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Traffic Engineering
Original Scientific Paper
Submitted: 21 May 2021
Accepted: 17 Aug. 2021

# SIMULATION MODELLING OF PERMITTED LEFT-TURN SATURATION FLOW RATE BASED ON OPPOSING THROUGH-FLOW DEGREE OF SATURATION

***ABSTRACT***

*This paper presents the modelling of the saturation flow rate of the permitted left turn in an exclusive lane. In the proposed model, the total permitted left-turn saturation flow rate is determined as a sum of saturation flow rates during the effective green time and the intergreen period. Primarily, the permitted left-turn saturation flow rate during the effective green time is modelled based on the opposing through-flow degree of saturation and the number of opposing through-flow lanes. The relation between the permitted left-turn saturation flow during the effective green time and these variables was examined using data from the simulation experiments in VISSIM. To our knowledge, this is the first study of the permitted left-turn saturation flow modelling based on the opposing through-flow degree of saturation instead of the opposing through-flow rate and signal-timing parameters. The proposed model was validated based on data collected at seven intersections with a permitted left turn served in an exclusive lane. The permitted left turn saturation flow rate could be accurately determined based on the opposing through-flow degree of saturation and the number of opposing lanes according to the RMSE of 58.4 pcu/h.*

***KEYWORDS***

*permitted left turn; saturation flow rate; degree of saturation; signalised intersection; simulation.*

## 1. INTRODUCTION

Saturation flow rate is a crucial input parameter for signal-timing optimisation, due to its effect on capacity and control delay estimation. The saturation flow rate is an hourly rate at which previously queued vehicles can traverse an intersection approach, assuming that green signal is available at all times and no lost times are experienced [1].

The saturation flow rate, besides various geometric and traffic conditions, depends on a movement type. Hence, the left-turn lane saturation flow is different from the through lane due to the lower turning speed and movement radius [2]. Also, the left-turn saturation flow rate is affected by the left-turn mode whereby left turns can be served as permitted, protected or permitted-protected. The permitted mode requires a left-turn vehicle to yield to both opposing vehicles and pedestrians, and to wait for an available acceptable gap in the opposing flow. So, permitted left turns, either served in shared lanes or exclusive lanes, seriously affect intersection operations [3]. Hence, estimating the left-turn saturation flow rate when it operates in the permitted mode is a complex problem.

Therefore, the discharge characteristics of the permitted left turns require considering several components of the permitted left-turn saturation flow rate (PLTSF). Wu in [4] suggests that there are three components of left-turn departures. The first component, named jumpers, consists of vehicles that turn in front of the first opposing vehicle. The second component refers to vehicles that turn during the green time, while the third component, named sneakers, consists of vehicles that pass an intersection during the intergreen period.

The majority of authors have developed the PLTSF model considering second and third components, so the PLTSF is estimated as a sum of the PLTSF during the green time and the PLTSF during the intergreen period (PLTSFs) [1, 2, 4–6]. Existing models do not consider the first component because the number of jumpers is usually small [4]. On the other hand, some studies have proposed models for the PLTSF estimation regardless of the mentioned PLTSF components [7–9].

The second and most important component, the PLTSF during the green time, is usually examined in two ways: during the effective green time [6] or during the unsaturated green time [1, 2, 4, 5]. Thereby, the effective green time refers to the effective green time for the opposing traffic flow [6], while the unsaturated green time represents the portion of green time after the clearance of the opposing queue [4]. The second approach requires defining the method for the unsaturated green time estimation, which additionally complicates the PLTSF model development and application.

The second PLTSF component depends on the opposing through-flow rate, the number of opposing lanes, and signal-timing parameters as variables that affect left turns during the (effective or unsaturated) green time. Still, some models included only the opposing through-flow rate [1], while others combined the opposing through-flow rate and the number of opposing lanes [2, 4]. On the other hand, Akçelik in [5] incorporated the opposing through-flow rate and signal-timing parameters in the model. Only the Canadian Capacity Guide model includes all listed variables [6].

The third PLTSF component is a result of sneakers during the intergreen period (PLTSFs). This PLTSF component can be of great importance, especially if the opposing flow is saturated during the entire green time since there are no available gaps to turn left during the green time. The PLTSFs depends on the number of sneakers per cycle and the number of cycles per hour. The number of sneakers per cycle is defined in several ways. In papers [1, 2, 5], the number of sneakers is given as a fixed, experimentally determined value. In [4], the number of sneakers is a function of the opposing through-flow rate, while in [6], it is a function of the intersection waiting space length. It is expected that the number of sneakers varies among intersections depending on the intersection geometry, so the approach with fixed predefined values could not be reliable.

The PLTSF model, presented in this paper, considers the second and third PLTSF components, while the second component is examined during the effective green time (PLTSFe). It is important to develop the PLTSFe model that includes all variables affecting left turns. However, using signal-timing parameters in the PLTSFe estimation is complicated, considering that they are the output from signal timing optimisation. This paper proposes PLTSFe modelling based on the opposing through-flow degree of saturation since it unites the effects of the opposing through-flow rate and signal-timing parameters. Hence, those variables are indirectly incorporated. Including the opposing flow degree of saturation, instead of separate variables, simplifies the PLTSFe estimation. The number of opposing flow lanes should not be neglected, considering its established effect on the PLTSFe. So, this paper hypothesises that the PLTSFe depends on the opposing through-flow degree of saturation and the number of opposing through-flow lanes. Also, this paper proposes that the number of sneakers per cycle, influencing the third component (PLTSFs), could be estimated based on the waiting space length if field data are not available.

## 2. LITERATURE REVIEW

In this section, PLTSF models proposed by the Highway Capacity Manual (HCM 2016), the Australian Road Research Broad (ARRB), and the Canadian Capacity Guide (CCG 3rd edition) are presented in more detail. These relevant models will be used for comparison with the model proposed in this paper. *Table 1* summarises the model formulations, considered PLTSF components, research methodology, and the critical headway suggested by authors.

$S_L$ – total permitted left-turn saturation flow rate in vehicles per hour or passenger car unit per hour [veh/h or pcu/h],

$s_{L_u}$ – permitted left-turn saturation flow rate during unsaturated green time in vehicles per second [veh/s],

$S_{L_e}$ – permitted left-turn saturation flow rate during effective green time [veh/h],

$S_{L_u}$ – permitted left-turn saturation flow rate during unsaturated green time [veh/h],

$S_b$ – base saturation flow rate [veh/h],

$Q_O$ – opposing through-flow rate [pcu/h, veh/h or veh/h/lane],

$q_O$ – opposing through-flow rate [veh/s],

$c$ – cycle length [s],

*Table 1 – Review of the relevant PLTSF models*

| Model | Permitted left-turn saturation flow models | | $N_O$ | $n_{L_s}$ [veh/cycle or pcu/cycle] | $t_h$ [s] | Methodology | Reference |
|---|---|---|---|---|---|---|---|
| ARRB model | $s_{L_u}$ | $\frac{q_O \cdot e^{-5.0 \cdot q_O}}{1 - e^{-3.0 \cdot q_O}}$ | - | 1.5 | 5 | Gap-acceptance theory | [5] |
| | $S_L$ | $\frac{3600}{g_e} \cdot (s_{L_u} \cdot g_u + n_{L_s})$ | | | | | |
| CCG 3 model | $S_{L_e}$ | $S_b \cdot 1.05 \cdot e^{-0.00121 \cdot \frac{1 \cdot Q_O \cdot c}{g_e}} - 0.05$ | 1 | 0.5 – 3.0* | - | Field research | [6] |
| | | $S_b \cdot 1.05 \cdot e^{-0.00121 \cdot \frac{0.625 \cdot Q_O \cdot c}{g_e}} - 0.05$ | 2 | | | | |
| | | $S_b \cdot 1.05 \cdot e^{-0.00121 \cdot \frac{0.51 \cdot Q_O \cdot c}{g_e}} - 0.05$ | 3 | | | | |
| | | $S_b \cdot 1.05 \cdot e^{-0.00121 \cdot \frac{0.44 \cdot Q_O \cdot c}{g_e}} - 0.05$ | 4 | | | | |
| HCM 2016 model | $S_{L_u}$ | $\frac{Q_O \cdot e^{-4.5 \cdot Q_O/3600}}{1 - e^{-2.5 \cdot Q_O/3600}}$ | - | 2.0 | 4.5 | Gap-acceptance theory | [1] |

** The number of sneakers depends on the number of opposing through lanes or waiting space length.*

$g_e$ – effective green time [s],
$g_u$ – unsaturated green time [s],
$N_O$ – number of opposing through lanes,
$n_{L_s}$ – number of sneakers that turn during the intergreen period [veh/cycle or pcu/cycle],
$t_h$ – critical headway [s].

All of the presented PLTSF models consider the second and third PLTSF components. However, in the CCG 3 model, the second component is examined during the effective green time, while in the HCM 2016 and ARRB models, this component is examined during the unsaturated green time. The methodology applied in the modelling of the second PLTSF component differs among the studies. Namely, the CCG 3 model is developed based on field data. This model includes all variables affecting left turns, such as the opposing through-flow rate, the number of opposing lanes, and signal-timing parameters. The HCM 2016 and ARRB models are based on the gap-acceptance theory, and these models include only the opposing through-flow rate as a model variable. However, the ARRB model includes signal timing parameters in the total PLTSF estimation.

The suggested number of sneakers varies between the presented models. The ARRB and HCM 2016 models define one fixed value, while the CCG 3 model defines the range of values depending on the waiting space length. It is important to note that the ARRB model recommends using the defined number of sneakers if the field data are not available [5].

# 3. METHODOLOGY

Based on the literature review, it is concluded that the PLTSF model should consider second and third PLTSF component. In this study, the PLTSF is determined as a sum of the PLTSF during the effective green time (PLTSFe) and the PLTSF during the intergreen period (PLTSFs), as given by *Equation 1*

$$S_L = S_{L_e} + S_{L_s} \quad (1)$$

where $S_{L_s}$ is permitted left-turn saturation flow rate during the intergreen period [pcu/h] and other variables are previously defined in pcu/h.

The PLTSFs depends on the number of cycles per hour, i.e. on the cycle length and the number of sneakers per cycle. On an hourly basis, the maximum number of sneakers represents the permitted left-turn capacity during the intergreen periods. Considering that the capacity is equal to the saturation flow rate multiplied by the effective green time ratio [1], the PLTSFs is calculated using *Equation 2*

$$S_{L_s} = \frac{C_{L_s}}{\lambda} = \frac{n_{L_s} \cdot \frac{3600}{c}}{\frac{g_e}{c}} = \frac{n_{L_s} \cdot 3600}{g_e} \quad (2)$$

where:

$C_{L_s}$ – permitted left-turn capacity during the intergreen period [pcu/h],
$\lambda$ – effective green time ratio, $\lambda = \frac{g_e}{c}$
and other variables are previously defined.

It is important to determine the number of sneakers precisely since the higher number of sneakers demonstrates that more left-turn vehicles can pass through the intersection [2]. Considering assumption that the number of sneakers varies among intersections depending on the intersection geometry, this paper recommends measuring the number of sneakers in the field or estimating it by dividing the waiting space length by the average passenger car length, as given by *Equation 3*. Thereby, the intersection waiting space represents a space between the stop line and the point where left-turn vehicles stop and wait for an acceptable gap.

$$n_{L_s} = \frac{L_{ws}}{l_{pc}} \tag{3}$$

where:

$L_{ws}$ – waiting space length [m],
$l_{pc}$ – average passenger car length [m].

The main objective of this paper is to model the PLTSFe based on the number of opposing lanes and the opposing through-flow degree of saturation, which is a function of the opposing through-flow rate and signal-timing parameters, as given by *Equation 4*

$$x_O = \frac{Q_O}{N_O \cdot \lambda \cdot S_O} = \frac{Q_O}{N_O \cdot \frac{g_e}{c} \cdot S_O} \tag{4}$$

where:

$x_O$ – opposing through-flow degree of saturation,
$S_O$ – opposing through-flow saturation flow rate [pcu/h]

and other variables are previously defined.

Measuring the PLTSFe in the field is difficult due to a limited sample of research sites and numerous combinations of variables affecting left turns. However, the permitted left-turn saturation flow has already been examined using simulation [2, 4, 7, 10, 11]. Simulation is a widely used technique that allows imitating the operations of various types of real-world facilities or processes by using computers [12]. So, simulation is an alternate, easier way to simulate various traffic and geometric conditions and measure PLTSFe.

The following two sections describe the experimental design and procedure used in this research. The next section describes the PLTSFe model development based on data collected by simulation experiments. The last section presents the methodology for the PLTSF model testing and comparison.

### 3.1 Experimental system design

In this paper, the PLTSFe was analysed and modelled using the microsimulation model PTV VISSIM. For the research purpose, VISSIM was set to represent local conditions according to the research presented in [13].

In VISSIM, the through-lane saturation flow rate depends on the parameters of the Wiedemann 74 model, which is given by *Equations 5 and 6*. Values of the parameters were adopted to represent local conditions based on the through-lane saturation flow of 1,850 pcu/h [13]. So, *bx_mult* and *bx_add* were 3.48 m and 2.48 m, respectively. Adopted values of parameters led to the left-turn lane saturation flow rate of 1,650 pcu/h. This saturation flow rate is valid in local conditions [14], but it is also close to values cited in [5, 6, 15].

$$d = ax + bx \tag{5}$$

$$bx = (bx\_add + bx\_mult \cdot z) \cdot \sqrt{v} \tag{6}$$

where:

*d* – distance between two following vehicles [m],
*ax* – average standstill distance [m], variation ± 1 m,
*bx* – desired safety distance [m],
*bx_add* – additive part of desired safety distance [m], default value 2 m,
*bx_mult* – multiplicative part of desired safety distance [m], default value 3 m,
*v* – vehicle speed [m/s],
*z* – constant, in range [0, 1] which is normally distributed around 0.5 with a standard deviation of 0.15.

Since the PLTSFe depends on the availability of critical headways in the opposing flow, it was necessary to modify this parameter in VISSIM. Different values of the critical headway were defined depending on the number of opposing through lanes. When the left turn was opposed by one-lane flow, the 5-second critical headway was used, based on the research conducted in local conditions [13]. The accepted value is close to values cited in the literature as 5 s [5, 16], 4.95 [4], 5.1 s [17], 5.7 [8], 4.5 s [1] and 6 s [18]. When the left turn was opposed by the two-lane flow, the critical headway of 6 seconds was accepted according to [16], due to the lack of appropriate research in local conditions.

To represent urban driver behaviour, the desired speed distribution was defined in the range from 35 to 58 km/h. The desired speed for the left turn was in the range from 20 to 25 km/h, with a maximum deceleration of 2 $m/s^2$. Applied left-turn desired speed is close to value cited in the literature of 22 feet/s ≈ 24 km/h [4].

The PLTSFe examination requires the experimental system to meet the following prerequisites:
- Permitted left-turn mode,
- Exclusive left-turn lane,
- One or two opposing through lanes,
- A 3.5-meter lane width,
- A flat approach grades,
- Constant left-turn demand,
- A passenger car flow (no heavy vehicles),
- Absence of parking, public transport and pedestrians,
- No effects of weather conditions and other factors on the saturation flow.

Hypothetic three-leg signalised intersection with an exclusive left-turn lane was used in VISSIM simulation experiments. The three-leg intersection was applied since one left-turn and opposing through flow are sufficient for the PLTSFe measuring. Hence, other flows were not the subject of the research. The intersection layout and the signal phasing sequence applied in experiments are shown in *Figure 1*. The traffic lane width was set to 3.5 m, as mostly applied on the urban street network, to exclude the lane width effect on the PLTSF. In experiments, the number of opposing through (approach and receiving) lanes was changed from one to two to analyse its effect on the PLTSFe. Two data collection points were placed at the intersection. The data collection point in front of the stop line measured the opposing through-flow rate, while another one, in the lane receiving left-turn vehicles, measured the left-turn discharge flow rate during the effective green time. So, the sneakers were excluded given the purpose of modelling only the PLTSFe using the simulation.

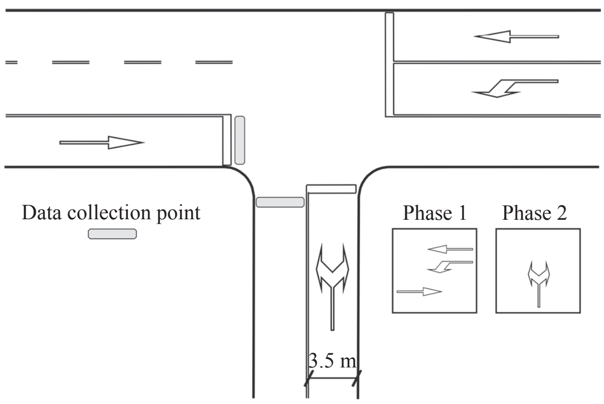


*Figure 1 – Intersection layout and signal phasing sequence used in experiments*

Traffic composition was set to passenger cars to exclude the effect of heavy vehicles on the PLTSFe. The left-turn flow rate was set to provide constant saturated conditions during the simulation.

## 3.2 Experimental procedure

The investigation of the PLTSFe dependence on the opposing through-flow degree of saturation requires various values of the opposing flow degree of saturation during simulation. Different values of the opposing degree of saturation, representing different traffic conditions in the opposing flow, were achieved by changing the following variables:
- The opposing through-flow rate: from 0 to 1,900 pcu/h/lane, with increment step 100 pcu/h.
- Signal-timing parameters: defined to provide the effective green time ratio in the range 0.1–0.9 with increment step 0.1 (the fixed cycle length of 100 s).

Combining the preceding values of variables resulted in 531 experimental scenarios in total. Namely, there were 180 scenarios for one opposing lane (20 opposing through-flow rates and 9 effective green time ratios) and 351 scenarios for two opposing lanes (39 opposing through-flow rates and 9 effective green time ratios). For each of 531 scenarios, ten simulation runs were executed using different random number seeds. A warm-up period of 600 s was followed by a one-hour simulation experiment (3,600 s). Collected data were averaged from ten simulation runs for each scenario.

## 3.3 PLTSFe model development

The average opposing through-flow rate was used for calculating the opposing through-flow degree of saturation, applying *Equation 4*. Due to the constant left-turn demand, the average left-turn flow represents a left-turn capacity during the effective green time. According to [1], the capacity is used for the PLTSFe calculation for each of 531 scenarios, applying *Equation 7*

$$S_{Le} = \frac{C_{Le}}{\lambda} \tag{7}$$

where $C_{L_e}$ is permitted left-turn capacity during the effective green time [pcu/h] and other variables are previously defined.

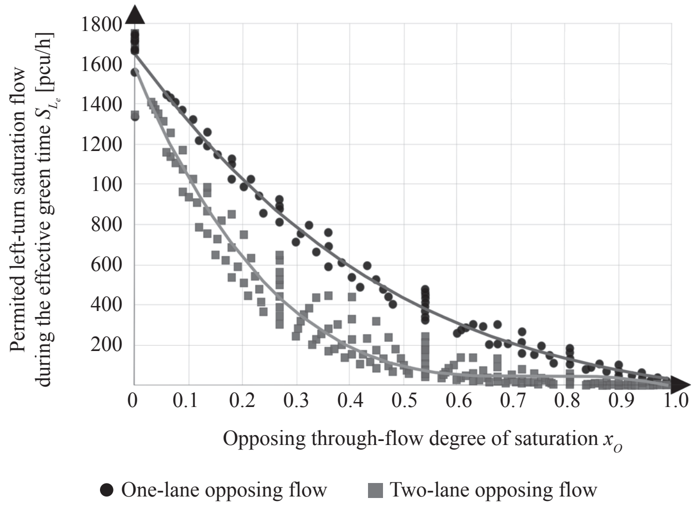


*Figure 2 – The relationship between the PLTSFe, the degree of saturation and the number of opposing lanes*

Hence, the database consists of the PLTSFe and the opposing through-flow degree of saturation for each simulation experiment. The PLTSFe and the corresponding values of the opposing flow degree of saturation are plotted in *Figure 2*. The figure shows that the PLTSFe decreases with the increase of the degree of saturation. However, data points are concentrated along two curves depending on the number of opposing lanes. Results of the two-sample *t*-test ($p$=0.0005<0.05) showed that the PLTSFe statistically differs when left turns are opposed by the one-lane flow and by the two-lane flow.

Based on the previous result, the relationship between the PLTSFe and the opposing flow degree of saturation is modelled separately depending on the number of opposing lanes. The model for the PLTSFe estimation is given by *Equation 8*, with $R^2$ of 0.99 and 0.97 when the left turn is opposed by one-lane and two-lane flow, respectively.

$$S_{Le} = \begin{cases} 1658.8 - 3661.5 \cdot x_O + 2868.5 \cdot x_O^2 - 835.2 \cdot x_O^3, & N_O = 1 \\ 1589.6 - 6200.1 \cdot x_O + 8269.5 \cdot x_O^2 - 3662.1 \cdot x_O^3, & N_O = 2 \end{cases} \quad (8)$$

where all variables are as previously defined.

The values of $R^2$ indicate that 99% and 97% of the variance of the PLTSFe is explained with the opposing flow degree of saturation, when the left turn is opposed by one-lane and two-lane flow, respectively.

The results confirm the hypothesis that the PLTSFe depends on the opposing through-flow degree of saturation and the number of opposing through-flow lanes. Hence, the PLTSFe can be estimated based on the opposing flow degree of saturation instead of the opposing flow rate and signal-timing parameters.

## 3.4 Data collection for PLTSF model testing and comparison

The final step of the defined methodology refers to the collecting of data for model testing and comparison with prominent widely used models, such as the ARRB model, the CCG 3 model and the HCM 2016 model. These PLTSF models, as the proposed model, consider the second and third PLTSF components.

The testing and comparison were conducted regarding real data collected in local conditions. Seven four-leg intersections in Belgrade were selected based on requirements defined during the experimental system design (permitted left-turn mode, exclusive left-turn lane, constant left-turn demand, one or two opposing through lanes, mostly a passenger car flow, closely a 3.5-meter lane width, approximately a flat approach grades, no parking, no public transport, no pedestrians or a negligible number of pedestrians and sunny weather). Video camera was installed at intersections to record left-turn flow, opposing through flow and signal-timing parameters for one hour. The waiting space length was measured in the field for each intersection. Collected data, given in *Table 2* for each intersection, represent the input data for model testing and comparison.

The opposing through flow ($Q_O$), the total left-turn capacity ($C_L$), the maximum number of sneakers ($n_{L_s}$) and signal-timing parameters ($c$, $g_e$) were obtained manually from the recordings and given in *Table 2* for each intersection. The opposing through flow represents the total flow rate on the approach, not per lane. The total permitted left-turn capacity is determined as a sum of left-turn vehicles during the effective green time and sneakers during the intergreen period. The maximum number of sneakers represents the maximum number of left-turn vehicles that pass the intersection during the intergreen period.

The total opposing through flow and signal-timing parameters were used to calculate the effective green time ratio ($\lambda$), the opposing through-flow degree of saturation ($x_O$) and the unsaturated green time ($g_u$), which are given in *Table 2*. The opposing through-flow degree of saturation was calculated using *Equation 4*, while the unsaturated green time was estimated according to [5].

Data in *Table 2* were further used for the PLTSF estimation (*Table 3*). As explained earlier, the capacity is equal to the saturation flow rate multiplied by the effective green time ratio, so the observed PLTSF was calculated by applying *Equation 2*, but using the total permitted left-turn capacity in *Table 2* instead of the capacity during the intergreen period. It is important to emphasise that the permitted left-turn capacity at intersection 6 consists only of the capacity during the intergreen period due to the constant saturation in the opposing flow during the green time. Other data in *Table 2* were used for the PLTSF estimation using the proposed model and aforementioned relevant models. It is important to note that the number of sneakers was adopted as referred to in each model. In the CCG 3 model, the number of sneakers was estimated using the waiting space length. Given that the waiting space is longer than 9 m at each intersection, the accepted number

*Table 2 – Collected data for model testing and comparison*

| Intersection | $Q_O$ [pcu/h] | $C_L$ [pcu/h] | $n_{L_s}$ [pcu/cycle] | $g_e$ [s] | $c$ [s] | $g_u$ [s] | λ | $x_O$ | $N_O$ | Waiting space [m] |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 80 | 360 | 3 | 21 | 90 | 17.9 | 0.23 | 0.19 | 1 lane | 16.5 |
| 2 | 451 | 297 | 4 | 34 | 90 | 15.9 | 0.38 | 0.65 | 1 lane | 21.3 |
| 3 | 640 | 200 | 4 | 34 | 90 | 4.4 | 0.38 | 0.92 | 1 lane | 21.3 |
| 4 | 495 | 360 | 3 | 60 | 120 | 38.1 | 0.50 | 0.54 | 1 lane | 18.4 |
| 5 | 645 | 225 | 3 | 60 | 120 | 27.9 | 0.50 | 0.70 | 1 lane | 18.4 |
| 6 | 900 | 180 | 5 | 43 | 100 | 24.7 | 0.43 | 0.57 | 2 lanes | 22.7 |
| 7 | 920 | 200 | 4 | 43 | 90 | 27.4 | 0.48 | 0.52 | 2 lanes | 20.1 |

*Note: The waiting space represents space in the intersection between the stop line and the point where left-turn vehicles stop and wait for an acceptable gap.*

*Table 3 – Estimated permitted left-turn saturation flow rates and comparison*

| Intersection | | Observed | Proposed model | HCM 2016 model | CCG 3 model | ARRB model |
|---|---|---|---|---|---|---|
| 1 | $S_L$ [pcu/h] | 1,543 | 1,627 | 1,682 | 1,679 | 1,459 |
| | Observed-to-modelled ratio | | 0.95 | 0.92 | 0.92 | 1.06 |
| 2 | $S_L$ [pcu/h] | 786 | 717 | 1,166 | 675 | 784 |
| | Observed-to-modelled ratio | | 1.10 | 0.67 | 1.16 | 1.00 |
| 3 | $S_L$ [pcu/h] | 526 | 521 | 1,013 | 472 | 506 |
| | Observed-to-modelled ratio | | 1.01 | 0.52 | 1.11 | 1.04 |
| 4 | $S_L$ [pcu/h] | 720 | 614 | 1,037 | 663 | 647 |
| | Observed-to-modelled ratio | | 1.17 | 0.69 | 1.09 | 1.11 |
| 5 | $S_L$ [pcu/h] | 450 | 438 | 918 | 488 | 474 |
| | Observed-to-modelled ratio | | 1.03 | 0.49 | 0.92 | 0.95 |
| 6 | $S_L$ [pcu/h] | 419 | 444 | 796 | 551 | 699 |
| | Observed-to-modelled ratio | | 0.94 | 0.53 | 0.76 | 0.60 |
| 7 | $S_L$ [pcu/h] | 419 | 423 | 784 | 604 | 640 |
| | Observed-to-modelled ratio | | 0.99 | 0.53 | 0.69 | 0.65 |

of sneakers was 3 pcu/cycle. On the other hand, in the HCM 2016 model, the number of sneakers is the fixed value of 2 pcu/cycle. The number of sneakers in the ARRB model was measured in the field and given in *Table 2*, while in the proposed model, the number of sneakers was determined using *Equation 3*, whereby the waiting space length was divided by the average passenger car length of 5 m.

## 4. RESULTS AND DISCUSSION

The model testing and comparison is based on the ratio of the observed PLTSF to the modelled PLTSF and the root mean square error (RMSE). Furthermore, the proposed model is validated based on the results of the paired two-sample *t*-test.

The observed-to-modelled ratio is determined and given in *Table 3* for each intersection and each model.

Results show that the proposed, CCG 3 and ARRB models slightly underestimate or overestimate PLTSF values when left turn is opposed by the one-lane flow. The proposed model is the most precise at intersections 1, 3 and 5, while the ARRB or the CCG 3 model is more precise at intersections 2 and 4. Hence, the proposed model determines the PLTSF more accurately at lower and higher values of the opposing flow degree of saturation than at median values. Despite different approaches in the PLTSFe estimation (effective or unsaturated green time), those three models estimate the PLTSF close to observed values at one-opposing-lane intersections.

However, all models overestimate observed PLTSF values when the left turn is opposed by the two-lane flow (intersections 5 and 6). The proposed model overestimates observed values at these intersections the least, with the observed-to-modelled ratio of 0.94 and 0.99. The ARRB model overestimates the PLTSF at two-opposing-lanes intersections since it does not consider the number of opposing lanes and overestimates the unsaturated green time. The unsaturated green time overestimation is evident at intersection 6, where the opposing flow is saturated during the entire green time, and the PLTSF is entirely the result of sneakers during the intergreen periods. The CCG 3 model considers all variables affecting left turns but underestimates the effect of the number of opposing lanes.

On the other hand, the HCM 2016 model overestimates observed PLTSF at each intersection, regardless of the number of opposing lanes. This model overestimates the observed PLTSF with the observed-to-modelled ratio from 0.53 to 0.92. Among the considered models, the HCM 2016 model is the only one that does not include signal-timing parameters in the PLTSF model, which leads to the PLTSF overestimation.

Also, the RMSE is calculated considering intersections with one or two opposing lanes and considering all intersections (*Table 4*). The RMSE confirms that all analysed models, except the HCM model, determine the PLTSF close to observed values for one-opposing-lane intersections. At these intersections, the ARRB and the proposed model determine the PLTSF with the lowest RMSE of 51.5 pcu/h and 68.1 pcu/h, respectively. Although all models overestimate observed PLTSF values when the left turn is opposed by the two-lane flow, the proposed model overestimates with the lowest RMSE of 17.9 pcu/h.

In summary, the proposed model estimates the PLTSF in local conditions more precisely than other models even though the ARRB and CCG 3 models estimate PLTSF more precisely for some one-opposing-lane intersections. Considering all intersections, the proposed model, on average, determines the PLTSF with the lowest RMSE of 58.4 pcu/h (*Table 4*). According to the results, the proposed model is more precise for two-opposing-lanes intersections than for one-opposing-lane intersections. However, a reliable conclusion needs further research due to the inclusion of only two intersections with two opposing lanes and similar values of the degree of saturation at these intersections.

The finding that the proposed model determines the PLTSF with low deviations is supported by the result of the paired two-sample *t*-test. The result in-

*Table 4 – Model comparison*

| RMSE [pcu/h] | Proposed model | HCM 2016 model | CCG 3 model | ARRB model |
|---|---|---|---|---|
| One-opposing-lane intersections | 68.1 | 379.6 | 87.5 | 51.5 |
| Two-opposing-lanes intersections | 17.9 | 371.4 | 160.4 | 252.5 |
| All intersections | 58.4 | 377.2 | 113.3 | 141.8 |

*Table 5 – Number of sneakers comparison*

| $\eta_{L_s}$ [pcu/cycle] | Observed | Proposed model | HCM 2016 model | CCG 3 model | ARRB model |
|---|---|---|---|---|---|
| Intersection 1 | 3 | 3.3 | 2 | 3 | 1.5 |
| Intersection 2 | 4 | 4.3 | 2 | 3 | 1.5 |
| Intersection 3 | 4 | 4.3 | 2 | 3 | 1.5 |
| Intersection 4 | 3 | 3.7 | 2 | 3 | 1.5 |
| Intersection 5 | 3 | 3.7 | 2 | 3 | 1.5 |
| Intersection 6 | 5 | 4.5 | 2 | 3 | 1.5 |
| Intersection 7 | 4 | 4.0 | 2 | 3 | 1.5 |

dicates that the PLTSF determined by the proposed model is not significantly different from the observed PLTSF due to the $p$-value of 0.6426>0.05.

There is a brief review of the suggested number of sneakers (*Table 5*). The assumptions that the number of sneakers varies among intersections and that the predefined values could not be suitable for every intersection were confirmed. Values suggested by the HCM 2016 and ARRB models statistically differ from the observed values with $p$ of 0.0010 and 0.0002, respectively. Hence, the recommendation in the ARRB model to use the observed values instead of suggested is justified. It was also confirmed by previous results of model comparison. The CCG 3 model defines the number of sneakers depending on the intersection waiting space, and the number of sneakers statistically differs at $\alpha$=0.05 with $p$ of 0.0465. Still, this model estimates PLTSF close to observed values based on previous results. However, this method should be more sensitive to values of waiting space length. The number of sneakers determined by the proposed method does not statistically differ from the values measured in the field, based on the paired two-sample $t$-test ($p$=0.1458>0.05).

## 5. CONCLUSIONS

Accurate estimation of the saturation flow rate is essential, considering that it is an input variable in the signal timing optimisation and the capacity and delay estimation. The modelling of the PLTSF is an intricate procedure, so researchers applied different approaches (simulation, field study, gap-acceptance theory) to model the PLTSF considering different affecting variables. In this study, the PLTSF is determined as a sum of the PLTSF during the effective green time (PLTSFe) and the PLTSF during the intergreen period (PLTSFs). To our knowledge, this is the first time that the PLTSFe is modelled based on the number of opposing lanes and the opposing through-flow degree of saturation instead of the opposing through-flow rate and signal-timing parameters. In the PLTSFe modelling procedure, data were collected by 531 simulation experiments in VISSIM. Besides, the PLTSFs is considered depending on the number of cycles per hour and the number of sneakers per cycle, which should be estimated based on the length of the intersection waiting space if field data are not available.

The proposed model was evaluated using data collected at seven intersections, and the presented results lead to the following conclusions.

The paper hypothesis that PLTSFe depends on the opposing through-flow degree of saturation and the number of opposing lanes is confirmed. Firstly, the results of the two-sample $t$-test confirmed that the PLTSFe statistically differs when left turns are opposed by the one-lane and the two-lane flow. This result supports the earlier finding that PLTSF depends on the number of opposing lanes [2, 4, 6, 15]. Secondly, the dependency of the PLTSFe on the opposing flow degree of saturation is confirmed with $R^2$ of 0.99 and 0.97 for the opposing one-lane and two-lane flow, respectively.

The RMSE of 58.4 pcu/h indicates that the PLTSF could be accurately determined based on the opposing through-flow degree of saturation and the number of opposing lanes. According to model comparison, the proposed model is the most precise for local conditions. The proposed model is applicable in signal timing optimisation regardless of the number of opposing lanes, unlike other analysed models. Also, the proposed model allows PLTSF estimation in the signal timing optimisation based on the desired opposing flow degree of saturation instead of the signal-timing parameters, which are the output from this procedure.

The PLTSFs is especially important when the opposing flow is saturated during the entire green time, so this PLTSF component must not be neglected. Namely, at intersections where the PLTSF is mostly the result of sneakers, the saturation flow would be significantly underestimated if this PLTSF part is not considered. The paired two-sample *t*-test showed that the number of sneakers could be accurately estimated based on the waiting space length.

According to the presented results, simulation modelling of the PLTSF can result in a precise model. Hence, the accepted critical headways of 5 s and 6 s were suitable when left turn is opposed by the one-lane and two-lane flow, respectively.

Further research should be focused on the proposed model validation for its wide application. Validation should be performed with an expanded sample of intersections with various traffic and geometric characteristics, such as opposing flow rates, signal-timing parameters and the number of opposing lanes. Based on various traffic conditions in the opposing flow, the proposed model accuracy at different values of the opposing flow degree of saturation could be tested.

## ACKNOWLEDGEMENT

This research was carried out within the Project TR36021, "The Effects of Global Challenges on Transportation Planning and Traffic Management in Cities", financed by the Ministry of Education, Science and Technological Development of the Republic of Serbia.

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## *PRIMENA SIMULACIJE PRI MODELIRANJU ZASIĆENOG SAOBRAĆAJNOG TOKA NEZAŠTIĆENOG LEVOG SKRETANJA NA OSNOVU STEPENA ZASIĆENJA U KONFLIKTNOM TOKU PRAVO*

### *REZIME*

*U ovom radu je predstavljeno modeliranje zasićenog saobraćajnog toka nezaštićenog levog skretanja iz ekskluzivne saobraćajne trake. Ukupan zasićeni saobraćajni tok nezaštićenog levog skretanja se, u predloženom modelu, određuje kao suma zasićenog toka tokom efektivnog zelenog vremena i zasićenog toka tokom međuzelenog perioda. Zasićeni saobraćajni tok tokom efektivnog zelenog vremena je modeliran na osnovu stepena zasićenja u konfliktnom toku pravo i broja traka konfliktnog toka pravo. Zavisnost između zasićenog saobraćajnog toka tokom efektivnog zelenog vremena i ovih promenljivih je analizirana na osnovu podataka prikupljenih primenom simulacije u VISSIM-u. Koliko nam je poznato, ovo je prva studija o modeliranju zasićenog saobraćajnog toka nezaštićenog levog skretanja na osnovu stepena zasićenja u konfliktnom toku pravo, umesto intenziteta konfliktnog toka pravo i parametara rada svetlosnih signala. Predloženi model je testiran na osnovu realnih podataka prikupljenih na sedam raskrsnica sa nezaštićenim levim skretanjem iz ekskluzivne saobraćajne trake. Vrednost RMSE od 58.4 PAJ/h pokazuje da zasićeni saobraćajni tok nezaštićenog levog skretanja može biti precizno određen na osnovu stepena zasićenja u konfliktnom toku i broja saobraćajnih traka konfliktnog toka.*

### *KLJUČNE REČI*

*nezaštićeno levo skretanje; zasićeni saobraćajni tok; stepen zasićenja; signalisana raskrsnica; simulacija.*

## REFERENCES

[1] *HCM 2016: Highway Capacity Manual*. Washington, DC: National Research Council, Transportation Research Board; 2016.

[2] Wang M-H, Benekohal RF. Analysis of left-turn saturation flow rates and capacity at signalized intersections. *Transportation Research Record*. 2010;2173(1): 96–104. doi: 10.3141/2173-12.

[3] Bai Q, et al. A method for determining the capacity of an exclusive left lane with a permitted phase under nonstrict priority. *Advances in Civil Engineering*. 2019;2019. doi: 10.1155/2019/2357437.

[4] Wu J-WJ. *Evaluation and enhancement of opposed left-turn flow analysis methodology*. PhD thesis. University of Florida; 1995.

[5] Akçelik R. *Traffic signals: Capacity and timing analysis*. Australian Road Research Board. Research report ARR No. 123, 1981.

[6] Teply S, Allingham DI, Richardson DB, Stephenson BW. *Canadian capacity guide for signalized intersections*. 3rd Edition. Toronto, Ontario, Canada: Canadian Institute of Transportation Engineers; 2008.

[7] Niittymäki J, Pursula M. Saturation flows at signal-group-controlled traffic signals. *Transportation Research Record*. 1997;1572(1): 24–32. doi: 10.3141/1572-04.

[8] Vejdirektoratet. [*Manual for capacity and level of service*]. 2015.

[9] Bhuiyan NF. *Towards performance measure analysis: Development of a left turn saturation flow rate model at signalized intersections*. PhD thesis. Claremont Graduate

University and California State University Long Beach; 2018.

[10] Chang G-L, Chen C-Y, Perez C. Hybrid model for estimating permitted left-turn saturation flow rate. *Transportation Research Record.* 1996;1566(1): 54–63. doi: 10.1177/0361198196156600107.

[11] Radičević V, Krstanoski N, Subotić M. New approach to estimating the saturation flow rate of a shared lane with permitted left turns. *Promet – Traffic&Transportation*. 2020;32(4): 573–583. doi: 10.7307/ptt.v32i4.3458.

[12] Mousa RM. Simulation modeling and variability assessment of delays at traffic signals. *Journal of Transportation Engineering*. 2003;129(2): 177–185. doi: 10.1061/(ASCE)0733-947X(2003)129:2(177).

[13] Kocić A, Čelar N, Stanković S. [VISSIM calibration for permitted left turn on signalized intersections in Belgrade]. *Put i saobraćaj*. 2018;64(4): 49–53. doi: 10.31075/PIS.64.04.07.

[14] Kocić A, Čelar N, Kajalić J, Stanković S. [Estimation of saturation flow rates on dual left-turn lanes]. *Tehnika*. 2018;65(2): 254–261. doi: 10.5937/tehnika1802254K.

[15] Al-Mistarehi B, Alomari A, Al Zoubi M. Investigation of saturation flow rate using video camera at signalized intersections in Jordan. *Open Engineering*. 2021;11(1): 216–226. doi: 1515/eng-2021-0021.

[16] Webster FV, Cobbe BM. *Traffic signals*. London: Her Majesty's Stationery Office; 1966.

[17] Fambro DB, Gaston GD, Hoff CM. *Comparison of two protected-permitted lead-lag left-turn phasing arrangements.* Texas Transportation Institute, Texas A&M University System, College Station Texas. Report: 989-1F, 1991.

[18] Aoyama E, Yoshioka K, Shimokawa S, Morita H. Estimating saturation flow rates at signalized intersections in Japan. *Asian Transport Studies*. 2020;6. doi: 10.1016/j.eastsj.2020.100015.