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ORDER-PARAMETER MODEL FOR SYNERGETIC THEORY-BASED RAILWAY FREIGHT SYSTEM AND EVOLUTION IN CHINA

ABSTRACT

The paper has investigated the synergetic nature and complexity of the railway freight system and selected thirteen parameters (railway fixed asset investment, GDP, railway revenue kilometres, etc.) as the system's state variables. By using the method of least square and the method of external function, an order-parameter model for synergetic theory-based railway freight system has been built, which will potentially support the studies on the railway freight system evolution. The result shows that railway fixed asset investment is the order-parameter that governs the evolution of the railway freight system: the average random fluctuation parameter ω is 0.7060, which means that the mean fluctuation period of railway freight system is nine years. The evolution of the railway freight system is a gradual process with abrupt changes from time to time.

KEY WORDS

railway freight, synergetics, order-parameter, railway freight transport, order-parameter modelling, MATLAB

1. INTRODUCTION

The railway freight system in China is the primary means for the delivery of coals, petroleum, iron, steel, non-metallic ore, metallic ore, grains and other key materials which are vital to the development of the nation's economy. Railway freight system is a complicated open system, involving economy, transport and social aspects that are mutually beneficial and restrictive. Research on the targeted-control of the order-parameter of railway freight system will guide the development of railway freight system into a more efficient and orderly way. Healthy growth of the complex transport system will better serve the society and economy. Synergetics was proposed by the famous physicist Herman Haken in 1970s and is now the leading theory in varied complex systems [1]. Synergetics is used to discover the self-organization of a complex system [2-5],to discuss the influence of order-parameter on a system [6-8],to study the rules according to which the complex systems evolve from disordered status to ordered status [9-15]. Few studies have been conducted on synergetics in the field of transport, mostly about the sustainable transport development [16-18], traffic administration [19-22], multi-modal transport [23-24] and other applications [25-28]. Most of these studies focus on local issues without understanding the rules of evolution of the transport system. This article uses the synergetics theory to construct the order-parameter model of the railway freight system. By locating the order-parameters that influence the system, the evolution of the railway freight system is analyzed.

2. INTERPRETATION OF RAILWAY FREIGHT SYSTEM SYNERGETIC NATURE

Railway freight system is a complex system combining the natural system and the manual system. With the time going on, the synergetic result is achieved by the movements between the sub-systems inside. As an open system, the railway freight system evolves from a disordered status to an ordered status, exchanging substance and energy with the external world. In this large system, thousands of people gather, engage in political, economic and cultural activities. It is really a mega system featured with complexity. The synergetic theory is applied to research on the structure and order of the complicated railway freight system, because the railway freight system has the following synergetic characteristics:

(1) Railway freight system developed in a self-organized way. It is designed and operated by certain rules, in other words, it runs in an ordered way. China is a typical continental country, characterized by asymmetric resource distribution and industrial distribution. The economic activities cover a wide area. With the large-scale transportation of product and production elements, the railway freight system is born as an outcome of the synergetic coordination between transportation supplies and transportation demands. The synergetic effects of the stream of people, logistics stream and information stream result in the spatial clusters of railway freight system. The result is: elements of the railway freight system meet and part in the space. The railway freight system forms by self-organization, and once it comes out, it develops a structure of its own nature and rules. It is in essence an organized organization. It operates by using its own rules, which are in fact the order of the railway freight system.

(2) The self-organization of the railway system is more complicated than the natural system.

Self-organization of the natural system is an optimized way for evolution after a long-time natural maturation. It is a relatively efficient way for the recycling of natural resources and matter energies. The railway freight system comes out spontaneously and is driven by many factors, such as the environment of transport market, demands on the transport market, social and economic development, the competitiveness of railway freight service and management. The development of the railway freight system thus will exhibit different structures and complexities than the nature system. The railway freight service involves trains, machines, engineering, power supply and vehicles. Its growth is closely related to the society and economy, environment and resources. The departments and elements in the system's development are mutually cooperative and competitive, which is a synergetic phenomenon. It increases the complexity of the self-organization of railway freight system. Railway freight system exchanges substances and energies with the external world, a self-organization process dynamically adapting to the outside world.

(3) Railway freight system based on synergetics is an organized continuum. The development of the railway freight system includes three processes: the first process is the evolution of railway freight system from a non-organized status to an organized one, from disorder to orders. It is the origin of an organization. The second is the process featuring with the railway freight system's in-shot. The system starts running according to certain rules, and the orderliness is improved by the in-shot. This process features complexity. The third process is about the railway freight system's organizational structure and function: from simplicity to complexity. The railway freight system synergetics is a self-organized process, with synergetic process running through the whole development process.

The railway freight system is an open and complicated system, which exchanges substances, energies and information with external world to keep it running on the right track.

Through the position movement of cargoes, it is evolving. Railway freight system is characterized by non-equilibrium; with the distribution of substances and energies in an unbalanced way within the system. From the perspective of time, the system is in the growth stage, and from the perspective of space, the system exhibits regional differences (relatively developed in coastal areas and lagging in western regions). The elements inside the railway freight system follow the non-linear mechanism: random fluctuation of any element might result in minor changes of the system status, and then be amplified by the non-linear feedback mechanism, which in turn leads to major changes of the system. Fluctuations thus bring abrupt changes to the system on the whole, which results in a more coordinated and orderly status.

3. ORDER-PARAMETER MODEL FOR SYNERGETICS-BASED RAILWAY FREIGHT SYSTEM

3.1 Selection of railway freight system status variable

In the social and economic system, the development of railway freight system is not a separate process; instead, it is closely related to the growth of the national economy, the adjustment of the industrial structure and some other primary social features.

To locate the order-parameters of railway freight system, it is necessary to find out first the major factors that influence the transport of railway cargos. Based on the synergetic theory, there are a variety of parameters that can be used to evaluate and indicate the system development within the railway freight system. The railway freight system interconnects the national economy system and the transport system, while being under the influence of other systems. Therefore, the article defines thirteen status variables for railway freight system. For these 13 parameters, data from 1991 to 2009 are indicated in *Table 1*.

Table 1 - The state variable of the railway freight transportation system in China

Year	Railway freight volume X_1/10,000 tons	Railway freight turnover X_2/1,000,000,000 ton kilometers	Service mileage X_3/10,000 kilometres	Railway goods vehicle X_4/unit	Railway freight transport revenue X_5/1,000,000,000 yuan	GDP X_6/1,000,000,000 yuan	First industry X_7/1,000,000,000 yuan	Second industry X_8/1,000,000,000 yuan	Quantity of railway locomotives X_9/unit	Railway fixed asset investments X_{10}/1,000,000,000 yuan	Society's goods turnover X_{11}/1,0000,000,000 ton kilometres	Society's freight turnover X_{12}/10,000 tons	Total investments in transport industry X_{13}/1,000,000,000 yuan
1991	152,893	10,972.0	5.8	370,054	291.4	21,781.5	5,342.2	9,102.2	13,906	135.5	27,987.0	985,793	330.6
1992	157,627	11,575.6	5.8	373,233	300.9	26,923.5	5,866.6	11,699.5	14,083	168.6	29,218.0	1,045,899	448.3
1993	162,794	12,090.9	5.9	390,097	315.5	35,333.9	6,963.8	16,454.4	14,397	317.4	30,646.81	1,115,902	901.2
1994	163,216	12,632.0	5.9	415,919	346.0	48,197.9	9,572.7	22,445.4	14,694	456.7	33,435.48	1,180,396	1,373.0
1995	165,982	13,049.5	6.2	432,731	357.6	60,793.7	12,135.8	28,679.5	15,146	462.9	35,908.9	1,234,938	1,587.5
1996	171,024	13,106.2	6.5	443,893	406.2	71,176.6	14,015.4	33,835.0	15,403	467.9	36,589.8	1,298,421	1,847.1
1997	172,149	13,269.9	6.6	437,686	460.8	78,973.0	14,441.9	37,543.0	15,335	496.0	38,384.7	1,278,218	2,197.5
1998	164,309	12,560.1	6.6	439,326	502.7	84,402.3	14,817.6	39,004.2	15,176	634.4	38,088.7	1,267,427	3,252.2
1999	167,554	12,910.3	6.7	436,236	522.4	89,677.1	14,770.0	41,033.6	14,480	680.2	40,567.8	1,293,008	3,429.3
2000	178,581	13,770.5	6.9	439,943	566.6	99,214.6	14,944.7	45,555.9	14,472	672.3	44,320.5	1,358,682	3,642.0
2001	193,189	14,694.1	7.0	449,921	688.6	109,655.2	15,781.3	49,512.3	14,955	689.1	47,709.9	1,401,786	4,116.4
2002	204,956	15,658.4	7.2	446,707	704.7	120,332.7	16,537.0	53,896.8	15,159	717.8	50,685.9	1,483,447	4,394.0
2003	224,248	17,246.7	7.3	503,868	784.3	135,822.8	17,381.7	62,436.3	15,456	616.4	53,859.2	1,564,492	4,892.7
2004	249,017	19,288.8	7.4	520,101	943.7	159,878.3	21,412.7	73,904.3	16,066	829.5	69,445.0	1,706,412	7,091.5
2005	269,296	20,726.0	7.5	541,824	1,105.7	184,937.4	22,420.0	87,598.1	16,547	1,267.7	80,258.1	1,862,066	8,860.4
2006	288,224	21,954.4	7.7	558,483	1,281.0	216,314.4	24,040.0	103,719.5	16,904	1,966.5	88,839.85	2,037,060	11,224.5
2007	314,237	23,797.0	7.8	571,078	1,392.5	265,810.3	28,627.0	125,831.4	17,311	2,492.7	101,418.8	2,275,822	12,997.1
2008	330,354	25,106.3	8.0	584,961	1,593.2	314,045.4	33,702.0	149,003.4	17,336	4,073.2	110,300.0	2,585,937	15,700.5
2009	333,348	25,239.2	8.6	594,388	1,623.5	340,506.9	35,226.0	157,638.8	17,825	6,660.9	122,133.3	2,825,222	23,271.3

Data source: China Statistic Yearbook 1992~2010

3.2 Description of order-parameter modelling

Suppose a non-equilibrium system characterized by n varying parameters. Thus it can be indicated with n-dimension vectors as follows:

$$\vec{q} = (q_1, q_2, \ldots, q_i, \ldots, q_n)$$

Generally, the motion form of $\vec{q}$ is expressed by the generalized Langevin equation:

$$\dot{q} = k_j(\vec{q}) + \xi_j(t) \quad (1)$$

where $\xi_j(t)$ is the random fluctuation generated by perturbation; $k_j(\vec{q}) = k_j(q_1, q_2, \ldots, q_i, \ldots, q_n)$ suggests the non-linear function of the varying parameters in the system. With $\xi_j(t)$ left out, equation (1) can be expressed as:

$$\dot{q} = \sum_{k=1}^{n} a_{jk} q_k + f_j(\vec{q}) \quad (2)$$

In equation (2), $f_j(\vec{q})$ is a non-linear function, and equation (2) is expressed as:

$$\dot{q}_1 = \gamma_1 q_1 + \sum_{k=1, k\neq 1}^{n} a_{jk} q_k + f_1(q_1, q_2, \ldots, q_i, \ldots, q_n)$$

$$\dot{q}_2 = \gamma_2 q_2 + \sum_{k=1, k\neq 2}^{n} a_{jk} q_k + f_2(q_1, q_2, \ldots, q_i, \ldots, q_n)$$

....

$$\dot{q}_n = \gamma_n q_n + \sum_{k=1, k\neq n}^{n} a_{jk} q_k + f_n(q_1, q_2, \ldots, q_i, \ldots, q_n) \quad (3)$$

In equation (3), $(\gamma_1, \gamma_2, \ldots, \gamma_n)$ is the relaxation coefficient, and $\gamma_i = a_{ii}$. According to Harken's synergetics theory, the order-parameter, on one hand, is the outcome of the subsystems' collective movements within the system (mutually competitive and synergetic). On the other hand, once it comes into being (supports or uses the subsystem), it dominates the overall evolving process. Order-parameter, as a slow relaxation parameter, takes quite a long time, or even infinite time, to achieve the new steady relaxation state. In the system's evolving process, it always plays a decisive role. Therefore, γ_i can be used to judge the order-parameter: the lower the γ_i is, the role of status variable it plays is more important. The status variable of the minimum value of γ_i is the order parameter. For the order-parameter u of a general non-linear transition system, the system's evolution is dominated by u and all other variables used by u.

3.3 Construction and solution of the model

Suppose $X_1(t)$, $X_2(t)$, $X_3(t)$, $X_4(t)$, $X_5(t)$, $X_6(t)$, $X_7(t)$, $X_8(t)$, $X_9(t)$, $X_{10}(t)$, $X_{11}(t)$, $X_{12}(t)$, $X_{13}(t)$ $(t = 1, 2, \ldots, 19)$ as railway freight volume, railway freight turnover, service mileage, railway goods vehicle, railway freight transport revenue, GDP, first industry, second industry, quantity of railway locomotives, railway fixed asset investments, society's goods turnover, society's freight turnover and total investments in transport industry. For the detailed data, refer to the Table.

Modelling processes are described as follows:

(1) Logarithm transformation of status variable. The calculation of the logarithm of the detailed status variables does not change the nature and the relationship of the data. Data obtained can easily solve the problem of heteroscedasticity. Suppose status variable

$$X_1(t),\ X_2(t),\ X_3(t),\ X_4(t),\ X_5(t),\ X_6(t),\ X_7(t),$$
$$X_8(t),\ X_9(t),\ X_{10}(t),\ X_{11}(t),\ X_{12}(t),\ X_{13}(t)$$
$$(t = 1, 2, \ldots, 19)$$

With the logarithm transformation, $x_i^{(0)}(t) = \ln X_i(t)$ changes into

$$x_1^{(0)}(t),\ x_2^{(0)}(t),\ x_3^{(0)}(t),\ x_4^{(0)}(t),\ x_5^{(0)}(t),\ x_6^{(0)}(t),$$
$$x_7^{(0)}(t),\ x_8^{(0)}(t),\ x_9^{(0)}(t),\ x_{10}^{(0)}(t),\ x_{11}^{(0)}(t),\ x_{12}^{(0)}(t),$$
$$x_{13}^{(0)}(t).$$

(2) For $x_1^{(0)}(t)$, $x_2^{(0)}(t)$, $x_3^{(0)}(t)$, $x_4^{(0)}(t)$, $x_5^{(0)}(t)$, $x_6^{(0)}(t)$, $x_7^{(0)}(t)$, $x_8^{(0)}(t)$, $x_9^{(0)}(t)$, $x_{10}^{(0)}(t)$, $x_{11}^{(0)}(t)$, $x_{12}^{(0)}(t)$, $x_{13}^{(0)}(t)$, conduct AGO (accumulating generation). It then changes into $x_1^{(1)}(t)$, $x_2^{(1)}(t)$, $x_3^{(1)}(t)$, $x_4^{(1)}(t)$, $x_5^{(1)}(t)$, $x_6^{(1)}(t)$, $x_7^{(1)}(t)$, $x_8^{(1)}(t)$, $x_9^{(1)}(t)$, $x_{10}^{(0)}(t)$, $x_{11}^{(0)}(t)$, $x_{12}^{(0)}(t)$, $x_{13}^{(0)}(t)$.

$$x_i^{(1)}(t) = \left(\sum_{t=1}^{1} x_i^{(0)}(t), \sum_{t=1}^{2} x_i^{(0)}(t), \ldots, \sum_{t=1}^{n} x_i^{(0)}(t)\right).$$

(3) Suppose the rate of change of $x_i^{(1)}(t)$ is $dx_i^{(1)}(t)/dt$. Single $dx_i^{(1)}(t)/dt$ follows the logistic rules, which is determined by its development and control effects. The development item is $a_{ii}x_i^{(1)}(t)$, the control item is $-b_{ii}(x_i^{(1)}(t))^2$. In the process of multi-variable system's evolvement, there is a synergetic effect working between different parameters within the system, that is, the synergetic and competitive relationship between variables. The synergetic item is $a_{ij}x_j^{(1)}(t)$ (parameter j has a synergetic effect on parameter i), the competitive item is $-b_{ii}(x_i^{(1)}(t))^2$, (parameter j has a competitive effect on parameter i), the average rate of change of $x_i^{(1)}(t)$ is λ_i (λ_i is a constant);the system fluctuation is recorded $\xi_i(t)$. The overall changing rate of $x_i^{(1)}(t)$ is

$$\frac{dx_i^{(1)}(t)}{dt} = a_{ii}x_i^{(1)}(t) - \left(b_{ii} + \sum_{j=1, j\neq i} b_{ij}\right)\left(x_i^{(1)}(t)\right)^2 +$$
$$+ \sum_{j=1, j\neq i} a_{ij}x_j^{(1)}(t) + \lambda_i + \xi_i(t) \quad (i = 1, 2, \ldots, 13)$$

(4) Suppose

$$-\left(b_{ii} + \sum_{j=1, j\neq i} b_{ij}\right) = b_i,$$

put it in equation (4), the non-linear different equation is:

$$\frac{dx_i^{(1)}(t)}{dt} = a_{ii}x_i^{(1)}(t) + b_i\left(x_i^{(1)}(t)\right)^2 + \sum_{j=1, j\neq i} a_{ij}x_j^{(1)}(t) +$$
$$+ \lambda_i + \xi_i(t) = b_i\left(x_i^{(1)}(t)\right)^2 + \sum_{j=1} a_{ij}x_j^{(1)}(t) +$$

$$+\lambda_i + \xi_i(t) \quad (i = 1, 2, \ldots, 13) \tag{4}$$

(5) Define parameters a_i, b_i, a_{ij} with the least square method. As

$$\frac{dx_i^{(1)}(t)}{dt} = x_i^{(1)}(t) - x_i^{(1)}(t-1) = x_i^{(0)}(t) \tag{5}$$

means:

$$x_i^{(0)}(t) = b_i(x_i^{(1)}(t))^2 + \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) + \lambda_i + \xi_i(t) \tag{6}$$

$$\xi_i(t) = x_i^{(0)}(t) - b_i(x_i^{(1)}(t))^2 - \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) - \lambda_i \tag{7}$$

According to the Least Square Method principle:

$$S = \min\frac{1}{2}\|f_i(t)\|_2^2 =$$

$$= \min\frac{1}{2}\sum_{t=2}^{19}\left[x_i^{(0)}(t) - b_i(x_i^{(1)}(t))^2 - \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) - \lambda_i\right]^2 \tag{8}$$

Based on the necessary condition for extreme value of multi-variable functions, the following is obtained:

$$\frac{\partial S}{b_i} = -\sum_{t=2}^{19}\left[x_i^{(0)}(t) - b_i(x_i^{(1)}(t))^2 - \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) - \lambda_i\right]\cdot$$

$$\cdot(x_i^{(1)}(t))^2 =$$

$$= -\sum_{t=2}^{19}\left[x_i^{(0)}(t)(x_i^{(1)}(t))^2 - b_i(x_i^{(1)}(t))^4 -\right.$$

$$\left.-\sum_{j=1}^{13} a_{ij}x_j^{(1)}(t)(x_i^{(1)}(t))^2 - \lambda_i(x_i^{(1)}(t))^2\right] =$$

$$= -\sum_{2}^{19} x_i^{(0)}(t)(x_i^{(1)}(t))^2 + b_i\sum_{t=2}^{19}(x_i^{(1)}(t))^4 +$$

$$+a_{ij}\sum_{j=1}^{13}\sum_{t=2}^{19} x_j^{(1)}(t)(x_i^{(1)}(t))^2 + \lambda_i\sum_{t=2}^{19}(x_i^{(1)}(t))^2 =$$

$$= 0 \tag{9}$$

$$\frac{\partial S}{a_{ik}} = -\sum_{t=2}^{19}\left[x_i^{(0)}(t) - b_i(x_i^{(1)}(t))^2 - \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) - \lambda_i\right]\cdot$$

$$\cdot x_k^{(1)}(t) =$$

$$= -\sum_{t=2}^{19}\left[x_i^{(0)}(t)x_k^{(1)}(t) - b_i(x_i^{(1)}(t))^2 x_k^{(1)}(t) -\right.$$

$$\left.-\sum_{j=1}^{13} a_{ij}x_j^{(1)}(t)x_k^{(1)}(t) - \lambda_i x_k^{(1)}(t)\right] =$$

$$= -\sum_{2}^{19} x_i^{(0)}(t)x_k^{(1)}(t) + b_i\sum_{t=2}^{19}(x_i^{(1)}(t))^2 x_k^{(1)}(t) +$$

$$+a_{ij}\sum_{j=1}^{13}\sum_{t=2}^{19} x_j^{(1)}(t)x_k^{(1)}(t) + \lambda_i\sum_{t=2}^{19} x_k^{(1)}(t) =$$

$$= 0 \tag{10}$$

$$\frac{\partial S}{\lambda_i} = -\sum_{t=2}^{19}\left[x_i^{(0)}(t) - b_i(x_i^{(1)}(t))^2 - \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) - \lambda_i\right] =$$

$$= -\sum_{t=2}^{19}\left[x_i^{(0)}(t) - b_i(x_i^{(1)}(t))^2 - \sum_{j=1}^{13} a_{ij}x_j^{(1)}(t) - \lambda_i\right] =$$

$$= -\sum_{2}^{19} x_i^{(0)}(t) + b_i\sum_{t=2}^{19}(x_i^{(1)}(t))^2 +$$

$$+a_{ij}\sum_{j=1}^{13}\sum_{t=2}^{19} x_j^{(1)}(t) + 18\lambda_i =$$

$$= 0 \tag{11}$$

Equations (9) (10) (11) change into matrix forms:

$$B_i = \begin{bmatrix} \sum_{t=2}^{19}(x_i^{(1)}(t))^4 & \sum_{t=2}^{19}x_1^{(1)}(t)(x_i^{(1)}(t))^2 & \sum_{t=2}^{19}x_1^{(1)}(t)(x_i^{(1)}(t))^2 & \cdots & \sum_{t=2}^{19}x_{13}^{(1)}(t)(x_i^{(1)}(t))^2 & \sum_{t=2}^{19}(x_i^{(1)}(t))^2 \\ \sum_{t=2}^{19}x_1^{(1)}(t)(x_i^{(1)}(t))^2 & \sum_{t=2}^{19}x_1^{(1)}(t)x_1^{(1)}(t) & \sum_{t=2}^{19}x_1^{(1)}(t)x_2^{(1)}(t) & \cdots & \sum_{t=2}^{19}x_1^{(1)}(t)x_{13}^{(1)}(t) & \sum_{t=2}^{19}x_1^{(1)}(t) \\ \sum_{t=2}^{19}x_2^{(1)}(t)(x_i^{(1)}(t))^2 & \sum_{t=2}^{19}x_1^{(1)}(t)x_2^{(1)}(t) & \sum_{t=2}^{19}x_2^{(1)}(t)x_2^{(1)}(t) & \cdots & \sum_{t=2}^{19}x_2^{(1)}(t)x_{13}^{(1)}(t) & \sum_{t=2}^{19}x_2^{(1)}(t) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sum_{t=2}^{19}x_{13}^{(1)}(t)(x_i^{(1)}(t))^2 & \sum_{t=2}^{19}x_1^{(1)}(t)x_{13}^{(1)}(t) & \sum_{t=2}^{19}x_2^{(1)}(t)x_{13}^{(1)}(t) & \cdots & \sum_{t=2}^{19}x_6^{(1)}(t)x_{13}^{(1)}(t) & \sum_{t=2}^{19}x_{13}^{(1)}(t) \\ \sum_{t=2}^{19}(x_i^{(1)}(t))^2 & \sum_{t=2}^{19}x_1^{(1)}(t) & \sum_{t=2}^{19}x_2^{(1)}(t) & \cdots & \sum_{t=2}^{19}x_{13}^{(1)}(t) & \sum_{t=2}^{19}1 \end{bmatrix}_{15\times15}$$

$$A_i = \begin{bmatrix} \sum_{t=2}^{19}x_i^{(0)}(t)(x_i^{(1)}(t))^2 \\ \sum_{t=2}^{19}x_i^{(0)}(t)x_i^{(1)}(t) \\ \vdots \\ \sum_{t=2}^{19}x_i^{(0)}(t)x_{13}^{(1)}(t) \\ \sum_{t=2}^{19}x_i^{(0)}(t) \end{bmatrix}$$

$$C_i = [b_i \;\; a_{i1} \;\; a_{i2} \;\; \ldots \;\; a_{i12} \;\; a_{i13} \;\; \lambda_i]^T$$

It changes into a matrix form:

$$B_i C_i = A_i \tag{12}$$

Matrix B_i can be proven as a symmetrical definite matrix; therefore, there is a unique solution

$$C_i = B_i^{-1}A_i \tag{13}$$

Insert the data specified in *Table 1* into equation(13). By MATLAB(R2010a)programming, it is obtained that Model (5) is:

$$\frac{dx_1^{(1)}(t)}{dt} = -3.4452\times10^{-4}x_1^{(1)}(t)^2 + 2.4711x_1^{(1)}(t) -$$

$$-0.9364x_2^{(1)}(t) + 1.5430x_3^{(1)}(t) + 0.4861x_4^{(1)}(t) -$$

$$-0.1345x_5^{(1)}(t) + 1.4412x_6^{(1)}(t) - 0.01176x_7^{(1)}(t) -$$

$$-0.9086x_8^{(1)}(t) - 2.3488x_9^{(1)}(t) + 0.05657x_{10}^{(1)}(t) +$$

$$+0.2681x_{11}^{(1)}(t) - 1.0935x_{12}^{(1)}(t) - 0.04862x_{13}^{(1)}(t) +$$

$$+11.7068 + \xi_1(t)$$

$$\frac{dx_2^{(1)}(t)}{dt} = -5.5698\times10^{-4}x_1^{(1)}(t)^2 + 1.3409x_1^{(1)}(t) +$$

$$+0.1301x_2^{(1)}(t) + 0.8407x_3^{(1)}(t) + 0.1652x_4^{(1)}(t) -$$

$$-0.0904x_5^{(1)}(t) + 1.3656x_6^{(1)}(t) - 0.1401x_7^{(1)}(t) -$$

$$-0.8567x_8^{(1)}(t) - 2.1702x_9^{(1)}(t) - 0.03573x_{10}^{(1)}(t) +$$

$$+0.06596x_{11}^{(1)}(t) - 0.3783x_{12}^{(1)}(t) + 0.1040x_{13}^{(1)}(t) +$$

$$+9.2552 + \xi_2(t)$$

$$\frac{dx_3^{(1)}(t)}{dt} = -2.1683\times10^{-3}x_3^{(1)}(t)^2 + 0.5059x_1^{(1)}(t) +$$

$$+0.4601x_2^{(1)}(t) + 1.4029x_3^{(1)}(t) + 0.3487x_4^{(1)}(t) -$$

$$-0.1168x_5^{(1)}(t) + 0.5828x_6^{(1)}(t) + 0.06998x_7^{(1)}(t) -$$

$$-0.5967x_8^{(1)}(t)-0.3266x_9^{(1)}(t)+0.1604x_{10}^{(1)}(t)+ +0.0700x_{11}^{(1)}(t)-0.6961x_{12}^{(1)}(t)-0.1391x_{13}^{(1)}(t)+ +1.6010+\xi_3(t)$$

$$\frac{dx_4^{(1)}(t)}{dt}=6.1545\times10^{-5}x_4^{(1)}(t)^2-0.1978x_1^{(1)}(t)+ +1.2653x_2^{(1)}(t)+0.2798x_3^{(1)}(t)+0.8176x_4^{(1)}(t)- -0.0446x_5^{(1)}(t)-0.0642x_6^{(1)}(t)+0.03218x_7^{(1)}(t)- -0.1098x_8^{(1)}(t)-0.6162x_9^{(1)}(t)+0.1456x_{10}^{(1)}(t)- -0.7175x_{11}^{(1)}(t)-0.4353x_{12}^{(1)}(t)-0.0448x_{13}^{(1)}(t)+ +12.8301+\xi_4(t)$$

$$\frac{dx_5^{(1)}(t)}{dt}=-1.6439\times10^{-3}x_5^{(1)}(t)^2-1.7507x_1^{(1)}(t)+ +2.0063x_2^{(1)}(t)+1.6411x_3^{(1)}(t)+0.4599x_4^{(1)}(t)+ +1.8930x_5^{(1)}(t)-0.6934x_6^{(1)}(t)+0.5860x_7^{(1)}(t)- -0.1102x_8^{(1)}(t)-1.9923x_9^{(1)}(t)+0.03702x_{10}^{(1)}(t)- -0.2072x_{11}^{(1)}(t)+0.5945x_{12}^{(1)}(t)-0.2005x_{13}^{(1)}(t)+ +5.0159+\xi_5(t)$$

$$\frac{dx_6^{(1)}(t)}{dt}=-2.4712\times10^{-4}x_6^{(1)}(t)^2+2.2698x_1^{(1)}(t)+ +0.4579x_2^{(1)}(t)-7182x_3^{(1)}(t)-0.2410x_4^{(1)}(t)- -0.4698x_5^{(1)}(t)+0.8826x_6^{(1)}(t)+0.06767x_7^{(1)}(t)- -0.5818x_8^{(1)}(t)-1.4178x_9^{(1)}(t)+0.1434x_{10}^{(1)}(t)- -0.3804x_{11}^{(1)}(t)-0.9285x_{12}^{(1)}(t)+0.1989x_{13}^{(1)}(t)+ +10.0694+\xi_6(t)$$

$$\frac{dx_7^{(1)}(t)}{dt}=-8.5670\times10^{-4}x_7^{(1)}(t)^2+6.0908x_1^{(1)}(t)+ +0.3719x_2^{(1)}(t)-0.3448x_3^{(1)}(t)-0.5434x_4^{(1)}(t)- -0.8146x_5^{(1)}(t)+2.3977x_6^{(1)}(t)+1.35476x_7^{(1)}(t)- -2.6728x_8^{(1)}(t)-3.7932x_9^{(1)}(t)+0.3009x_{10}^{(1)}(t)- -1.4307x_{11}^{(1)}(t)-2.1480x_{12}^{(1)}(t)+0.6360x_{13}^{(1)}(t)+ +9.0298+\xi_7(t)$$

$$\frac{dx_8^{(1)}(t)}{dt}=-6.8071\times10^{-5}x_8^{(1)}(t)^2+0.8954x_1^{(1)}(t)+ +0.5804x_2^{(1)}(t)-0.9554x_3^{(1)}(t)-0.1013x_4^{(1)}(t)- -0.3171x_5^{(1)}(t)+0.01356x_6^{(1)}(t)-0.0655x_7^{(1)}(t)+ +0.1038x_8^{(1)}(t)-1.0450x_9^{(1)}(t)+0.03889x_{10}^{(1)}(t)+ +0.03573x_{11}^{(1)}(t)-0.3414x_{12}^{(1)}(t)- -0.003227x_{13}^{(1)}(t)+8.7754+\xi_8(t)$$

$$\frac{dx_9^{(1)}(t)}{dt}=-5.6561\times10^{-4}x_9^{(1)}(t)^2-0.4884x_1^{(1)}(t)+ +1.4352x_2^{(1)}(t)-0.4388x_3^{(1)}(t)+0.2158x_4^{(1)}(t)- +0.2158x_5^{(1)}(t)+0.9925x_6^{(1)}(t)-0.2391x_7^{(1)}(t)- -0.6466x_8^{(1)}(t)-0.4832x_9^{(1)}(t)-0.02510x_{10}^{(1)}(t)- -0.1114x_{11}^{(1)}(t)-0.1662x_{12}^{(1)}(t)+0.1217x_{13}^{(1)}(t)+ +9.5070+\xi_9(t)$$

$$\frac{dx_{10}^{(1)}(t)}{dt}=-8.4188\times10^{-4}x_{10}^{(1)}(t)^2-6.9007x_1^{(1)}(t)+ +4.6439x_2^{(1)}(t)-3.2766x_3^{(1)}(t)-4.9434x_4^{(1)}(t)+ +2.9170x_5^{(1)}(t)-9.5013x_6^{(1)}(t)+0.3982x_7^{(1)}(t)- +5.5179x_8^{(1)}(t)+2.1470x_9^{(1)}(t)-1.0092x_{10}^{(1)}(t)- -0.5564x_{11}^{(1)}(t)+8.8208x_{12}^{(1)}(t)+1.8407x_{13}^{(1)}(t)+ +4.9111+\xi_{10}(t)$$

$$\frac{dx_{11}^{(1)}(t)}{dt}=-6.4666\times10^{-4}x_{11}^{(1)}(t)^2+1.4894x_1^{(1)}(t)+ +0.2846x_2^{(1)}(t)+0.9160x_3^{(1)}(t)-0.6118x_4^{(1)}(t)- -0.1504x_5^{(1)}(t)+0.4115x_6^{(1)}(t)+0.3983x_7^{(1)}(t)- -0.7104x_8^{(1)}(t)-2.6649x_9^{(1)}(t)-0.2317x_{10}^{(1)}(t)+ +0.4468x_{11}^{(1)}(t)+0.2872x_{12}^{(1)}(t)+0.6263x_{13}^{(1)}(t)+ +10.4104+\xi_{11}(t)$$

$$\frac{dx_{12}^{(1)}(t)}{dt}=-4.1914\times10^{-5}x_{12}^{(1)}(t)^2+1.2726x_1^{(1)}(t)- -0.5257x_2^{(1)}(t)-1485x_3^{(1)}(t)+0.08948x_4^{(1)}(t)- -0.2733x_5^{(1)}(t)+0.4040x_6^{(1)}(t)+0.09232x_7^{(1)}(t)- -0.3059x_8^{(1)}(t)-1.0707x_9^{(1)}(t)+0.00923x_{10}^{(1)}(t)+ +0.1800x_{11}^{(1)}(t)-0.2447x_{12}^{(1)}(t)+0.01867x_{13}^{(1)}(t)+ +13.7212+\xi_{12}(t)$$

$$\frac{dx_{13}^{(1)}(t)}{dt}=2.00295\times10^{-3}x_{13}^{(1)}(t)^2-7.6656x_1^{(1)}(t)+ +2.0143x_2^{(1)}(t)-6.1427x_3^{(1)}(t)-2.8463x_4^{(1)}(t)+ +1.6289x_5^{(1)}(t)-6.6534x_6^{(1)}(t)+0.6603x_7^{(1)}(t)+ +3.2693x_8^{(1)}(t)+6.3337x_9^{(1)}(t)-1.3054x_{10}^{(1)}(t)- -0.2240x_{11}^{(1)}(t)+6.2739x_{12}^{(1)}(t)+0.7905x_{13}^{(1)}(t)+ +3.8372+\xi_{13}(t) \qquad (14)$$

(6) Define $\xi_i(t)$. When $\xi_i(t)=0$, model (5) simply reflects the average effects of non-linear actions between sub-systems, but does not indicate the random fluctuations of the system in the evolvement process. However, fluctuations are very important for the system's self-organized evolvement.

With fourth-order Runge-Kutta method, the numerical solution of non-linear differential equation (14) is obtained as $\bar{x}_i(t)$,

Let us assume

$$\xi_i(t)=x_i^{(0)}(t)-\bar{x}_i(t) \qquad (15)$$

With the error sequence, construct the periodical model for fluctuation $\xi_i(t)$. As all periodical functions can be Fourier extended, suppose

$$\xi_i(t)=a_{i0}+\sum_{k=1}^{n}(a_{1k}\cos\omega_i kt+b_{ik}\sin\omega_i kt) \qquad (16)$$

Cftool of MATLAB(R2010a) provides eight Fourier approximation functions, that is, $n = 1, 2, \ldots, 8$ in equation (16). When Cftool of MATLAB(R2010a)is applied for approximation, with the increase of n, the approximation becomes more precise. However, more data have to be learnt. When n-scale Fourier approximation function is applied, the data needed is $2n+2$. Data provided in *Table 1* is 18, which means the Model allows for highest approximation ($n = 8$). However when $n = 8$, the fluctuation $f_6(t)$, $f_{11}(t)$ of status variable X6 and X11 diffuses. So $n = 7$ is selected for a close approximation, and equation (16) changes into:

$$f_i(t) = a_{i0} + \sum_{k=1}^{7}(a_{1k}\cos\omega_i kt + b_{ik}\sin\omega_i kt) \qquad (17)$$

Input the data, and find the solution with Cftool of MATLAB(R2010a):

$$\begin{aligned}\xi_1(t) = [&-1.264\times10^{-2} - 0.3438\cos(\omega_1 t) + \\ &+0.004782\sin(\omega_1 t) - 0.1037\cos(2\omega_1 t) + \\ &+0.7454\sin(2\omega_1 t) - 1.729\cos(3\omega_1 t) + \\ &+0.4468\sin(3\omega_1 t) + 0.4457\cos(4\omega_1 t) + \\ &+1.954\sin(4\omega_1 t) + 1.129\cos(5\omega_1 t) - \\ &-2.05\sin(5\omega_1 t) - 0.3699\cos(6\omega_1 t) - \\ &-0.04521\sin(6\omega_1 t) + 0.5353\cos(7\omega_1 t) - \\ &-0.8808\sin(7\omega_1 t)]\times10^{-3} \quad (\omega_1 = 0.7383)\end{aligned}$$

$$\begin{aligned}\xi_2(t) = [&6.196\times10^{-3} + 0.126\cos(\omega_2 t) + \\ &+0.1536\sin(\omega_2 t) + 0.007855\cos(2\omega_2 t) + \\ &+0.00317\sin(2\omega_2 t) - 0.3026\cos(3\omega_2 t) - \\ &-0.1885\sin(3\omega_2 t) + 0.8323\cos(4\omega_2 t) + \\ &+1.136\sin(4\omega_2 t) + 1.1\cos(5\omega_2 t) - \\ &-0.7887\sin(5\omega_2 t) - 0.548\cos(6\omega_2 t) - \\ &-1.103\sin(6\omega_2 t) - 0.001549\cos(7\omega_2 t) + \\ &+0.8085\sin(7\omega_2 t)]\times10^{-3} \quad (\omega_2 = 0.7293)\end{aligned}$$

$$\begin{aligned}\xi_3(t) = [&-8.451\times10^{-2} + 0.1689\cos(\omega_3 t) + \\ &+0.01483\sin(\omega_3 t) - 0.517\cos(2\omega_3 t) + \\ &+0.8501\sin(2\omega_3 t) - 0.1206\cos(3\omega_3 t) + \\ &+0.2074\sin(3\omega_3 t) - 2.641\cos(4\omega_3 t) + \\ &+2.764\sin(4\omega_3 t) - 0.4522\cos(5\omega_3 t) - \\ &-2.223\sin(5\omega_3 t) + 0.2548\cos(6\omega_3 t) - \\ &-0.8233\sin(6\omega_3 t) + 1.894\cos(7\omega_3 t) \\ &+2.062\sin(7\omega_3 t)]\times10^{-3} \quad (\omega_3 = 0.586)\end{aligned}$$

$$\begin{aligned}\xi_4(t) = [&-0.1718 - 0.3738\cos(\omega_4 t) + \\ &+0.5402\sin(\omega_4 t) + 1.267\cos(2\omega_4 t) + \\ &+1.468\sin(2\omega_4 t) + 2.138\cos(3\omega_4 t) + \\ &+2.59\sin(3\omega_4 t) + 1.697\cos(4\omega_4 t) - \\ &-3.089\sin(4\omega_4 t) - 6.719\cos(5\omega_4 t) + \\ &+9.464\sin(5\omega_4 t) + 2.636\cos(6\omega_4 t) + \\ &+3.635\sin(6\omega_4 t) - 0.4025\cos(7\omega_4 t) + \\ &+1.469\sin(7\omega_4 t)]\times10^{-3} \quad (\omega_4 = 0.6438)\end{aligned}$$

$$\begin{aligned}\xi_5(t) = [&-0.2533 + 0.5813\cos(\omega_5 t) + \\ &+1.431\sin(\omega_5 t) - 0.2086\cos(2\omega_5 t) - \\ &-3.091\sin(2\omega_5 t) - 9.238\cos(3\omega_5 t) + \\ &+1.114\sin(3\omega_5 t) + 7.691\cos(4\omega_5 t) + \\ &+5.175\sin(4\omega_5 t) - 0.5527\cos(5\omega_5 t) + \\ &+4.24\sin(5\omega_5 t) + 7.007\cos(6\omega_5 t) + \\ &+3.773\sin(6\omega_5 t) - 0.9221\cos(7\omega_5 t) - \\ &-2.504\sin(7\omega_5 t)]\times10^{-3} \quad (\omega_5 = 0.7405)\end{aligned}$$

$$\begin{aligned}\xi_6(t) = [&-4.409\times10^{-2} + 0.1253\cos(\omega_6 t) + \\ &+0.2705\sin(\omega_6 t) + 1.488\cos(2\omega_6 t) - \\ &-0.9713\sin(2\omega_6 t) - 2.30\cos(3\omega_6 t) - \\ &-1.417\sin(3\omega_6 t) - 6.215\cos(4\omega_6 t) + \\ &+3.22\sin(4\omega_6 t) + 4.605\cos(5\omega_6 t) + \\ &+5.213\sin(5\omega_6 t) + 2.495\cos(6\omega_6 t) - \\ &-2.171\sin(6\omega_6 t) + 1.494\cos(7\omega_6 t) - \\ &-1.88\sin(7\omega_6 t)]\times10^{-3} \quad (\omega_6 = 0.5460)\end{aligned}$$

$$\begin{aligned}\xi_7(t) = [&-5.754\times10^{-2} - 0.7616\cos(\omega_7 t) + \\ &+0.4331\sin(\omega_7 t) + 0.5457\cos(2\omega_7 t) + \\ &+1.413\sin(2\omega_7 t) - 12.03\cos(3\omega_7 t) + \\ &+5.852\sin(3\omega_7 t) + 2.665\cos(4\omega_7 t) + \\ &+2.763\sin(4\omega_7 t) + 3.617\cos(5\omega_7 t) - \\ &-2.59\sin(5\omega_7 t) + 4.959\cos(6\omega_7 t) - \\ &-1.823\sin(6\omega_7 t) + 1.399\cos(7\omega_7 t) - \\ &-1.929\sin(7\omega_7 t)]\times10^{-3} \quad (\omega_7 = 0.7415)\end{aligned}$$

$$\begin{aligned}\xi_8(t) = [&-8.105\times10^{-3} + 0.08607\cos(\omega_8 t) - \\ &-0.4468\sin(\omega_8 t) + 0.6612\cos(2\omega_8 t) - \\ &-0.767\sin(2\omega_8 t) - 2.911\cos(3\omega_8 t) + \\ &+2.844\sin(3\omega_8 t) - 2.242\cos(4\omega_8 t) + \\ &+1.364\sin(4\omega_8 t) - 2.776\cos(5\omega_8 t) + \\ &+1.71\sin(5\omega_8 t) - 0.1972\cos(6\omega_8 t) - \\ &-0.3539\sin(6\omega_8 t) + 1.468\cos(7\omega_8 t) + \\ &+0.299\sin(7\omega_8 t)]\times10^{-3} \quad (\omega_8 = 0.7393)\end{aligned}$$

$$\begin{aligned}\xi_9(t) = [&-5.033\times10^{-2} + 0.5548\cos(\omega_9 t) + \\ &+0.503\sin(\omega_9 t) - 1.679\cos(2\omega_9 t) - \\ &-0.07\sin(2\omega_9 t) - 1.774\cos(3\omega_9 t) + \\ &+4.79\sin(3\omega_9 t) + 0.2397\cos(4\omega_9 t) - \\ &-4.463\sin(4\omega_9 t) + 5.136\cos(5\omega_9 t) - \\ &-1.314\sin(5\omega_9 t) - 1.172\cos(6\omega_9 t) + \\ &+0.093\sin(6\omega_9 t) + 1.003\cos(7\omega_9 t) + \\ &+1.607\sin(7\omega_9 t)]\times10^{-3} \quad (\omega_9 = 0.7403)\end{aligned}$$

$$\begin{aligned}\xi_{10}(t) = [&0.371 + 1.447\cos(\omega_{10}t) + \\ &+ 1.795\sin(\omega_{10}t) + 2.923\cos(2\omega_{10}t) + \\ &+ 4.494\sin(2\omega_{10}t) - 2.108\cos(3\omega_{10}t) + \\ &+ 7.887\sin(3\omega_{10}t) - 7.609\cos(4\omega_{10}t) + \\ &+ 11.45\sin(4\omega_{10}t) + 4.488\cos(5\omega_{10}t) - \\ &- 4.932\sin(5\omega_{10}t) - 1.927\cos(6\omega_{10}t) + \\ &+ 10.40\sin(6\omega_{10}t) - 2.252\cos(7\omega_{10}t) + \\ &+ 3.754\sin(7\omega_{10}t)] \times 10^{-3} \quad (\omega_{10} = 0.7388)\end{aligned}$$

$$\begin{aligned}\xi_{11}(t) = [&-0.1442 + 0.1979\cos(\omega_{11}t) - \\ &- 0.5872\sin(\omega_{11}t) + 2.522\cos(2\omega_{11}t) - \\ &- 1.889\sin(2\omega_{11}t) + 6.17\cos(3\omega_{11}t) - \\ &- 0.4633\sin(3\omega_{11}t) - 0.8937\cos(4\omega_{11}t) - \\ &- 0.8937\sin(4\omega_{11}t) + 3.041\cos(5\omega_{11}t) + \\ &+ 5.027\sin(5\omega_{11}t) - 1.863\cos(6\omega_{11}t) + \\ &+ 6.695\sin(6\omega_{11}t) - 7.101\cos(7\omega_{11}t) + \\ &+ 1.731\sin(7\omega_{11}t)] \times 10^{-3} \quad (\omega_{11} = 0.7577)\end{aligned}$$

$$\begin{aligned}\xi_{12}(t) = [&1.836 \times 10^{-2} - 0.2839\cos(\omega_{12}t) + \\ &+ 0.3838\sin(\omega_{12}t) + 0.2687\cos(2\omega_{12}t) - \\ &- 0.4741\sin(2\omega_{12}t) + 1.131\cos(3\omega_{12}t) + \\ &+ 0.7914\sin(3\omega_{12}t) - 3.077\cos(4\omega_{12}t) + \\ &+ 3.677\sin(4\omega_{12}t) + 2.922\cos(5\omega_{12}t) - \\ &- 3.09\sin(5\omega_{12}t) - 1.802\cos(6\omega_{12}t) + \\ &+ 3.871\sin(6\omega_{12}t) - 2.097\cos(7\omega_{12}t) - \\ &- 1.291\sin(7\omega_{12}t)] \times 10^{-3} \quad (\omega_{12} = 0.7346)\end{aligned}$$

$$\begin{aligned}\xi_{13}(t) = [&-0.2226 - 0.2645\cos(\omega_{13}t) + \\ &+ 0.9499\sin(\omega_{13}t) - 0.8104\cos(2\omega_{13}t) + \\ &+ 0.1071\sin(2\omega_{13}t) - 0.8104\cos(3\omega_{13}t) + \\ &+ 14.12\sin(3\omega_{13}t) - 0.0005511\cos(4\omega_{13}t) - \\ &- 5.532\sin(4\omega_{13}t) + 3.895\cos(5\omega_{13}t) + \\ &+ 6.799\sin(5\omega_{13}t) - 11.63\cos(6\omega_{13}t) - \\ &- 8.933\sin(6\omega_{13}t) - 0.5441\cos(7\omega_{13}t) + \\ &+ 0.9401\sin(7\omega_{13}t)] \times 10^{-3} \quad (\omega_{13} = 0.7413) \quad (18)\end{aligned}$$

Input $\xi_i(t)$, $(i = 1, 2, \ldots, 13)$ from equation (18) into equation (14). Model (14) reflects the synergetic effects of the railway freight system and order-parameter model of fluctuations. The mean value of $\omega_i(t)$, $(i = 1, 2, \ldots, 13)$ in equation (18) is 0.7060, the fluctuation period is $2\pi/0.7060 \approx 9$, which means railways in China take on the average 9 years into a new development cycle.

(7) Order-parameter of system evolvement. Find solution of equation (14), to obtain the relaxation coefficients of status variable as follows:

$$\begin{aligned}&\gamma_1 = 2.4711,\ \gamma_2 = 0.1301,\ \gamma_3 = 1.4029,\\ &\gamma_4 = 0.8176,\ \gamma_5 = 1.8930,\ \gamma_6 = 0.8825,\\ &\gamma_7 = 1.3548,\ \gamma_8 = 0.1038,\ \gamma_9 = -0.4832,\\ &\gamma_{10} = -1.0092,\ \gamma_{11} = 0.4468,\\ &\gamma_{12} = -0.2447,\ \gamma_{13} = 0.7920\end{aligned}$$

By sequencing of the relaxation coefficients, the following is concluded:

$$\begin{aligned}&\gamma_{10} < \gamma_9 < \gamma_{12} < \gamma_8 < \gamma_2 < \gamma_{11} < \gamma_{13} <\\ &< \gamma_4 < \gamma_6 < \gamma_7 < \gamma_3 < \gamma_5 < \gamma_1\end{aligned}$$

The results indicate that the relaxation coefficient of railway fixed assets is the minimum value, being followed by the quantity of railway locomotives. The relaxation coefficient of railway freight volume is the maximum.

3.4 Result analysis for the model solution

According to Haken's synergetic servo theory, fast relaxation variable obeys the slow relaxation variable, which in turn determines the system evolvement. The railway fixed asset investments are the major order-parameter for railway freight system evolvement, dominating the evolvement and development of railway freight system.

Railway fixed asset investments are primarily used for the construction and upgrading of railway lines, purchases of locomotives and vehicles. The investments are directly transformed into railway fixed assets for the improvement of railway system transport capacity. Therefore, fixed railway asset investments are the key features that determine the improvement of railway network structure, expansion of the production capacity, enhancement of transport efficiency and solution of transport bottleneck. Meanwhile, the railway infrastructure construction will support the development of other related industries, expand the domestic demands and push forward the national economy. As a result, it will promote the circulation of materials. Railway fixed asset investments also help in restructuring of regional industries, speeding up the growth of local economy and the urbanization process. Railway fixed asset investments, as an order parameter, influence the collective synergetic actions of the status variables of railway freight system. Investments dominate the whole system evolvement process and determine the result of system evolvement. Therefore, the government is expected to develop proper plans for railway development in line with the economic conditions, geographic difference, industrial arrangement and resource distributions in China, and in view of the railway developments at home and abroad.

The second key factor influencing railway freight system is the quantity of railway locomotives. The locomotives power up the railway transport and stand for the technological performance of the railway transport. Heavy-haul and fast transport are the two major trends of railway freight service, and the locomotive standards determine if China can achieve the two goals of heavy-haul transport and fast delivery. Therefore, locomotive quality is the basic condition for China's railway freight industry to adapt to the development condition. Locomotive quality is closely related to railway fixed asset investments. China is expected to import more advanced locomotive technologies from abroad and at the same time improve its innovation capacity for the integration of advanced locomotive technologies and breakthroughs.

According to the result of the model solution, society's goods turnover, second industry, railway freight turnover, society's freight turnover, and total investments in transport industry, quantity of railway locomotives, GDP, first industry, service mileage, railway freight transport revenue, railway freight volume influences on the evolvement of railway freight system decrease. Society's goods turnover on the whole reflects the social transport demands and the outputs of varied transport means. Therefore, it also has a major influence on the system. Second industry provides the basic sources of goods to be delivered by railway freight service, and is the primary service target of railway freight. Railway freight turnover is the result of railway freight turnover multiplied by average freight haul distance. It indicates the changes in service mileage and railway freight volume. Society's freight turnover reflects the total social demands, and investments in transport industry on the other hand show the railway investments and transport market climate. The quantity of vehicles influences the railway transport capacity. Meanwhile, it is also affected by the railway fixed asset investments. GDP reflects the macro-economic environment. Grains, wood and cotton from the first industry are the key service target for railway freight business. Service mileage is directly influenced by the railway fixed asset investments, and it is also a symbol of the railway transport capacity. Railway freight transport revenues show the changes in railway freight volume and freight service price.

From this it can be seen that under the influence of railway fixed asset investments, the status variables of railway freight system exert influences, to different degrees, on the system evolvement and its direction through synergetic actions.

4. ANALYSIS OF RAILWAY FREIGHT SYSTEM EVOLVEMENT PROCESS

4.1 Analysis of railway fixed asset investment potential function

Railway fixed asset investments, as the order-parameter, influence the evolvement of railway freight system and its direction. Potential function refers to the function of behavioural variables. Generally, there are two ways to construct the variable potential function: one is to construct the description model for variables based on qualitative analysis, and the other one is to change into the potential function for variables based on the features of system variables. This paper applies the second method to construct the potential function of railway fixed asset investments to analyze the evolvement of railway freight system.

First, analyze the features of railway fixed asset investments. By fitting the historical data, the evolvement equation is constructed as follows:

$$v(x) = ax^3 + bx^2 + cx + d \tag{19}$$

Make initial change: $z = x + \frac{b}{3a}$, then:

$$v(z) = az^3 + \left(c - \frac{b^2}{3a}\right)z + \left(\frac{2b^3}{27a^2} - \frac{bc}{3a} + d\right) \tag{20}$$

For formula (20) calculate the integral function of variable X, and obtain its function form:

$$v_0(z) = \frac{a}{4}z^4 + \left(\frac{c}{2} - \frac{b^2}{6a}\right)z^2 + \left(\frac{2b^3}{27a^2} - \frac{bc}{3a} + d\right)z \tag{21}$$

Suppose $V(z) = \frac{4}{a}v_0(z)$, then

$$V(z) = z^4 + \left(\frac{2c}{a} - \frac{2b^2}{3a^2}\right)z^2 + \left(\frac{8b^2}{27a^2} - \frac{4bc}{3a} + \frac{4d}{a}\right)z \tag{22}$$

Suppose $u = \frac{2c}{a} - \frac{2b^2}{3a^2}$ $\quad v = \frac{8b^2}{27a^2} - \frac{4bc}{3a} + \frac{4d}{a}$

Then formula (22) is simplified to:

$$V(z) = z^4 + uz^2 + vz \tag{23}$$

According to the Catastrophic Theory in mathematics, formula (23) suggests that railway fixed asset investment's potential function is a standard form with cusp catastrophe. Using the cusp catastrophe theory to analyze the equation for evolvement of railway fixed asset investments, it is found that the critical point of equation (23) is the solution of Equation C.

$$M: V'(z) = 4z^3 + 2uz + v = 0 \tag{24}$$

From equation (24), the equilibrium curve M is obtained. The 3D evolvement of railway freight system is indicated in *Figure 1*.

Figure 1 shows the 3D evolvement of railway fixed asset investments. As railway investments are the order-parameter of railway freight system, the curve also indicates the evolvement of railway freight system. In

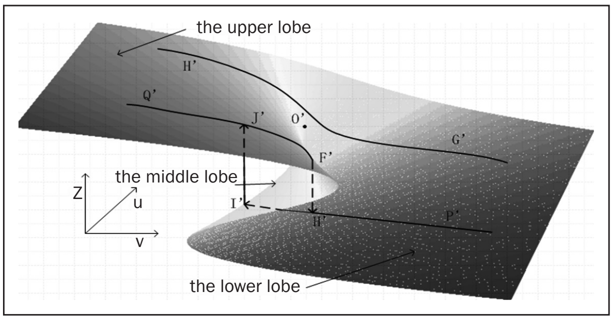

Figure 1 - 3D Evolvement of railway fixed asset investments

Figure 1 the curve exhibits smooth folds, which are divided into upper, middle and lower lobes that get narrower towards the back and finally disappear at point O'. O' is the point of origin in 3D coordinates, *u* and *v* represent the control coefficients beyond the system that have influence on the order-parameter of railway freight system. Intentional control of *u* and *v* guides the system into a more efficient and orderly direction. When $u > 0$ the system evolvement has a tendency of continuous smoothness like Curve H'G' in the Figure. The bigger *u*, the smoother the curve. When $u < 0$ the system evolvement exhibits an obvious catastrophe, and the railway investments are seen on the upper lobe, middle lobe and lower lobe along with the changes in v. However, as the middle lobe is an unstable status, final railway investments are observed on the balanced status (upper or lower lobe) after passing through the folded margin, and the tendency is indicated by Q'J'F'H'P'.

Mathematically, $u > 0$ changes in *v* will only result in smooth change in *z*, which makes *v* a regular parameter; $u < 0$ changes in *v* will only result in the discontinuous changes in *z* inside some M, which makes *u* a part parameter. In China, railway investments are primarily determined by two factors, government supportive policies for railway transport industry and development of national economy. Therefore, in the potential function of railway fixed asset investments, control factor *u* represents the development of national economy; when $u > 0$, the national economy is growing healthily and steadily; when $u < 0$, the national economy is in crisis or predicament; control coefficient *v* represents the government supportive policies for railway transport industry; when $v > 0$, the policy is relatively tight, when $v < 0$, the policy is relatively slack.

Project the evolvement process in *Figure 1* onto the control plane $C(u,v)$, as shown in *Figure 2*. Closed angle in *Figure 2* is the projection of the fold area in *Figure 1*. As equation (24) is a cubic expression, it can be learnt from algebra that it has one real root or three real roots. The number of real roots is determined by Cardano estimation formula $\Delta = 8u^3 + 27v^2$: when $\Delta < 0$, there are three different real roots; when

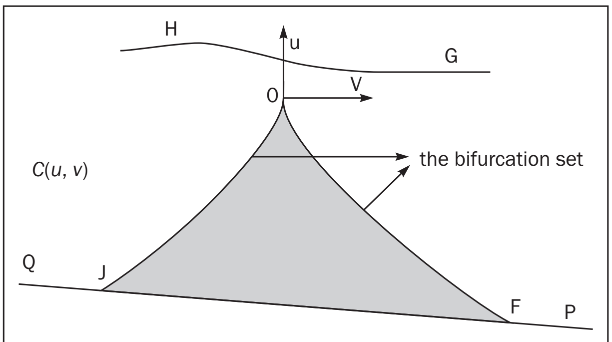

Figure 2 - Control plane of railway fixed asset investments

$\Delta = 0$, if neither *u* nor *v* is 0, there are three real roots, but if there is one double-root, and both *u* and *v* are 0, the three roots are all 0; when $\Delta > 0$, there is one real root and a couple of conjugate complex roots.

Singularity collection *S* is a sub-collection of *M* that is made up by all degenerated critical points of *V*, which means it has to meet the requirements of equation (24) and to meet the condition:

$$V''(z) = 12z^2 + 2u = 0 \qquad (25)$$

Project of *S* on the control plane $C(u,v)$ is termed as the bifurcation set. Remove *z* along with equation (24) and (25), and obtain:

$$8u^3 + 27v^2 = 0$$

It is the bifurcation set.

According to Cardano estimation formula, $\Delta < 0$ corresponds to the closed angle area *OJF* in the control plane $C(u,v)$ in *Figure 2*. Potential function (23) is indicated in *Figure 3*(a). The potential function has two minimum values, which means two balanced positions. It shows that when the national economy depresses, in order to expand the domestic demands, the government may adopt a slack policy to support the railway industry and expand the investments in railway infrastructure. Railway freight system is in the upper-lobe balancing status of heavy investments; the government may also adopt a tight or relatively steady policy in railway industry, and upgrade the investments in other infrastructure industries, then the system is in the lower-lobe balancing status of low investments. $\Delta = 0$ corresponds to the curves *OF* and *OJ* in the control plane $C(u,v)$ in *Figure 2*, the bifurcation set. It also corresponds to fold margins of the curves in *Figure 1*, where catastrophe happens during the system's evolving process. Potential function (23) is indicated in *Figure 3* (b) and (c). The potential function has two minimal values but only one minimum value. *Figure 3*(b) means that when the national economy is in sag, the government may expand the investments in railway infrastructure, so that the railway freight system completes a catastrophe from the low-investment balancing status to the high-investment balancing status. *Figure 3*(c) means that when the national economy is in sag, the government may reduce the investments in railway infrastructure, so that the railway freight system completes a catastrophe from the high-investment balancing status to the low- investment balancing status. $\Delta > 0$ corresponds to areas other than the closed angle area *OJF* in the control plane $C(u,v)$ in *Figure 2*. Potential function has only one smallest point, which means when the national economy is growing steadily and healthily, the government will keep its investments in the railway projects, and thus bring the system in a steady and balancing status, as indicated in *Figure 3*(d).

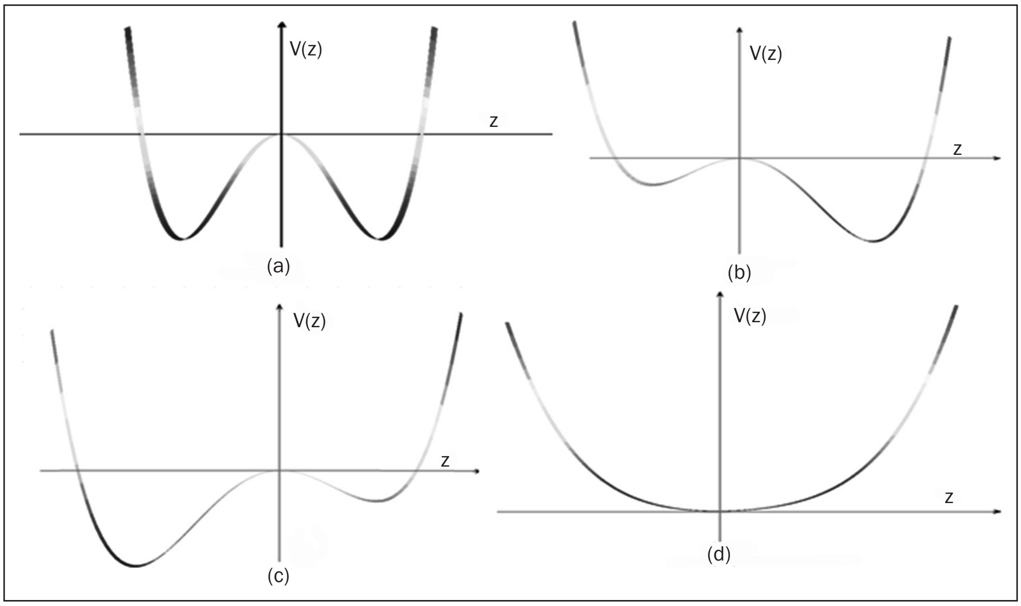

Figure 3 - Curve of potential function of railway fixed asset investments

4.2 Empirical analysis of railway freight system

The above analysis shows changes in control coefficients u, v in the control space $C(u,v)$ which may result in the gradual change or the catastrophe of railway investments. Then, railway investments, as the order-parameter, will result in the gradual change or the catastrophe of the overall railway freight system. When $u > 0$, the national economy grows steadily and healthily, the social demands for transport are increasing, and the government's policy for railway industry turns from relatively tight to slack. Gradual increase in railway investments will gradually push the railway system from the low-investment balancing status to the high-investment balancing status. When $u < 0$, the national economy is in crisis or in predicament, the government may adopt three strategies: first, to expand the domestic demands and step up infrastructure construction, the government's policy for railway industry turns from relatively tight to relatively slack, so as to increase investment in railway projects. The railway freight system completes a catastrophe from the low-investment balancing status to the high-investment balancing status. Second, the government may adopt a relatively tight policy for railway industry, invest less or even none in railway industry. The railway freight system completes a catastrophe from the high-investment balancing status to the low-investment balancing status. Third, the government may keep the original investment policy, and the railway freight system will keep the high-investment balancing status or the low-investment balancing status.

In view of the history of China's railway industry, the investments in railway industry are rising along with the development of the national economy. However under government administrations, the investments are kept at a relatively low balancing status. In 2008, the national economy of China slowed down due to the world economic crisis. To expand domestic demands, and to step up infrastructure constructions, and as a result of the steady constructions of high-speed railway projects as per middle- and long-term plan for railway industry, the investments in railways increased significantly. Railway freight system is observed to have a catastrophe from the low-investment balancing status to the high-investment balancing status. The system's transport capacity is improved for higher efficiency and higher regularity. With the economic crisis being solved, national economy picked up speed, and thus the government has adopted a tight investment policy, which put the railway freight system back onto the track of gradual continuity. Therefore, the railway freight system evolvement is a unification of gradual changes and catastrophes.

5. CONCLUSION

(1) Railway fixed asset investments, as the order-parameter of railway freight system evolvement, dominate and control the evolvement and development of railway freight system. Status variables are ordered in terms of their influences on railway freight system evolvement: quantity of railway locomotives, society's goods turnover, second industry, railway freight turnover, railway freight turnover, total investments in transport industry, quantity of vehicles, GDP, first industry, service mileage, total investments in transport industry, railway freight volume.

(2) In the random fluctuations, the mean value of parameter ω is 0.7060, which means the fluctuation

period is 9. It indicates that railways in China take on the average 9 years into a new development cycle.

(3) When $u > 0$, national economy is growing steadily and healthily, social demands for transport are increasing and the government's policy for railway industry turns from relatively tight to slack, which will put the railway system onto a continuous and gradually changing course from low-investment balancing status to high-investment balancing status. When $u < 0$, national economy is in crisis or in predicament, the government may adopt three strategies: first, expand the domestic demands and step up infrastructure construction, the government's policy for railway industry turns from relatively tight to relatively slack, so as to increase investment in railway projects. The railway freight system completes a catastrophe from the low-investment balancing status to the high-investment balancing status. Second, the government may adopt a relatively tight policy for railway industry. The railway freight system completes a catastrophe from the high-investment balancing status to the low-investment balancing status. Third, the government may keep the original investment policy, and the railway freight system will keep the high-investment balancing status or the low-investment balancing status.

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摘要

基于协同论的铁路货运系统序参量模型及演化

阐释铁路货运系统协同性质，针对铁路货运系统的复杂性，选取铁路固定资产投资、GDP、铁路营业里程等13个参数作为铁路货运系统的状态变量，建立基于协同学理论的铁路货运系统序参量模型，采用最小二乘法和极值函数法进行求解。对所求出的序参量进行势函数分析，探讨铁路货运系统演化过程。研究结果表明：铁路固定资产投资是影响铁路货运系统演化发展的序参量；随机涨落参数 ω 平均值为0.7060，即铁路货运系统的平均涨落周期为9年；铁路货运系统演化过程是渐变和突变的统一。

关键词

铁路货运；协同学；序参量；Logistic；最小二乘；函数极值

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