li et al. [19] forecasted the particulate matter 2.5 using hybrid convolution with long short-term memory neural networks (CNN-LSTM). The system used the seven days of air quality information from Beijing. The gathered details were examined using four models such as multivariate LSTM, univariate LSTM, multivariate CNN-LSTM and univariate CNN-LSTM that predicted the particulate matter. The hybrid model predicted the particulate matter efficiently with the minimum error rate and achieved the same with minimum training time.

Bandara et al. [20] introduced the recurrent neural network based clustering approach to forecast the time-series database information. The system used the time-series data from the CIF2016 forecasting competition dataset that has been analysed using the recurrent network, which examined the similar data to form the clusters. This model predicted the time-series information and the system efficiency was determined using the benchmark dataset, which ensured maximum accuracy.

Osipov et al. [21] introduced spiral structured layer-based recurrent neural network for forecasting the traffic flow. The proposed algorithm analysed the traffic flow details, and the information was saved in the neural network memory – the stored data was used to recognise future traffic information without creating any delay. During the analysis process, the system used the spiral structure of layers that predicted the traffic flow with maximum accuracy. Xu et al. [22] utilised ARIMA and Kalman filter for predicting the road traffic state. Initially, the ARIMA model was applied to analyse the time-series data. The Kalman filter was then combined with the ARIMA to predict the future forecast and the optimised parameters were selected by training the model.

Hosseini et al. [23] developed a traffic flow prediction system which was able to handle the data loss and noise data conditions while predicting the Minnesota traffic flow. During this process, the traffic data were analysed by considering the traffic congestion, weather conditions and accidents. From this information, mutual information-based features were selected that were processed by ARIMA and MLP model that successfully forecasted the traffic flow robustly. Lv et al. [24] applied big data and deep learning concepts to forecast the traffic flow. The collected traffic data were examined continuously for extracting the temporal and spatial correlations. The derived features were further analysed using a stacked autoencoder approach, and the training process was performed with the help of a greedy layer-wise method. With the help of the trained features, new traffic data were forecasted successfully using deep learning networks. The author introduced big data with a deep learning based traffic prediction system that ensured superior results.

Luo et al. [25] introduced the holiday-based traffic flow forecasting system using discrete Fourier transform and support vector regression approach. Initially, holiday-related traffic data were gathered from the Jiangsu province of China which were then analysed by Fourier transform. The analysis process had a specific threshold value used to derive the traffic features. Then the support vector regression was used to successfully predict the holiday traffic patterns on the residual time series. Kumar et al. [26] predicted the traffic flow from non-urban highways using artificial neural networks. The neural network examined the various traffic information like density, speed, traffic volume and time-related data. The neural network finally forecasted the traffic patterns, increasing from 5 minutes to 15 minutes interval.

Ardalani-Farsa et al. [27] proposed a prediction and residual analysis system by applying hybrid Elman-NARX networks. Initially, the time-series data were collected and then processed by embedding theorem that derived the various phase space points. The derived attributes were trained with the Elman networks. Then the residual time-series forecasting process was done by the NARX network. The hybrid network predicted the relationship between residual time-series data and successfully predicted original time-series data. Lu et al. [28] addressed the stochastic and complexity issues in short-term traffic prediction by applying recurrent neural network. This process combined the autoregressive integrated moving average (ARIMA) model with the recurrent neural network to examine the nonlinear traffic features. From the analysed information, a dynamic weighting sliding window helped to predict future traffic patterns. The system was evaluated using Highway AL215, Highway AL2206 and Highway AL2292 dataset information. Finally, the system attained promising results with reduced error rate.

Shen et al. [29] developed a deep learning based time-series forecasting system for overcoming the low forecasting accuracy issue. The system used the SeriesNet approach for examining the traffic data. The traffic data were initially analysed by a long, short-term network that extracted holistic features. The dimensionality of the derived features was minimised with the help of causal convolution networks. Later, the multi-level features and the range information were analysed by SeriesNet that predicted the traffic data successfully. Rajendran et al. [30] introduced the urban transportation short-term traffic prediction system by applying the structure pattern and regression model. The traffic data were analysed using a locally weighted learning process which derived the present and next traffic patterns. The forecasting model attained better results on both nonlinear and linear models.

Sharma et al. [31] forecasted the traffic flow on short-term basis using artificial neural networks. The traffic data were gathered for a two-lane highway from the Indian roads and processed by back-propagation networks. The derived features were examined using different layers, and the output was successfully forecasted. The neural network with K-nearest neighbour regression model predicted the traffic data with 0.9962 R2 values.

The traffic flow forecasting has a significant role in resolving traffic congestion issues as per the literature stated above. Statistical methods and machine learning methods are two coexisting approaches to time-series forecasting, each with its own set of strengths and limitations. Statistical methods like ARIMA have captured only linear relationships in data, that may not be the case in real-world data which may contain both linear and nonlinear patterns, limiting forecasting performance. Machine learning methods like neural networks have achieved better results with nonlinear time-series data. But the machine learning methods have achieved inconsistent results for purely linear time series, despite successfully overcoming the shortcoming of statistical models in nonlinear connections. As a result, neither ARIMA nor neural networks are adequate for modelling a real-world time series. Hence, by combining the best of statistical and machine learning methodologies, hybrid methods promise to advance time-series forecasting. This motivates the development of hybrid models that combine statistical ARIMA and nonlinear MLP and RNN models to efficiently forecast linear and nonlinear time series. The detailed working of the hybrid traffic flow forecasting system is discussed in Section 4.

3. TRADITIONAL TIME-SERIES FORECASTING MODELS

This section discusses the three traditional forecasting models, such as autoregressive integrated moving average (ARIMA), multilayer perceptron (MLP) and recurrent neural network (RNN) model. These models are trained for the given traffic flow data which later forecast the future flows. In this connection, the notations utilised in this paper are described in *Table 1*.

Figure 1 shows the process involved in time-series forecasting for traffic flow prediction. The following steps are involved in the prediction process: traffic data collection, data preprocessing, training

Table 1 – Notations and descriptions

Notations	Descriptions
T	Total number of traffic flow data
N	Total number of training data
$D(y_1,y_2,y_3,...,y_n,y_{n+1},...,y_{n+t-1},y_{n+t})$	Time series data (training data + test data)
$X(y_1,y_2,y_3,...,y_n)$	Training data (January 2020 – June 2020)
$Y(y_{n+1},...,y_{n+t-1},y_{n+t})$	Testing data (July 2020)
$\bar{Y}(\bar{y}_{n+1},...,\bar{y}_{n+t-1},\bar{y}_{n+t})$	Forecasted data from ARIMA model
$E=Y\text{-}\bar{Y}(e_{n+1},...,e_{n+t-1}, e_{n+t})$	Residual or error value
$\bar{E}$	The output of residual analysis by MLP/RNN in the hybrid model
$\hat{Y}=\bar{Y}+\bar{E}(\hat{y}_{n+1},...,\hat{y}_{n+t-1},\hat{y}_{n+t})$	Final traffic flow forecast from the hybrid model

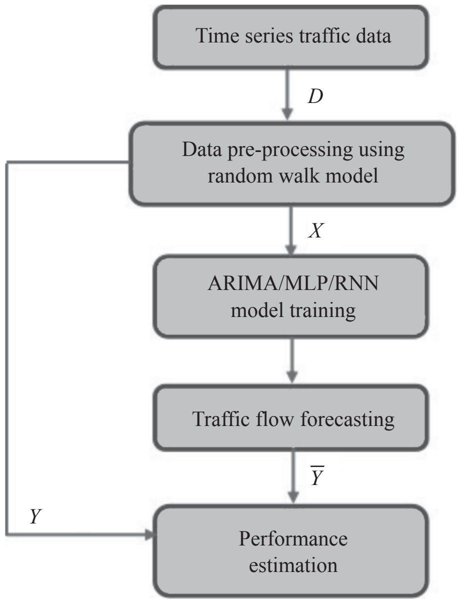

Figure 1 – Time-series forecasting model

the forecasting model and finally, forecasting the traffic flow. The detailed working process is discussed as follows.

3.1 Traffic data collection

The initial step is the traffic data collection process. The UK Highways dataset is utilised to build the proposed traffic flow forecasting model. The MIDAS Site – 5,825 at M48 westbound between M4 and J1 (102022401) – UK data are used. The dataset consists of traffic flow data collected in every 15-minute interval per day for each month. This work uses the 2020 traffic flow data for implementation purposes. The dataset consists of almost 20,449 records; in which, January 2020 to June 2020 records (17,473) are utilised for training the model, and July 2020 records (2,976) are used for testing the forecasts. In this dataset, weekday (Monday to Friday) traffic flow data for peak hour (9 AM) and non-peak hour (9 PM) are considered for analysis. The attributes of the dataset are described in *Table 2*.

Table 2 – Attribute description

Attributes	Description
Local Date	Date of the year
Local Time	Time of the day
DayTypeID	Day in the week
Total Carriageway Flow	Traffic flow data for every 15-minute interval

3.2 Traffic data preprocessing

The data retrieved from the dataset consist of several unwanted, irrelevant and missing information that ultimately affects the forecasting system. To overcome this issue, data preprocessing should be performed. After gathering the traffic data, they need to be analysed by applying a preprocessing technique called a random walk model. The random walk model is a simple and easy method for analysing time-series data. With this preprocessing, each set of data take an unexpected step that is independently and also identically distributed among the available data. The random walk model is used to find the presence of drift or no-drift in the time-series data. For the nth time period and k step forecast, the random walk model of variable Y for drift is defined in *Equation 1*.

$$\hat{Y}_{n+k} = Y_n \tag{1}$$

If there is no drift in the traffic flow data, prediction is carried out easily. If there is a drift in the traffic flow data, then average period-to-period observing points need to be computed to replace the drift value. The value is estimated by calculating the difference of last and first value and the given resultant value divided by n-1.

$$\hat{d} = \frac{Y_n - Y_l}{n - 1} \tag{2}$$

In *Equation 2*, $\hat{d}$ is defined as the difference value, n is referred as the number of time-series data, Y_l is defined as the first value of time-series data and Y_n is defined as the last value of time-series data. Based on the above computation, data preprocessing is carried out on the traffic flow data. The preprocessed data are used to train the time-series forecasting models, such as autoregressive integrated moving average (ARIMA), multilayer perceptron (MLP) and recurrent neural network (RNN) approaches.

3.3 Training the forecasting model

As discussed earlier, the forecasting model predicts future traffic flow data with minimum prediction errors. Here, traffic data from the UK Highways dataset – 1 January 2020 to 30 June 2020 – are used for training purposes. Each forecasting model is discussed as follows.

Autoregressive integrated moving average (ARIMA)

A mathematical model called the Box-Jenkins model is used to forecast data in a time series. The data in the time series can be stationary or nonstationary. The statistical properties of a stationary process do not vary throughout time. The properties of a nonstationary process/time series alter throughout time. The Box-Jenkins model uses the disparities between data points to make a nonstationary time series stationary. Autoregressive (AR) moving average (MA) or ARMA models can be used to model any stationary time series. To determine the greatest fit of a time-series model to past values of a time series, the Box-Jenkins technique uses autoregressive moving average ARMA or ARIMA models. This enables the models to detect patterns, with computations often involving auto regression (p), moving averages (q) and differencing (d).

The computed values fit the ARIMA model and X_t is calculated using *Equation 3*.

$$X_t = \Phi_1 X_{t-1} + \cdots + \Phi_p X_{t-p} + \ldots + \Phi_1 \varepsilon_{t-1} + \Phi_2 \varepsilon_{t-2} + \Phi_q \varepsilon_{t-q} + \cdots + \varepsilon_t \tag{3}$$

In *Equation 3*, time-series traffic data are defined as X_t, where the integer index denotes the time period t, Φ_p is an autoregressive model parameter, Φ_q is ARIMA model moving window parameter and ε_t is an error value. In addition to this, differencing parameters are computed for the consecutive observations as shown in *Equation 4*.

$$d = y_t - y_{t-1} \tag{4}$$

The significant orders are selected based on Akaike's Information Criteria (AIC) and Bayesian Information Criteria (BIC) Henry De et al. [32].

$$AIC = -2\log(L) + 2(p+q+k) \tag{5}$$

$$BIC = AIC + (log(T) - 2)(p+q+k) \tag{6}$$

In *Equations 5 and 6*, L denotes traffic data likelihood value, p and q represent an order of autoregressive and moving window part and k is the ARIMA model intercept value. If the computed AIC and BIC values are minimum, then the ARIMA model is a good selection.

The time-series values are said to be stationary if the ACF (autocorrelation factor)/PACF (partial autocorrection factor) of the time-series data abruptly shuts off or fades down. The ACF/PACF of the time-series data is said to be nonstationary if it either shuts off or fades down exceptionally slowly, Milenkovic et al. [33]. The autocorrelation and partial autocorrelation plots are shown in *Figures 2 and 3* with lags of 50. The plots make it evident that the series is nonstationary.

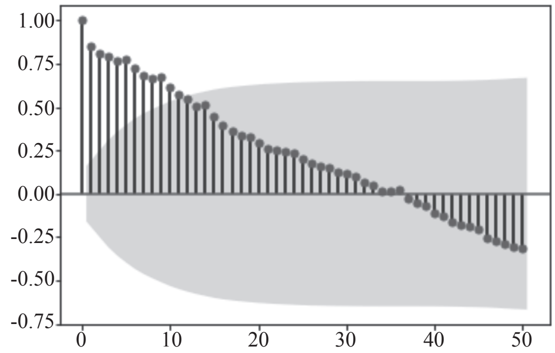

Figure 2 – Autocorrelation plot

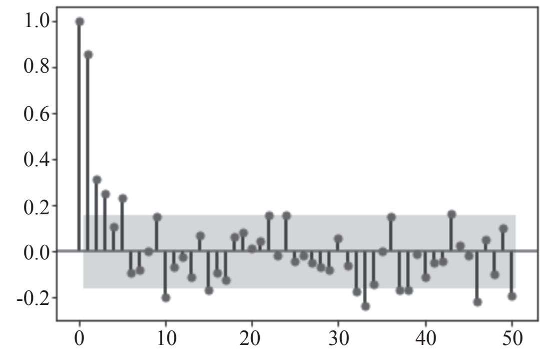

Figure 3 – Partial autocorrelation plot

The ADF test (augmented Dickey Fuller test) is a common statistical test used to determine whether a time series is stationary or not. It is one of the most extensively used statistical tests for determining if a series is stationary. *Figure 4* depicts the outcome of the ADF test. The p-value is more than the threshold, indicating that the series is nonstationary.

```
Test Statistic                  -1.462275
p-value                          0.552084
#Lags Used                       4.000000
Number of Observations Used    146.000000
Critical Value (1%)             -3.475953
Critical Value (5%)             -2.881548
Critical Value (10%)            -2.577439
dtype: float64
```

Figure 4 – ADF test for nonstationary series

The series is differenced to make it stationary. After differencing, the ADF test is performed, and the result is shown in *Figure 5*. As a result, the p-value is close to zero, indicating that the differenced series is stationary.

By comparing the results of various models of ARIMA with varying p and q values as shown in *Table 3*, it has been identified that ARIMA (4,1,2) is the best model with minimum AIC.

The residuals of ARIMA are tested to verify the presence of white noise. If the series is a random sequence of numbers that cannot be predicted then

```
Test Statistic                  -9.643256e+00
p-value                          1.501147e-16
#Lags Used                       3.000000e+00
Number of Observations Used      1.460000e+02
Critical Value (1%)             -3.475953e+00
Critical Value (5%)             -2.881548e+00
Critical Value (10%)            -2.577439e+00
dtype: float64
```

Figure 5 – ADF test for stationary series

Table 3 – Various ARIMA Models

Model	AIC
ARIMA (4, 1, 1)	1095.946
ARIMA (4, 1, 2)	1094.761
ARIMA (4, 1, 3)	1096.592
ARIMA (4, 1, 4)	1096.178
ARIMA (5, 1, 1)	1097.502
ARIMA (5, 1, 2)	1096.694
ARIMA (5, 1, 3)	1098.427

the series is said to contain white noise. The Ljung-Box test is conducted on the residuals to check the presence of white noise. *Figure 6* shows the result of Ljung-Box test on residuals. The first value is the test statistic and the second one is the *p*-value based on chi-square distribution. As shown in the result, *p*-value is greater than 0.05 which means that the residuals contain white noise and are uncorrelated. Hence, applying nonlinear models like multilayer perceptron (MLP) and recurrent neural network (RNN) would improve the forecasting outcomes even more.

```
(array([8.43004344e-05]), array([0.9926743]))
```

Figure 6 – Ljung-Box test results

The time-series data change over time. It means that the average value is not constant over time and also the variance is not constant over time. The R^2 expression is the ratio between the sum of squared errors and the estimated variance of the sample set taken for forecast. The larger the sum of squared errors, R^2 tends to 0. On the opposite, the smaller the sum of squared errors, R^2 tends to 1. Another factor that pushes R^2 to 1 is the growing variance. Indeed, for the nonstationary linear time series, the value of R^2 can quickly move close to 1, because of larger variance and not due to lower total error. R^2 may be high if the model is overfitting. Hence, R^2 is a biased estimate. Adjusted R^2 works better than R^2 in evaluating good fit in linear forecasting models. As shown in *Table 4*, R^2 for ARIMA model in peak hour forecast is 0.89 and for non-peak hour 0.82. The adjusted R^2 for ARIMA model is 0.865 and 0.792 for peak hour and non-peak hour forecast, respectively. Since there is less difference between R^2 and adjusted R^2, the model has a good fit.

The BDS test statistic is used to determine whether or not the series is linear, Cromwell et al. [34]. *Figure 7* shows the result of BDS test. Since the *p*-value is zero for all embedding dimensions (near points), the time series is nonlinear. The detailed descriptions of MLP and RNN are discussed in the following sections.

Multilayer perceptron (MLP)

Multilayer perceptron (MLP) is one among the intelligent techniques used to forecast the time-series traffic data. This model predicts the data effectively and extrapolates the generated data without requiring any assumptions. The multilayer perceptron with back propagation is used to analyse the time-series data and to efficiently perform the forecasting process. The network consists of three

```
               BDS Test

data:  y

Embedding dimension =  2 3

Epsilon for close points =  17.7998 35.5996 53.3993 71.1991

Standard Normal =
      [ 17.7998 ] [ 35.5996 ] [ 53.3993 ] [ 71.1991 ]
[ 2 ]     37.7470     39.3945     25.3685     16.0167
[ 3 ]     51.2712     47.7674     29.2289     18.0360

p-value =
      [ 17.7998 ] [ 35.5996 ] [ 53.3993 ] [ 71.1991 ]
[ 2 ]           0           0           0           0
[ 3 ]           0           0           0           0
```

Figure 7 – BDS test results

layers, namely the input, hidden and output layer. The input layer consumes time-series lagged values as input that is denoted as $y_{t-1},...,y_{t-p}$. From the notation, it means that the p nodes of the input layer are connected with the q nodes of the hidden layer. Each layer has specific weight and bias values that are used to compute the output values. The network uses the activation function $f(x)$ to interface the input and output layer. The sigmoid function is used as the activation function in the hidden layer which is computed using *Equation 7*.

$$f(x) = \frac{1}{1+\exp(-x)} \tag{7}$$

In *Equation 7*, x is the time-series input data. The intermediate result is transmitted from the hidden layer to the output layer. The softmax activation function is used at the output layer to predict future values. During this process, the output layer has one node due to the one-step-ahead forecasting process. According to the discussion, the input and the output relationship is estimated using *Equation 8*.

$$y_t = w_0 + \sum_{j=1}^{q} w_j \cdot f\left[w_{0,j} + \sum_{i=0}^{p} w_{ij} \cdot y_{t-1}\right] + \varepsilon_t \tag{8}$$

In *Equation 8*, the connection weights are denoted as $w_{ij}(i=0,1,\dots,p,\ j=1,2,\dots,q)$ and $w_j(j=0,1,2,\dots,q)$. Based on the computation, time-series input features are continuously examined with the number of neurons presented in the network. Later, the error is backpropagated into the network. The backpropagation is done to adjust the weights and to fine tune the parameters of the network. This process is repeated for the specified number of epochs which will finally reduce the training error. In addition to this, a recurrent neural network (RNN) is trained with the time-series traffic data. The RNN forecasting model is explained as follows.

Recurrent neural networks (RNN)

The next forecasting model is recurrent neural network (RNN). The plain RNN has gradient problems while training the time-series data. To resolve this issue, a long short-term memory network is utilised, which is a type of RNN network. This network has three layers – input, hidden and output layer. We consider the time-series traffic data defined as $X=x_1,x_2,\dots x_t$, RNN hidden layer inputs denoted as $H=h_1,h_2,\dots h_t$ and the output defined as $Y=y_1,y_2,\dots y_t$, where t is the prediction time. The recurrent neural network predicts the output from the update rule. Then the hidden layer output for every traffic data is computed using *Equation 9*.

$$h_t = f\left(h^{(t-1)}, x^{(t)}; \theta\right) \tag{9}$$

In *Equation 9*, $h(t)$ is denoted as current hidden state function and $h^{(t-1)}$ is the previous hidden state function and the parameter is represented as θ. Then the output for input is estimated using following *Equations 10–13*.

$$a^{(t)} = b + Wh^{(t-1)} + Ux^{(t)} \tag{10}$$

$$h^{(t)} = \tanh\left(a^{(t)}\right) \tag{11}$$

$$o^{(t)} = c + Vh^{(t)} \tag{12}$$

$$\hat{y}^{(t)} = softmax\left(o^{(t)}\right) \tag{13}$$

During the output estimation process, the loss rate is computed using *Equation 14*.

$$\begin{aligned} &L\left(\{x^{(1)},\dots,x^{(\tau)}\},\{y^{(1)},\dots,y^{(\tau)}\}\right)\\ &= \sum_t L^{(t)} = \sum_t -\log \hat{y}^{(t)}_{y^{(t)}} \end{aligned} \tag{14}$$

According to the activation function, the model is trained efficiently and validated by comparing the test data with the forecasted traffic flow. The traditional forecasting model efficiency is further improved by the proposed hybrid approaches. The ARIMA model is linear and works well on time-series data. Therefore, the ARIMA model is combined with nonlinear models MLP and RNN approaches to improve the performance of the traffic flow forecasting system. The detailed hybrid time-series forecasting process is explained as follows.

4. HYBRID TIME-SERIES FORECASTING MODELS

This section discusses about the proposed hybrid time-series forecasting models in traffic flow forecasting. In this paper, two hybrid models are introduced – hybrid autoregressive integrated moving average with multilayer perceptron (ARIMA-MLP) and hybrid autoregressive integrated moving average with recurrent neural network (ARIMA-RNN). The detailed working process of hybrid time-series forecasting model is illustrated in *Figure 8*.

Figure 8 illustrates the operational methodology of the hybrid time-series forecasting for traffic flow prediction. As discussed above in Section 2, the traffic data have been collected from the UK Highways dataset, and the detailed explanation of the dataset is discussed in Section 3.1. The collected traffic data are preprocessed using a random

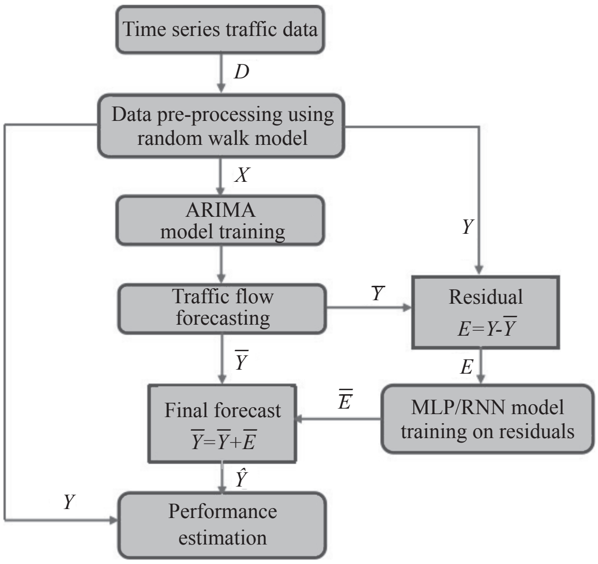

Figure 8 – Hybrid time-series forecasting model

walk model that eliminates the drift and no-drift information present in the data. The working process of the random walk model is explained in Section 3.2. The traffic data (*D*), after preprocessing, are split into training data (*X*) and test data (*Y*) as explained in Section 3.1. The ARIMA model is trained to forecast the future traffic flow ($\overline{Y}$). The residuals are computed as ($E=Y-\overline{Y}$). Multilayer perceptron (MLP) or recurrent neural network (RNN) is trained on these residuals to obtain residual analysis output ($\overline{E}$). This $\overline{E}$ is summed up with $\overline{Y}$ to obtain the final traffic flow forecast ($\hat{Y}=\overline{Y}+\overline{E}$). The following sections explain in detail the work and importance of the two hybrid models.

4.1 Hybrid ARIMA with MLP forecasting model (ARIMA-MLP)

This section discusses the autoregressive integrated moving average traffic flow forecasting model (ARIMA) with multilayer perceptron (MLP). The ARIMA model is linear, which is commonly used to predict the time-series data. The linear model produces nonlinear residuals. These residuals are analysed by a nonlinear model, multiple layer perception networks (MLP). The nonlinear model residual values will have a linear structure.

First, the ARIMA model is applied to the linear traffic flow data. Consider y_t to be the test data, $\bar{y}_t$ to be the forecasted traffic flow data from the ARIMA model and e_t to be the residual at time period *t*. The residual is computed as shown in *Equation 15*.

$$e_t = y_t - \bar{y}_t \quad (15)$$

The nonlinear residuals from ARIMA model are trained by the multilayer perceptron. The output of residual analysis from MLP is denoted as in *Equation 16*.

$$\overline{E} = f(e_{n+1}, e_{n+2}, e_{n+3}, \dots, e_{t-1}, e_t) \quad (16)$$

In *Equation 16*, the nonlinear function *f*, modelled using the multilayer perceptron is used to compute the output of residual data analysis which is denoted by $\overline{E}$. The final forecasted value $\hat{y}_t$, at time period *t*, from the hybrid model is calculated using *Equation 17*.

$$\hat{y}_t = \bar{y}_t + \overline{E} \quad (17)$$

Based on the discussion, the ARIMA-MLP traffic flow forecasting model is illustrated in *Figure 9*.

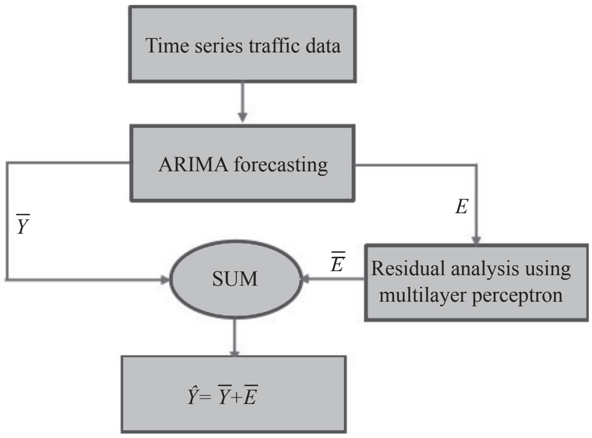

Figure 9 – ARIMA-MLP traffic flow forecasting

According to *Figure 9*, time-series traffic data have been continuously analysed using the ARIMA model, and respective nonlinear residual errors are successfully resolved with the help of the MLP model. Furthermore, the forecasting efficiency is improved by applying the ARIMA model with the RNN network. The detailed working process of this hybrid algorithm is discussed as follows.

4.2 Hybrid ARIMA with RNN forecasting model (ARIMA-RNN)

Another forecasting model is created by combining the autoregressive integrated moving average with recurrent neural network (ARIMA-RNN). As discussed, the ARIMA model effectively works on the time-series data. During this process, the ARIMA model computes the maximum likelihood value of time-series data. At the time of the computation process, it may produce the nonlinear residuals. Therefore, the original time-series data need to be smoothened. This is done by applying the recurrent neural networks. Based on the structure of the network, the output at time period t is estimated and the error rate is denoted as e_t. The residual error value is optimised by the function f as in the *Equation 18*, but the function is computed by recurrent neural network. Based on the discussion, the ARIMA-RNN traffic flow forecasting model is illustrated in *Figure 10*.

According to *Figure 10*, time-series traffic data have been analysed using the ARIMA model, and respective nonlinear residual errors are successively resolved with the help of the RNN model. With this setup, the proposed hybrid time-series forecasting model improves the overall traffic data prediction process. The efficiency of the hybrid models is evaluated and discussed in the following section.

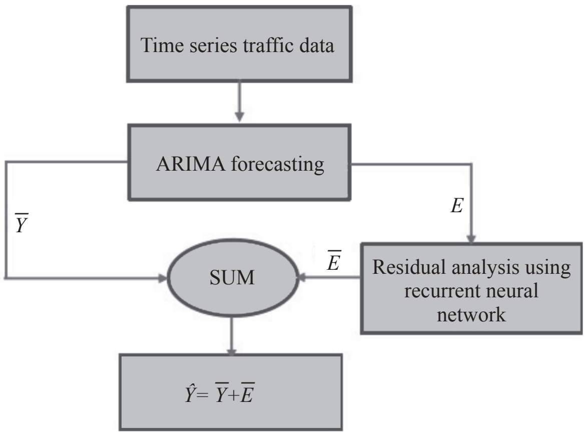

Figure 10 – ARIMA-RNN time-series forecasting

5. RESULTS AND DISCUSSIONS

This section discusses the excellence of the hybrid time-series traffic flow forecasting models, such as hybrid autoregressive integrated moving average with multilayer perceptron (ARIMA-MLP) and hybrid autoregressive integrated moving average with recurrent neural network (ARIMA-RNN). The traffic data are collected from the UK Highway dataset that consists of 20,449 records in the year 2020 till July. The data are preprocessed by the random walk model for eliminating the drift and no-drift information. The preprocessed data are split into training set (January to June – traffic flow) and testing set (July – traffic flow). The model implementation is carried out using R language. The competence of the system is evaluated using Mean Absolute Error (MAE), Mean Square Error (MSE), Root Mean Square Error (RMSE), and Coefficient of Determination (R^2). The metrics are computed as shown in *Equations 18–21*.

$$MAE = \frac{1}{n}\sum_{t=1}^{n} | y_t - \hat{y}_t | \tag{18}$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n} (y_t - \hat{y}_t)^2 \tag{19}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} (y_t - \hat{y}_t)^2} \tag{20}$$

$$R^2 = 1 - \frac{\sum_{t=1}^{n} (y_t - \hat{y}_t)^2}{\sum_{t=1}^{n} (y_t - \bar{y})^2} \tag{21}$$

In the above equations, n denotes the number of test data, y_t denotes the test data value at time period t, $\hat{y}_t$ denotes the forecasted traffic flow at time period t from the corresponding models and finally $\bar{y}$ denotes $\sum_{t=1}^{n} y_t$. Using these metrics, the time-series traffic data have been forecasted for the peak hour (9 AM) and non-peak hour (9 PM) on all weekdays. The efficiency of the proposed hybrid models ARIMA-MLP and ARIMA-RNN is compared with the traditional forecasting models such as ARIMA, MLP and RNN. The obtained results are illustrated in *Table 4*.

Table 4 clearly illustrates that the hybrid forecasting models ARIMA-MLP and ARIMA-RNN attain better results compared to other traditional forecasting models. Then the respective graphical analyses of all models for peak hour (PH) forecast are illustrated in *Figure 11*.

In addition to this, the model efficiency is evaluated using the non-peak hour (NPH) traffic data, and the obtained results are illustrated in *Figure 12*.

Figures 11 and 12 illustrate the performance analyses of all the five different forecasting models on peak hour (PH) and non-peak hour (NPH) traffic data, respectively. The results demonstrated that ARIMA-MLP and ARIMA-RNN approaches attain significant results. The effective training of ARIMA model for the given time-series data helps to identify the nonlinear residual information. The residual values are analysed by the MLP and RNN to reduce the deviation between the original time-series data and forecasted time-series data.

6. CONCLUSION

Traffic flow forecasting is a research of estimating the number of vehicles that may flow through a particular lane at a specified period in the future. The extrapolation of future traffic flows is quite

Table 4 – Performance Analysis

Models	Metrics	Peak Hour (PH) Forecast	Non-Peak Hour (NPH) Forecast
ARIMA	MAE	0.74	0.78
	MSE	1.17	1.57
	RMSE	1.08	1.25
	R^2	0.89	0.82
MLP	MAE	0.69	0.69
	MSE	0.87	1.04
	RMSE	0.93	1.02
	R^2	0.92	0.88
RNN	MAE	0.61	0.65
	MSE	0.78	0.83
	RMSE	0.88	0.91
	R^2	0.93	0.91
ARIMA - MLP	MAE	0.55	0.61
	MSE	0.71	1.06
	RMSE	0.84	1.03
	R^2	0.94	0.88
ARIMA - RNN	MAE	0.53	0.52
	MSE	0.66	0.61
	RMSE	0.81	0.78
	R^2	0.94	0.93

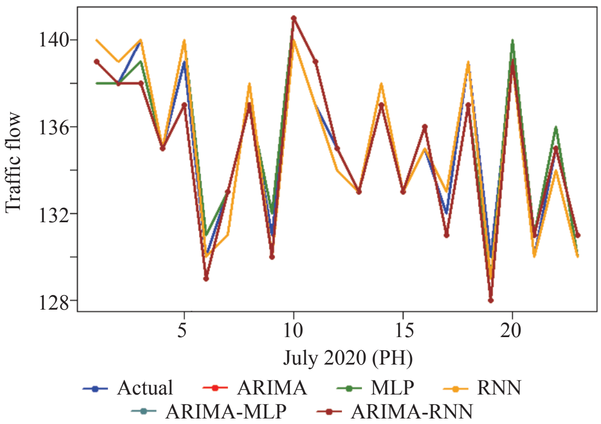

Figure 11 – Peak hour (PH) forecasting results

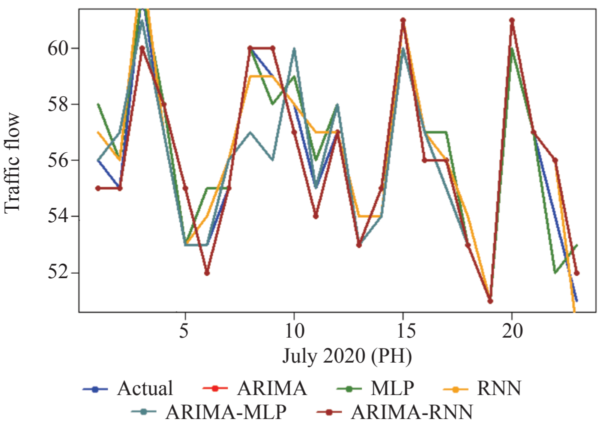

Figure 12 – Non-peak hour (NPH) forecasting results

challenging. This paper analysed the efficiency of the proposed hybrid autoregressive integrated moving average with multilayer perceptron (ARIMA-MLP) model and hybrid autoregressive integrated moving average with recurrent neural network (ARIMA-RNN) model in traffic data forecasting. Initially, the data are preprocessed and then the state-of-the-art models ARIMA, MLP and RNN are trained individually. Later the hybrid models ARIMA-MLP and ARIMA-RNN are trained to forecast the future traffic flows. The efficiency of the models is evaluated using MAE, MSE, RMSE and R^2 metrics. From the results, the ARIMA-MLP and ARIMA-RNN attain the superior results compared to ARIMA, MLP and RNN. The R^2 values clearly state that the residual error is reduced to a greater extent while forecasting is carried out with the proposed hybrid models. In the future, the traffic flow forecasting can be enhanced with bio-inspired optimisation techniques.

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போக்குவரத்து ஓட்டம் முன்னறிவிப்புக்கான ஹைப்ரிட் டைம் சீரிஸ் முன்கணிப்பு மாதிரிகள்

சுருக்கம்
இன்றைய போக்குவரத்து அமைப்பில் போக்குவரத்து முன்னறிவிப்பு மிகவும் முக்கியமானது, ஏனெனில் பயண வழியை தீர்மானிக்க ஒரு போக்குவரத்து திட்டத்தை உருவாக்குவது அவசியம். இந்த ஆராய்ச்சியின் குறிக்கோள், சாலைவழிகளில் போக்குவரத்து நெரிசலைக் குறைப்பதற்காக எதிர்கால போக்குவரத்தை மதிப்பிடுவதற்கு நேர-தொடர் முன்கணிப்பு மாதிரிகளைப் பயன்படுத்துவதாகும். கணிப்புப் பிழையைக் குறைப்பது போக்குவரத்துக் கணிப்பில் மிகவும் கடினமான பணியாகும். எதிர்கால போக்குவரத்து ஓட்டத்தை எதிர்நோக்க, கணினிக்கு வாகனங்கள் மற்றும் சாலைவழிகளில்

இருந்து நிகழ்நேர தரவு தேவைப்படுகிறது. மல்டிலேயர் பெர்செப்ட்ரான் (அரிமா-எம்எல்பி) உடன் ஆட்டோரெஃப்ரெஸ்ஸிவ் இன்டகிரேடட் மூவிங் ஆவெரேஜ் மற்றும் ரெக்கரண்ட் நியூரல் நெட்ஒர்க் (அரிமா-ஆர்என்என்) உடன் ஆட்டோரெஃப்ரெஸ்ஸிவ் இன்டகிரேடட் மூவிங் ஆவெரேஜ் ஆகியவை இந்தத் தாளில் முன்மொழியப்பட்டுள்ளன. யூகே நெடுஞ்சாலைகள் தரவுத்தொகுப்பில் இருந்து போக்குவரத்து தரவு பயன்படுத்தப்படுகிறது. நேரத் தொடர் தரவு ரேண்டம் வாக் மாடலைப் பயன்படுத்தி முன்கூட்டியே செயலாக்கப்படுகிறது. ஆட்டோரெஃப்ரெஸ்ஸிவ் இன்டகிரேடட் மூவிங் ஆவெரேஜ் (அரிமா), ரெக்கரண்ட் நியூரல் நெட்ஒர்க் (ஆர்என்என்) மற்றும் மல்டிலேயர் பெர்செப்ட்ரான் (எம்எல்பி) ஆகியவை பயிற்சியளிக்கப்பட்டு சோதிக்கப்படுகின்றன. முன்மொழியப்பட்ட ஹைபிரிட் அரிமா-எம்எல்பி மற்றும் அரிமா- ஆர்என்என் மாடல்களில், எம்எல்பி மற்றும் ஆர்என்என் மாடல்களைப் பயிற்றுவிக்க அரிமா மாதிரியின் ரெசிடுயல்ஸ் பயன்படுத்தப்படுகின்றன. பின்னர் எம்எஇ, எம்எஸ்இ, ஆர்எம்எஸ்இ மற்றும் R^2 என்ற அளவீடுகளைப் பயன்படுத்தி ஹைபிரிட் அமைப்பின் செயல்திறன் மதிப்பிடப்படுகிறது. (அரிமா-எம்எல்பி மாதிரியில் பீக் ஹவர் முன்னறிவிப்பு-0.936763 மற்றும் நான் பீக் ஹவர் முன்னறிவிப்பு-0.87638 பதிவாகியுள்ளது. அரிமா-ஆர்என்என் மாதிரியில் பீக் ஹவர் முன்னறிவிப்பு-0.9416466, மற்றும் நான் பீக் ஹவர் முன்னறிவிப்பு-0.931917 பதிவாகியுள்ளது).

முக்கிய வார்த்தைகள்
நேரத் தொடர் பகுப்பாய்வு; போக்குவரத்து ஓட்டம் முன்னறிவிப்பு; ரேண்டம் வாக் மாடல்; ரெசிடுயல் பகுப்பாய்வு; அரிமா-எம்எல்பி; அரிமா-ஆர்என்என்.

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