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A THRESHOLD POLICY FOR DISPATCHING VEHICLES IN DEMAND-RESPONSIVE TRANSIT SYSTEMS

ABSTRACT

This paper considers vehicle dispatching for a flexible transit system providing doorstep services from a terminal. The problem is tackled with an easy-to-implement threshold policy, where an available vehicle is dispatched when the number of boarded passengers reaches or exceeds a certain threshold. A simulation-based approach is applied to find the threshold that minimizes the expected system-wide cost. Results show that the optimal threshold is a function of demand, which is commonly stochastic and time-varying. Consequently, the dispatching threshold should be adjusted for different times of the day. In addition, the simulation-based approach is used to simultaneously adjust dispatching threshold and fleet size. The proposed approach is the first work to analyse threshold dispatching policy. It could be used to help improve efficiency of flexible transit systems, and thereby make this sustainable travel mode more economical and appealing to users.

KEY WORDS

dispatching vehicles; flexible transit; threshold policy; fleet sizing; ride sharing;

1. INTRODUCTION

The passage of the Americans with Disabilities Act of 1990 has encouraged the implementation of flexible transit systems, which provide doorstep services to passengers who are unable to use the conventional transit with fixed routes and schedules. Such trends have prompted a lot of effort towards developing models for planning new, or optimizing operations of the existing flexible transit systems. At the planning level, different models were developed to determine the system capacity needed to meet the demand. Examples of such models include analytical (Stein [1], Daganzo [2], Diana et al. [3]), simulation (Fu [4], Shinoda et al. [5], Quadrifoglio et al. [6]), and statistical (Fu [7], Markovic et al. [8,9]). At the operational level, a lot of work has focused on the development and implementation of vehicle routing and scheduling algorithms. Examples of such algorithms include various heuristics, metaheuristics and exact methods for routing prescheduled requests (Jaw [10], Toth and Vigo [11], Cordeau [12]), as well as techniques for real-time scheduling of dynamic requests (Teodorovic and Radivojevic [13], Attanasio et al. [14], Coslovich et al. [15]). However, it appears that very little effort was made towards studying the optimal dispatching control of flexible one-to-many transit systems, which is a relevant problem for transportation companies providing doorstep services from hospitals, airports, metro and train stops.

The problem of dispatching vehicles in flexible transit systems is now explained. Consider a transportation company providing doorstep services from a terminal (e.g., a metro stop or an airport) to a neighbouring area. In particular, the company has a fleet of vehicles which collect passengers at the terminal and deliver them to their home locations. The passenger arrivals are stochastic, as well as their drop-off locations. Given this randomness inherent to real-world operations, we would expect to have routes of considerably different durations, and therefore non-uniform arrivals of vehicles at the terminal to pick up new passengers. In such a one-to-many demand responsive system with stochastic route durations, proposing a fixed schedule for vehicle departures would imply some obvious drawbacks. For example, a fixed schedule may lead to unnecessary passenger waiting times when a vehicle with "enough" boarded passengers is held at the terminal until the scheduled departure time. On the other hand, a fixed schedule may result in dispatching underused vehicles if passenger arrival intensity reduces unexpectedly at some point during the operations. Thus, instead of a fixed schedule, we consider a vehicle dispatching approach which could cope with the aforementioned issues as they arise in the real-time operations.

Our objective is to control vehicle dispatching with a policy (i.e., a set of rules) that could be easily implemented in the field. That is, a policy that does not assume availability of real-time information that may be difficult or expensive to acquire (e.g., the time when the next vehicle will become available, or the time when the next passenger will arrive at the terminal). Thus, we consider a threshold policy, which is stated as follows.

Threshold Policy: If there are Q or more passengers waiting at the terminal when the next vehicle becomes available, then this vehicle is dispatched with as many passengers as can fit within its capacity c. Otherwise, the vehicle departure is delayed until the Q-th passenger arrives to the terminal and boards the vehicle.

Note that we would not know the optimal threshold Q^* beforehand, but would intuitively expect it to be a function of demand, fleet size, and various types of costs. Here, we propose a simulation-based approach to find Q^* which minimizes the total expected cost, including the fleet operating and passenger time cost. Our numerical experiments show that Q^* is indeed a function of demand, which is typically stochastic and time-varying. As a result, the threshold should be adjusted for different times of the day (e.g., peak vs. off-peak). Since the considered dispatching policy is described with a single parameter, threshold Q can be easily adjusted with other features of the system (e.g., fleet), to achieve additional savings.

The remainder of this paper is organized into five sections. We proceed by reviewing related literature on demand-responsive transit systems. In the following two sections we mathematically formulate the problem and present a simulation-based solution method. After discussing numerical experiments, we draw conclusions and discuss possible extensions of this work.

2. LITERATURE REVIEW

Flexible bus services, also known as demand responsive, paratransit or dial-a-ride services, have been widely studied since the 1970s. Some of the initial work involved continuous approximation of a bus tour length, which was later incorporated into analytical models used to optimize paratransit systems (e.g., fleet size, vehicle capacity, service headways). Specifically, Stein [16] explored the tour length for flexible bus services, while assuming that service zones are fairly compact and convex. Daganzo [17] extended this analysis to zones of different shapes, and showed that a tour length could be approximated as a function of (a) area of the service zone, (b) number of passengers within the zone, and (c) a constant that depends on the type of distances between stops (e.g., 1.15 for rectilinear spaces). This continuous tour length approximation was later employed to analytically optimize the flexible bus services, including fleet size, vehicle capacity, and service headways (Chang and Schonfeld [18]).

Conventional transit (with fixed routes/schedules) and flexible demand-responsive services have their advantages and disadvantages. In general, conventional transit is more economical in areas with high demand densities (e.g., downtowns), while flexible services represent a better alternative in regions with sparse demand (e.g., rural areas). Interestingly, the demand in certain suburban areas may vary considerably during the peak and off-peak hours, so researchers have also considered the possibility of alternating between the conventional and flexible transit modes (Kim and Schonfeld [19]). To this end, (Kim and Schonfeld [20]) explored integration of conventional and flexible bus services, and found that such a service integration is especially promising when demand is heterogeneous over time and space. This line of work was extended in Kim and Schonfeld [21] to consider transit integration with timed passenger transfers. It is worth noting that the above studies provide useful insights for planning transit services; however, their flexible service formulations are based on continuous approximation of tour lengths, which we try to overcome in this paper by taking a simulation-based approach.

The aforementioned simulation was widely applied to analyse various aspects of flexible transit systems. For example, simulation was employed to validate analytical models for (a) deriving the critical demand density for operating feeder transit services (Quadrifoglio and Li [22]), (b) determining the optimal design of feeder service zones (Li and Quadrifoglio [23, 24]), and (c) estimating the optimal cycle length of demand-responsive feeder transit services (Chandra and Quadrifoglio [25]). Comprehensive simulation studies were also conducted to (a) evaluate the effect of emerging technologies on operations of paratransit systems

(Fu [4]), (b) compare flexible with conventional transit (Shinoda [5]), (c) explore the impact of zoning strategies and time windows on system-wide performance measures (Quadrifoglio et al. [6]), and (d) study feasibility of a flexible transit system with electric vehicles (Jung et al. [26]) that would be particularly suitable for serving smaller communities (Wang and Gonzalez [27]). In systems where demands and capacities are coordinated by dynamic prices based on real-time data, there is need for the information systems and the operations modelled in order to create the service (Földes et al. [28]). Another real time service is a concept of Flexible Mobility on Demand, a demand-responsive system in which a list of travel options is provided in real-time to each passenger request (Atasov et al. [29]). Lastly, we refer the interested reader to Sin, C. Ho et al [30], which provides an excellent survey of research on the dial-a-ride problem since 2007.

In spite of rich literature concerned with simulation-based analysis of flexible transit systems, this appears to be the first paper to consider the problem of optimal vehicle dispatching for one-to-many flexible transit services. It tackles the problem with an easy-to-implement threshold policy, which does not require any real-time information. The policy is described with a single parameter Q, which makes it easy to adjust the dispatching threshold together with other features of the system. This is demonstrated by simultaneously optimizing Q and the fleet size. The proposed work could help improve efficiency of flexible transit systems (with ride sharing), and thereby make this sustainable travel mode more economical and appealing to users.

3. PROBLEM FORMULATION

We discretize the time of paratransit operations into intervals with relatively steady demand, which are indexed by $t \in T$(e.g., 8:00 - 10:00, 10:00 - 16:00, 16:00 - 18:00, 18:00 - 22:00). Let ξ_t be a vector denoting random arrival rates of passengers to the terminal, as well as their random drop-off locations. As argued before, we consider the policy π which is completely characterized with threshold Q_t for every period $t \in T$. We let x_t be the number of vehicles utilized in period $t \in T$ from a homogeneous fleet of m vehicles with capacity c. Let C_t^π be the cost of operating the flexible transit system in period $t \in T$, given the state S_t of the system at the beginning of period $t \in T$ (i.e., passengers waiting at the terminal, position of vehicles, passengers loaded (if any) and their destinations). Our problem,

$$V^\pi = \min_{Q_t, x_t \in \mathbb{Z}^+} \mathbb{E}\Big[\sum_{t \in T} C_t^\pi\left(Q_t, x_t, \xi_t \middle| S_t\right)\Big] \tag{1}$$

is to find Q_t^* and x_t^* for every $t \in T$, such that the total expected cost is minimized. The cost per time period is defined as the sum of fleet operating cost F_t^π, passenger waiting time cost W_t^π, and passenger in-vehicle time cost I_t^π,

$$C_t^\pi\left(Q_t,x_t,\xi_t \middle| S_t\right) = F_t^\pi\left(Q_t,x_t,\xi_t \middle| S_t\right) + W_t^\pi\left(Q_t,x_t,\xi_t \middle| S_t\right) + I_t^\pi\left(Q_t,x_t,\xi_t \middle| S_t\right) \tag{2}$$

Due to the complexity, we cannot compute F_t^π, W_t^π and I_t^π analytically; however, we can estimate them via simulation. If we let $\omega \in \Omega$ denote a realization of all ξ_t, then we can rewrite *1* as

$$\hat{V}^\pi = \min_{Q_t, x_t \in \mathbb{Z}^+} \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \sum_{t \in T} C_t^\pi\left(Q_t,x_t,\xi_t \middle| S_t\right) \tag{3}$$

where

$$C_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right) = F_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right) + W_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right) + I_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right) \tag{4}$$

represents the cost computed based on a single simulation run. In the following section we discuss the simulation-based approach used to solve problem *3*. Also, for convenience, all the notation is summarized in *Table 1*.

Table 1 – Notation summary

$t \in T$	Time index
ξ_t	Pass arrival rates
π	Threshold policy
Q_t	Dispatching threshold
m	Fleet size
x_t	Number of vehicles utilized
$\omega \in \Omega$	Realization of all ξ_t
c	Vehicle capacity
S_t	State of the system
C_t^π	Cost of operating the system
F_t^π	Fleet operating cost
W_t^π	Passenger waiting cost
I_t^π	Passenger in-vehicle time cost
V^π	Total expected cost

4. SOLUTION APPROACH

A simulation model is developed to evaluate $C_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right)$ defined in *4*. The model employs the heuristics for solving the travelling salesman problem (TSP), including the convex hull insertion algorithm for TSP construction and 2-opt exchange procedure for TSP improvement (Kay [31]). The solution to the TSP is needed to simulate efficient multi-stop routes, based on which we compute $F_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right)$, $W_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right)$ and $I_t^\pi\left(Q_t,x_t,\xi_t(\omega) \middle| S_t\right)$. The outline of the simulation model written in MATLAB is provided in Algorithm 1. At the beginning, we simulate passenger arrival times and their drop-off destinations. Looking at vehicle availability and the number of queued passengers, we proceed by dispatching vehicles according to the proposed threshold policy and the widely applied First-In-First-Out (FIFO) rule. It should be noted that the simulation clock is governed by the vehicle dispatching time and that the flexible transit system is simulated over a specified simulation horizon.

Algorithm 1: Outline of a simulation model used to estimate $C_t^{\pi}\left(Q_t, x_t, \xi_t(\omega) \middle| S_t\right)$

Initialize variables based on S_t;
Simulate new passengers, including their arrivals and drop-off locations;
While clock < simulation horizon *do*
- Get the earliest possible departure time given availability of x_t vehicles in the fleet;
- Get the number of queued passengers at the time of the earliest possible departure;
- If there are Q_t or more passengers, dispatch the vehicle right away. Otherwise, post-pone its departure until arrival of the Q_t-th passenger;
- Given destinations of the boarded passengers, solve the TSP and store the corresponding statistics (e.g., veh-mi, veh-h, passenger time);
- Update vehicle schedules (i.e., earliest availability), the list of unserved passengers, and clock;

end While
Return system-wide cost computed based on veh-mi, veh-h and passenger waiting/in-vehicle time;

Algorithm 1 allows us to estimate the cost given Q_t and x_t. However, we still need to determine the values of these variables that minimize the expected cost over all time periods, i.e., Q_t^* and x_t^* for all $t \in T$. In this regard, it should be noted that x_t is bounded by the available pool of m vehicles (i.e., $x_t \leq m$). Moreover, we make the following observation.

Remark 1: It is suboptimal to delay a vehicle when there are more than c passengers waiting at the terminal, because all types of costs are non-decreasing for $Q_t > c$. Thus, we can let $Q_t \leq c$ without altering the optimization problem 3.

The above remark is very intuitive and reduces the domain of our optimization problem to $c \cdot m \cdot |T|$ feasible solutions. In real-world paratransit operations, capacity c is typically 8 to 12 passengers. Moreover, $|T|$ would be about 5 because we would discretize the day of operations into periods with relatively equal passenger arrival rates. Since this translates into a rather small number of feasible solutions, we can find Q_t^* and x_t^* for a realistically-sized problem through enumeration. This is yet another advantage of a simple (and easy-to-implement) threshold dispatching policy which is described only by Q_t.

Finally, we make another intuitive observation which relies on the earlier remark. This observation will be useful in interpreting some numerical results shown in the following section.

Remark 2: The system-wide cost becomes insensitive to threshold Q_t when demand exceeds system capacity and queues of passengers form.

Proof: Based on the threshold policy, we dispatch a vehicle with load L=max(Q_t, min(y,c)) where y is the number of queued passengers and $Q_t \leq c$ based on Remark 1. If demand exceeds system capacity and queues start to form, then $y>c$ and we will be dispatching vehicles with a constant load L=c regardless of the specified value for Q_t.

5. NUMERICAL EXPERIMENTS

We begin by presenting the parameters used in the numerical examples, which include realistic cost estimates for Washington metropolitan area. Afterwards, we proceed by exploring (a) how Q^* varies with different demand levels for a fixed number of utilized vehicles, and (b) how (Q^*, x^*) varies with demand and passenger time cost. To facilitate exploration of trade-offs, we will consider a single time period (e.g., an off-peak period from 10:00 to 16:00) and examine the threshold policy that minimizes the expected hourly cost of operating the considered flexible transit system.

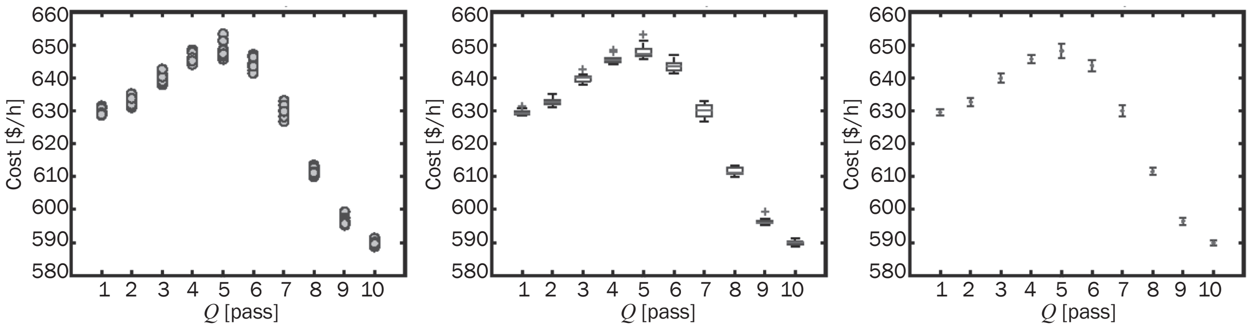
5.1 Parameters and assumed simulation settings

The assumed vehicle capacity is 10 passengers (i.e., c=10). The fleet operating cost includes $20/veh-h for the driver and vehicle depreciation, and $0.5/veh-mi for gasoline, oil, etc. (Markovic et al. [32]). The average passenger waiting and in-vehicle time costs are $12/pass-h (Kim and Schonfeld [21]), which allows us to compute the total passenger cost. In our case studies, we assume that the terminal is located in the middle of a service area of 15x15 mi (like in *Figure 1*) and that the average vehicle speed is 25 mi/h. Moreover, we assume availability of historical information about demand. In our experiments, we simulate demand while assuming uniform arrivals of passengers to the terminal, as well as uniform distribution of their drop-off locations across the service area. The passengers are assumed to be travelling from the terminal to locations scattered around the service region, and are boarding vehicles based on the FIFO rule. Lastly, it should be noted that the proposed simulation-based methodology could be extended to tackle other settings, including: different distribution of arrivals and drop-offs, different location of the terminal relative to demand, different service rule, and bi-directional travel of passengers.

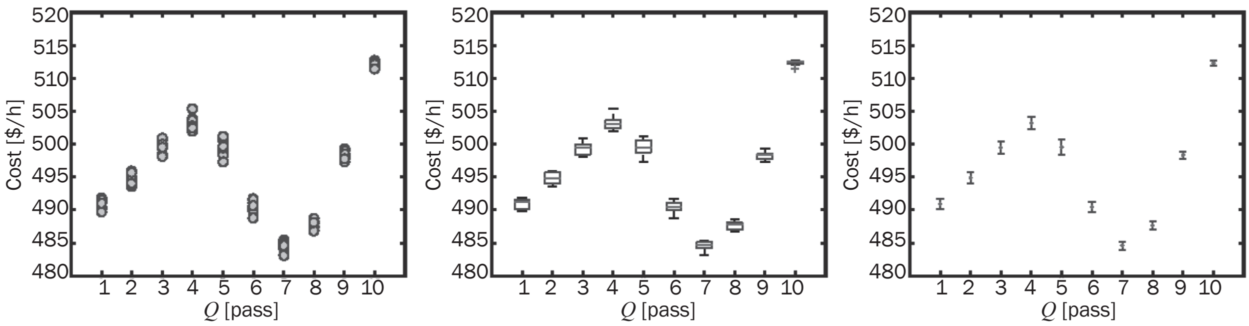
5.2 Exploring Q^* under different scenarios

After fixing the number of vehicles to six, we simulated the flexible transit system over 5,000 hours and compute the hourly cost of operating the system. This procedure is repeated 10 times using different seeds for the random number generator. In *Figure 1a*, we plot $C(Q,6,\xi(\omega)|S)$ for $Q\in$ [1,10], and observe little variance in hourly cost given different $\omega\in\Omega$(i.e., simulation run). In this particular case Q^*=10, which means that we are better off delaying the vehicle dispatch until all of its capacity is used up. However, if the demand changes (e.g., during the evening off-peak period from 18:00 till 22:00), the optimal dispatching threshold may change as well. To illustrate this, we reduce demand from 25 to 21 pass/h, and perform the same type of analysis. This time Q^*=7, which can be observed from *Figure 1b*. The relation between optimal dispatching threshold and demand is further explored in *Figure 1c,* which indicates that Q^* is non-decreasing with respect to demand. Specifically, Q^*=1 for relatively low demand (16-18 pass/h), because in such an underused system, delaying a ready vehicle would increase the passenger waiting cost. As the demand increases from 19 to 24 pass/h, Q^* takes values between six and nine passengers. Lastly, for a relatively high demand of more than 25 pass/h, Q^* reaches its maximum which corresponds to the capacity of 10 passengers per vehicle (i.e.,Q^*=c). In the base case (*Figure 1a*) Q^*=10 and it statistically dominates other solutions; however, when demand is reduced by 16%, then Q^*=7 (*Figure 1b*). Further relation between Q^* and demand is shown in *Figure 1c*.

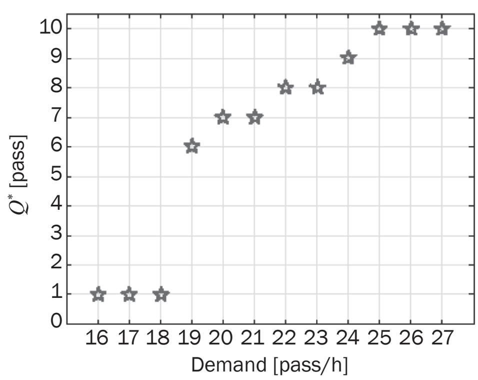
Recall from Remark 2 that system-wide cost becomes insensitive to threshold Q when demand exceeds the system capacity and queues of passengers



a) Costs obtained in 10 simulation runs given 25 pass/h demand



b) Costs obtained in 10 simulation runs given 21 pass/h demand



c) Q^* vs. demand

Figure 1

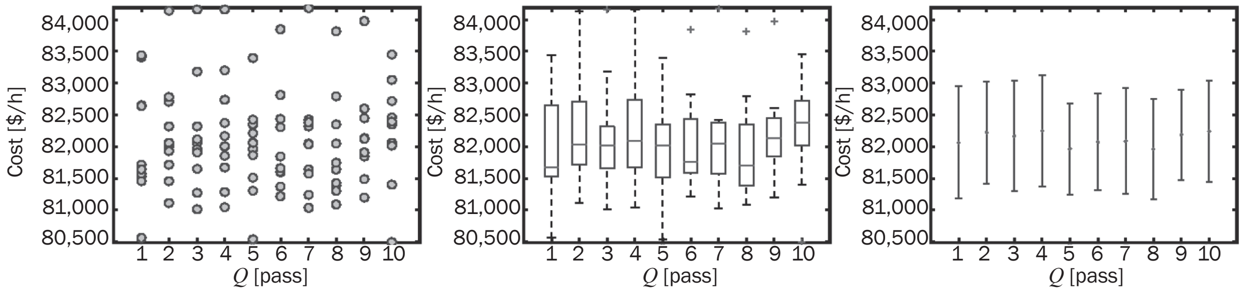
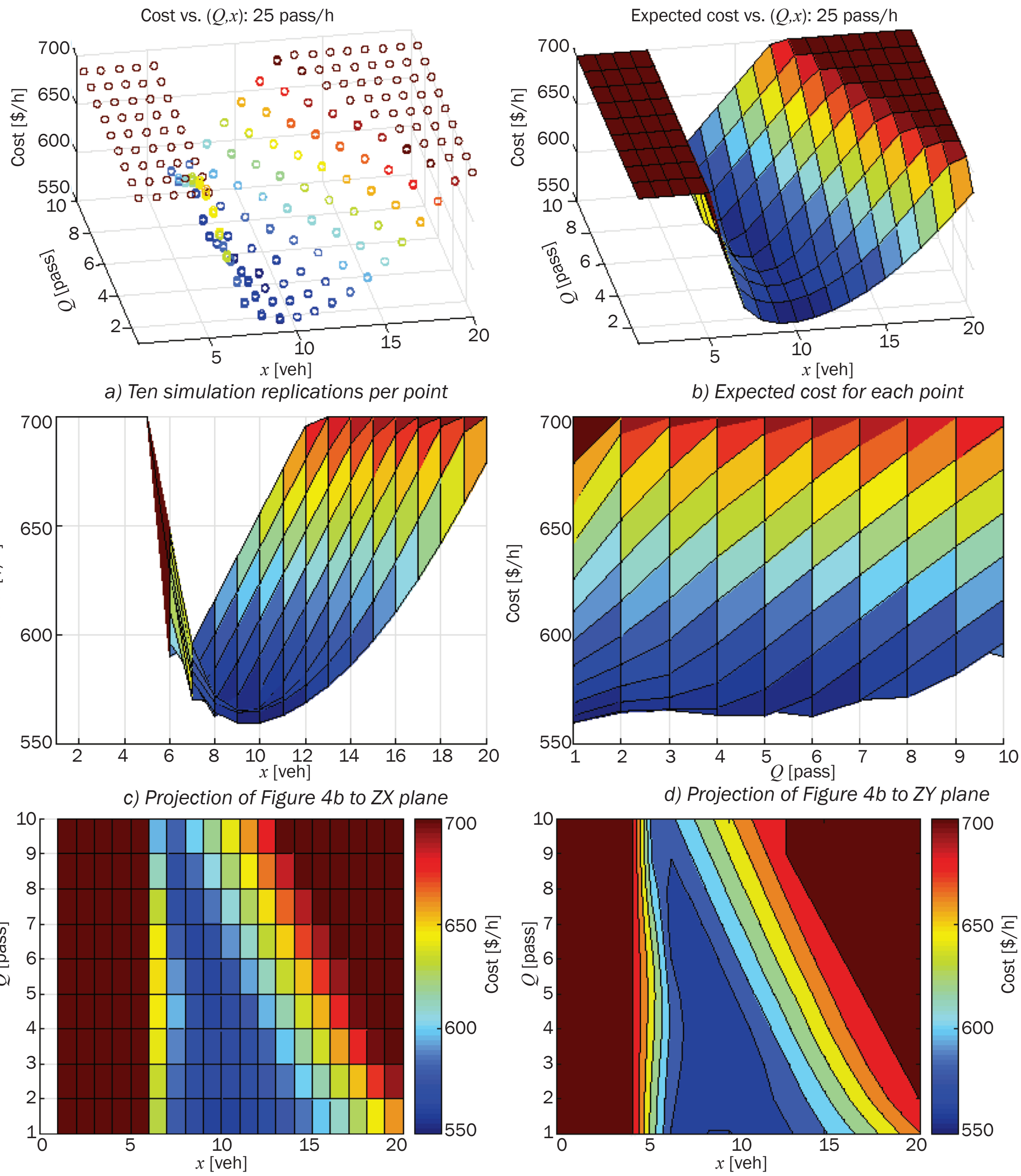


Figure 2 – Cost vs. Q given 30 pass/h demand



a) Ten simulation replications per point

b) Expected cost for each point

c) Projection of Figure 4b to ZX plane

d) Projection of Figure 4b to ZY plane

e) Projection of Figure 4b to YX plane

f) Contour plot for Figure 4e

Figure 3 – The expected hourly cost is minimized for x^*=10 vehicles and Q^*=1 passenger
Note: In these visuals, costs above 700 are treated as a constant in order to emphasize values closer to optimal

start to form. As argued before, the reason for this phenomenon is that load $L=c$ regardless of Q, which is very intuitive. Such a situation is illustrated in *Figure 2*, where the assumed demand is 30 pass/h and the number of vehicles is six. In this particular case, the average hourly cost remains statistically indifferent for Q=1,...,10 (i.e., the error bars overlap), and additional replications would not change the described pattern. Since the over-saturated operations are simulated over a 5,000-hour horizon during which queues of passengers continue to increase, the accumulated passenger waiting times produce excessive average hourly cost. When demand exceeds the system capacity and queues start to form, a system-wide cost becomes insensitive to dispatching threshold (Remark 2) and statistically there is no difference in costs for $Q \in [1,c]$.

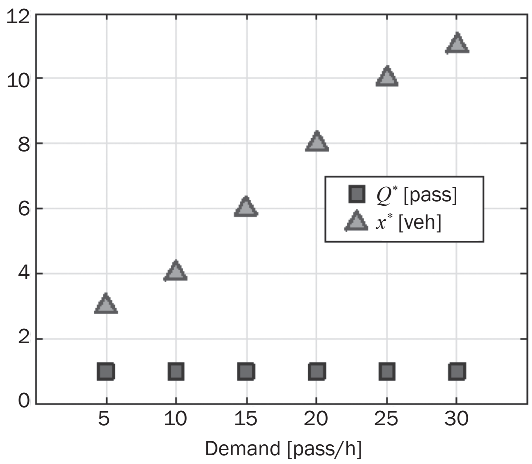
5.3 Exploring (Q^*, x^*) under different scenarios

Thus far, we optimized only the dispatching threshold and studied the relation between Q^* and the demand. Here, we proceed by simultaneously optimizing the dispatching threshold and fleet size. *Figure 3a* illustrates the costs for $Q \in$ [1,10] and $x \in$ [1,20], where very little variance is observed in different simulation replications because hourly cost is obtained by simulating the system for 5,000 hours and taking the average. Thus, hourly costs pertaining to different replications mostly overlap in *Figure 3a*. Furthermore, *Figures 3b-3d* illustrate the expected costs for $Q \in$ [1,10] and $x \in$ [1,20]. Moreover, *Figures 3e* and *3f* indicate that the expected cost is minimized for x^*=10 vehicles and Q^*=1 passenger. Such a solution clearly minimizes the passenger waiting and in-vehicle time, at the cost of operating more vehicles. In *Figure 3f*, we explore the aforementioned trade-off between the passenger and fleet cost. We observe, namely, that similar system-wide cost is maintained by increasing the fleet size x (which increases the fleet cost), while at the same time reducing the dispatching threshold Q (which reduces the passenger waiting and in-vehicle time cost).

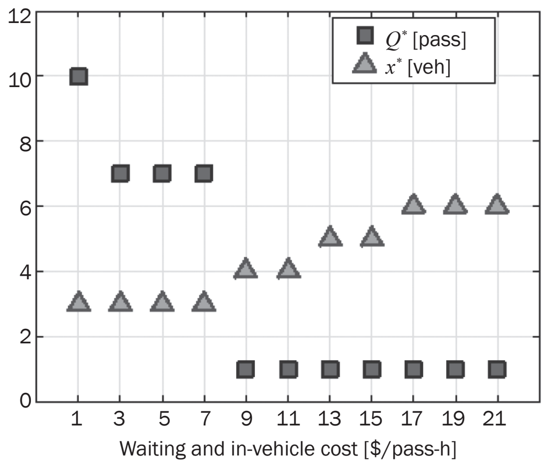
The effect of demand on (Q^*, x^*) is explored in *Figure 4*a. It shows that only x^* changes with demand, whereas Q^*=1 for different demand levels. Such a pattern is due to relatively high passenger time cost with respect to fleet operating cost. Intuitively, we would expect Q^* to increase for smaller passenger time cost (e.g., \$12/h assumed so far may be excessive for other counties or countries). This is explored in *Figure 4b*, which indeed shows the suggested trend. Specifically, when the passenger time cost is only \$1/h, then Q^*=10 and x^*=1, which is a solution that reduces the fleet operating cost to its minimum. As the passenger time cost is increased to \$7/h, then Q^*=7 and x^*=3, which balances between the two types of costs. For greater passenger time costs, the system-wide cost is minimized by letting Q^*=1 and by increasing the fleet as passenger time cost is increased. As argued earlier, such solutions seek to minimize relatively expensive passenger time, at the cost of operating more vehicles.

6. CONCLUSION

We address the optimal dispatching control of a flexible one-to-many transit system providing doorstep services from a terminal. The dispatching problem is tackled with an easy-to-implement threshold policy, which allows the operator to control the system by adjusting just a few parameters. A simulation-based approach is proposed to simultaneously optimize the dispatching threshold and fleet size, and extensive numerical experiments indicate that:



a) Q^* =1 pass given \$12/pass-h waiting and in-vehicle cost, and different demand



b) Q^* and x^* vary given 10 pass/h demand, and different passenger time cost

Figure 4 – Sensitivity analyses showing (Q^*, x^*) vs. demand and passenger time cost

- the optimal dispatching threshold is a function of time-varying demand. Thus, the dispatching threshold should be adjusted for times of the day with different demand (peak vs. off-peak) in order to minimize the total system cost.
- the optimal dispatching threshold is also a function of the fleet size. In some cases, the system operator may be better off reducing the fleet size during off-peak periods, while simultaneously adjusting the dispatching threshold.

The two findings are very intuitive, and the presented approach would allow an operator to determine (Q^*, x^*) given the local demand and costs. In a real-world application, historical data about the passenger arrivals and their destinations would be used to determine the optimal dispatching control of the system for different times of the day. In particular, a historical day of operations would represent a scenario (i.e., $\xi(\omega)$), and would be used within the proposed simulation-based approach to find system parameters that minimize the total cost averaged over all the historical days considered. The proposed method could also be applied at the planning level, primarily to determine the pool of vehicles that should be acquired to minimize the expected cost. This application would require a preliminary analysis of demand (i.e., arrival rates of passengers and spatial distribution of their destinations), which is required as an input to our model. At the planning level, we could also optimize vehicle capacity, by simply treating as a variable and by accounting for costs associated with different vehicle types.

Future work may include numerical comparison of the considered threshold policy with some other, potentially more complex policies defined with more than one parameter per time period (e.g., a policy that would also limit the amount of time that a passenger can spend waiting on board). It would be of particular interest to prove analytically that a policy (e.g., threshold or any other) is optimal (e.g., see Pepeyne and Cassandras [33]). Future extensions may also include bi-directional travel of passengers, which would require simulating passenger arrivals not only at the terminal but across the service region as well. In addition, one may consider bulk arrivals of passengers at the terminal or group requests for transportation to the terminal.

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