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# MICROSCOPIC MODEL OF AUTOMOBILE LANE-CHANGING VIRTUAL DESIRE TRAJECTORY BY SPLINE CURVES

#### ABSTRACT

With the development of microscopic traffic simulation models, they have increasingly become an important tool for transport system analysis and management, which assist the traffic engineer to investigate and evaluate the performance of transport network systems. Lane-changing model is a vital component in any traffic simulation model, which could improve road capacity and reduce vehicles delay so as to reduce the likelihood of congestion occurrence. Therefore, this paper addresses the virtual desire trajectory, a vital part to investigate the behaviour divided into four phases. Based on the boundary conditions,  $\beta$ -spline curves and the corresponding reverse algorithm are introduced firstly. Thus, the relation between the velocity and length of lane-changing is constructed, restricted by the curvature, steering velocity and driving behaviour. Then the virtual desire trajectory curves are presented by Matlab and the error analysis results prove that this proposed description model has higher precision in automobile lane-changing process reconstruction, compared with the surveyed result.

#### **KEY WORDS**

traffic simulation, lane-changing model, virtual desire trajectory,  $\beta$ -spline curves, driving behaviour

# **1. INTRODUCTION**

Lane-changing behaviour induces traffic conflicts and reduces the efficiency of the normal road traffic and in some serious conditions, even results in traffic accident. By far, China has also witnessed a substantial rise in the number of traffic crashes, injuries and fatalities, especially since 1998, related to unsafe lane-changing behaviour. A recent government report surveyed in four typical cities, including Beijing, Harbin, Dalian and Shenzhen, by the National Transportation Agency shows lane-changing behaviour involved crashes account for more than 30% of all crashes and more seriously in Northeast China. Therefore, it is important to investigate the characteristics of lane-changing behaviour in order to improve driving safety, which leads to increased scientific attention focused on lane-changing virtual desire trajectory as a vital component of the behaviour during the last decade.

Over the past decades, lane-changing virtual desire trajectory has been subject to extensive research. Chee (1994) proposed the conception of VDT (virtual desire trajectory), according to the highway vehicles operating system for lateral control. Gao et al. (2002) focused on the principle of solving vehicle gravity centre trajectory with the vehicle velocity integral method experimental test. Papadimitriou et al. (2003) presented a fast lane-changing algorithm for Intelligent Vehicle Highway Systems, using polynomial for trajectory planning. Dynamic constraints were also taken into account, which showed the main advantages of the algorithm especially in the case of lane-changing with multiple obstacles. Hidas (2005) introduced simulation of Intelligent Transport Systems (SITRAS), a massive multi-agent simulation system in which driver-vehicle objects were modelled as autonomous agents. Ma et al. (2006) concentrated on trajectory simulation by using VB.NET graphic image processing functions and Matlab curve fitting. Liu et al. (2007) used VTC software systems cultivating independent to convert the coordinates between the ground and the video, in which the lane-changing track could be extracted from video data. Ammoun et al. (2007) drew an area of the virtual desire trajectory by using the polynomial trajectory models through adding some constraints, such as speed, acceleration, vertical acceleration and maximum curvature, and so on.

Wang et al. (2008) concentrated on experimental simulation technology in offshore engineering. Despite various research efforts, however, many problems related to lane-changing safety have remained unsolved today. Most existing studies investigated acquisition and curve fitting. Chung et al. (2009) used a dynamic structural equation model (SEM) to describe the relation of activity participation and travel behaviour. Hang et al. (2010) measured the space-mean-speed (SMS) by the derived ITS data from inductive loop detectors (ILDs), and the analysis results showed that the proposed method can be used to estimate lane-change behaviour. Actually, lane changing has different characteristics due to numerous environmental factors as well as other geometric and traffic effects. Therefore, it is necessary to explore driving behaviours while travelling through construction areas and to use ITS measures.

Upon urgent requirements, the purpose of this paper is to make a deep insight into the construction of a virtual desire trajectory with continuous curvature by ITS ideas for traffic management. It is organized as follows: lane-changing behaviour is analyzed first and then  $\beta$ -spline curves are introduced to describe the virtual desire trajectory by close-range photogrammetry principle with linear transformation. The proposed model is verified by simulation experiment in the end.

## 2. LANE-CHANGING PROCESS DIVISION

In 1999 Winsum et al. published a research paper and stated that lane-changing manoeuvre could be divided into three phases: steering wheel reaches a maximum angle from nearly zero in the first phase, and then it is turned into the opposite direction; during the second phase, it comes to a stopping point when the steering wheel angle passes through zero and thus the vehicle heading approaches the maximum; in the third phase "the steering wheel turns to a second maximum steering wheel angle in opposite direction to stabilize the vehicle in the new lane".

The existing description of characteristics of lanechanging virtual desire trajectory shows the curve of trajectory is continuous without inflexion. Accordingly, this paper postulates that the rule of change of the curvature is linearity. In the initial situation, the vehicle velocity is V with zero as the acceleration. At the same

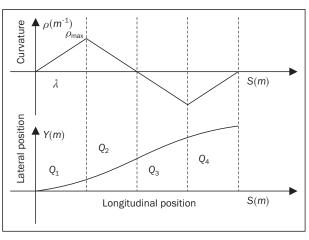


Figure 1 - Sequential phases of lane-changing

time, the vehicles are in the terminal of the straight line, so the curvature is also zero.

Therefore, the whole lane-changing process in this research can be seen as two completely opposite driving behaviours, consisting of four phases, as shown in *Figure 1*. The subject vehicle transfers from the initial point to maximum curvature point during the first phase. The second phase ends when the vehicle moves across the lane line, during which the vehicle is considered as a particle with head angle. For the third phase, the vehicle moves from the lane line (curvature is zero) to the curvature minimum point (negative absolute maximum or the opposite maximum at the opposite direction to the first phase). The last phase ends when all the lane-changing behaviours are finished (curvature is zero).

At the initial and terminal position, the subject vehicle has the same traffic characteristics, including velocity and acceleration, except for the dynamic planar position. Each phase of lane-changing can be considered as a segment spline curve. Thus the entire lane-changing virtual desire trajectory consists of four connected  $\beta$ -spline curves.

# 3. VDT MODEL VIA SPLINE CURVES

Previously, the trapezoidal acceleration trajectory was proposed as VDT considering passenger's ride comfort and transition time during lane-changing manoeuvres (Feng et al., 2007). Chee (1994) combined the sliding mode controller and the state estimator to study lane change manoeuvres at low speeds. But the algorithm was too complicated and it overlooked the curvature restrictions.

The  $\beta$ -spline curves have the advantage of dealing with each segment separately, which is only controlled by the four adjacent control vertices. Therefore, the change of polygon vertices will only affect the adjacent four curves, while other curves will never change.

#### 3.1 β-spline curves

A  $\beta$ -spline is a parametric piece-wise curve which presents continuity of the derivatives of a given order (first and second order, for instance) even at the joints between adjacent segments. This type of curves is specified by a set of points called control vertices. These vertices connected in sequence build up a control polygon. The curve tends to mimic the overall shape of the control polygon, presented by *Figure 2*.

Let the control polygon be composed of the sequence of control vertices  $[V_0, V_1, ..., V_m]$ , specified by each Cartesian coordinate, and the curve is composed of m - 2 segments. Thus, the *i*-th curve segment (denoted  $Q_i$ ) is determined by four control vertices  $V_{i+r}$ , r = -1, 0, 1; 2; and any point on this segment is a weighted average of these vertices. The coordinates of the point  $Q_i(u)$  on the *i*-th curve segment are defined as [10]

$$Q_i(u) = \sum_{r=-1}^{2} b_r(\beta_1, \beta_2, u) V_{i+r}$$
(1)

where u is a characterizing parameter within [0,1], and i = 1, 2, ..., m - 2.

subject to,

$$\begin{cases} b_{\cdot1}(u;\beta_1,\beta_2) = \frac{2\beta_1^3}{\delta}(1-u)^3 \\ b_0(u;\beta_1,\beta_2) = \frac{1}{\delta}[2\beta_1^3u(u^2-3u+3) + \\ + 2\beta_1^2(u^3-3u^2+2) + 2\beta_1(u^3-3u+2) + \\ + \beta_2(2u^3-3u^2+1)] \\ b_1(u;\beta_1,\beta_2) = \frac{1}{\delta}[2\beta_1^2u^2(-u+3) + \\ + 2\beta_1u(-u^2+3) + \beta_2u^2(-2u+3) + \\ + 2(-u^3+1)] \\ b_2(u;\beta_1,\beta_2) = \frac{2}{\delta}u^3 \end{cases}$$

and

$$\beta_1 > 0, \beta_2 \ge 0, \ \delta = 2\beta_1^3 + 4\beta_1^2 + 4\beta_1 + \beta_2 + 2.$$

In our model we have to choose  $\beta_1$  and  $\beta_2$  as the shape parameters to determine the way that the curve

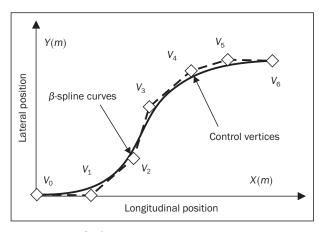


Figure 2 -  $\beta$ -spline curves and control vertices

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tends to mimic the shape of the control polygon. However, the shape parameters are enabled to smooth the curve and control its shape. In another way, they also induce subtle differences (Cuesta et al., 2008).  $\beta_1$  presents relative displacement and if  $\beta_1 = 1$  the curve has no offset. With the increase of  $\beta_1$ , the curve leads to slant and mimics the control polygon in an asymmetric way. If  $\beta_1$  decreases, we find that the curve propagates towards the opposite direction at the same offset. Parameter  $\beta_2$  represents the tension, and when  $\beta_2 = 0$ , the curve is called tension free or non-taut curve. With the increase of  $\beta_2$ , the curve becomes tense and uniform when it mimics the control polygon.

#### 3.2 Restrictions

The  $\beta$ -spline curves can amend curves without moving control vertices, by changing the value of the shape parameters. The nature presents of the  $\beta$ -spline curves would be more flexible and more extensive to be applied. It also has some of the following restrictions in generating virtual desire trajectory using  $\beta$ -spline curves.

1) *Curvature*: In actual condition, the curvature radius is continuous. Therefore, the change of the curvature along the trajectory curves should also be continuous. So it requires that  $\beta$ -spline curves function has continuous first and second order derivatives. Thus:

$$\begin{array}{l}
Q_{i+1}(0) = Q_{i}(1) \\
Q_{i+1}''(0) = Q_{i}''(1)
\end{array}$$
(2)

2) Driving Behaviour: From vehicle operation characteristics, we know that the curvature is zero at the trajectory terminal. Thus the curvature formula reaches as follows:

$$\left|\rho\right| = \left|\frac{d\varphi}{ds}\right| = \frac{\left|y''\right|}{\left(1+y^{2}\right)^{3/2}} = \frac{\left|x'y''-y'x''\right|}{\left(1+y^{2}\right)^{3/2}}$$
(3)

where

$$x_i(u) = \sum_{r=-1}^{2} b_r(\beta_1, \beta_2, u) \cdot x_{i+r}$$

is parametrics formula for x-component of the point from  $\beta$ -spline curve,  $x_{i+r}$  is constant coordinate from the control vertix  $V_{i+r}$  and  $x' = \frac{dx}{du}$ . Analogue for x'', y' and y''.

Assuming  $\rho = 0$  obtains the boundary conditions as:

$$\begin{cases} 2\beta_1^3 V_0 - (2\beta_1^3 + 2\beta_1^2 + \beta_2)V_1 + (2\beta_1^2 + \beta_2)V_2 = 0\\ (2\beta_1 + \beta_2)V_{m-2} - (2\beta_1 + \beta_2 + 2)V_{m-1} + 2V_m = 0 \end{cases}$$
(4)

3) Steering Velocity: Although  $\beta$ -spline curves are continuous, the curvature change is smooth for path tracking. It can be experimentally illustrated that changes in curvature along  $\beta$ -spline curves can be approximated by the linear relationship (Cong, 2007;

Yang et al., 2005). From *Figure 1*, the above mentioned satisfies:

$$\frac{d\rho}{dx} = \frac{\rho_{\text{max}}}{\lambda} \tag{5}$$

where  $\lambda$  is the longitudinal offset when the vehicle trajectory curve achieves the maximum value of the curvature. Then, we define the longitudinal velocity v of operating vehicle as v = dx/dt, and obtain the curvature change rate along the spline as follows:

$$\dot{\rho} = \frac{d\rho}{dt} = \frac{d\rho}{dx} \cdot \frac{dx}{dt} = \frac{\rho_{\text{max}}}{\lambda} \cdot v \le \dot{\rho}_{\text{max}} \quad (6)$$

Locally, the vehicle is positioned on the arc of the circle with radius  $1/|\rho|$ . If  $\varphi$  is the angle between centroidal trace and lane marker (i.e. if it is the angle which changes with steering the wheel), then one can see the wheelbase I from the centre of the imagined circle of radius  $1/|\rho|$  in the same angle  $\varphi$ . The simple geometry gives

$$\operatorname{tg} \phi = \frac{I}{\frac{1}{\rho}} = I\rho.$$

Differencing this equation after time parameter t, we obtain that curvature changing rate should depend on the steering system:

$$\dot{\rho} = \frac{1 + \mathrm{tg}^2 \varphi}{I} \dot{\varphi} \tag{7}$$

where: *I* is wheelbase, and  $\varphi$  is the steering wheel angle (front wheel).

If  $\dot{\varphi}_{max}$  is the maximum angular velocity of the steering system, each value of  $\varphi$  presents a maximum value for the curvature change velocity  $\dot{\rho}_{max}(\varphi)$  and we only consider the steering wheel angle  $\varphi$  in the following research work for describing the lane-changing virtual desire trajectory. After having organized Eq. (7), this value is further limited by:

$$\dot{\rho}_{\text{max}} = \frac{1 + \text{tg}^2 \varphi}{I} \dot{\phi}_{\text{max}} = (1 + \text{tg}^2 \varphi) \frac{\dot{\varphi}_{\text{max}}}{I}$$
(8)

Generally, steering wheel angle  $\varphi$  is a little value. Thus, let us combine Eq (6) and Eq (8), and we restrict the vehicle longitudinal velocity by spline curves as:

$$v \le (1 + tg^2 \varphi) \frac{\lambda \dot{\varphi}_{max}}{I \rho_{max}} \approx \frac{\lambda \dot{\varphi}_{max}}{I \rho_{max}}$$
(9)

where  $\rho_{\rm max}$  is the maximum curvature, which has a minimum curvature radius  $1/|\rho_{\rm max}|$ . The maximum radius tends to  $\infty$  when  $\rho$  tends to 0 and the curve becomes straight. Here, the maximum curvature radius could be defined as:

$$\lambda_{\max} = \frac{L}{k} \tag{10}$$

where *L* is the length of lane-changing and *k* is obtained from the experimental observation with the range  $k \in (3.5, 5.3)$ .

Thus, rearranging Eq (9) and Eq (10), it yields the length limitation of lane-changing during the process:

$$L = k\lambda_{\max} \ge k\lambda \ge k \frac{V \rho_{\max}}{\dot{\varphi}_{\max}}$$
(11)

$$L \ge \frac{k v l \rho_{\max}}{\dot{\varphi}_{\max}} \tag{12}$$

This method mines information on the location of lane-changing behaviour from video data. Its appealing features include: 1) virtual lane-changing can be located and sized by spline curves; 2) lanechanging trajectory can be easily estimated without solving a series of tedious equations of a system; and 3) disposal procedure is simple and on-line operation is feasible.

#### 3.3 Result

From the basic characteristics of spline curves, we can see the four segments spline curves will bring five nodes and seven control vertices, in order,  $V_0, V_1, \ldots, V_6$ , as marked in Figure 3. There are two steps of solving the model: constructing the control vertices coordinates under the combination of boundary conditions and node equation with choosing reasonable values of  $\beta_1$ ,  $\beta_2$ , based on restriction (4). The four segments of the curves are determined with five notes as shown in Figure 3:  $P_0$ ,  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$ , where  $P_0(0,0)$  and  $P_4(L,N)$ are the terminals,  $P_2(L/2, N/2)$  is the intersection of the dividing line and the vehicle,  $P_1(\lambda, \gamma)$ ,  $P_3(L - \lambda, N - \gamma)$ are the curvature maximum points. Here, L is the length of lane-changing and N represents the width of the lane. From a large number of experimental data, the value range of  $\gamma$  is located as (L/4, N/4).

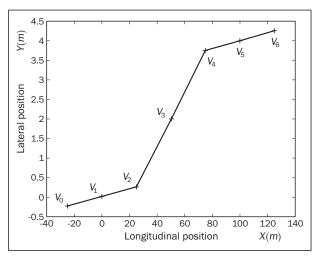
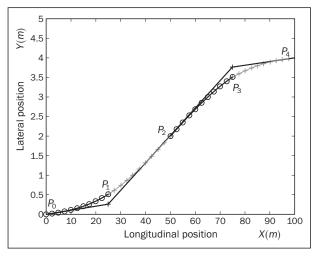


Figure 3 - Control polygon of trajectory curves

By choosing the appropriate trajectory parameters (*L*, *N*,  $\lambda$ , *r*) exactly, we obtain the control polygon through MATLAB for  $\beta_1 = 1$  and  $\beta_2 = 0$ . Figure 3 shows the control polygon with seven control vertices. Figure 4 is the final result of the lane-changing virtual desire trajectory based on  $\beta$ -spline curves. Different symbols along the curves present four segments spline curves. Thus a smooth and continuous path without inflexion point is obtained.





# 4. EXPERIMENTAL EXAMPLE

Choose typical lane-changing behaviours in the video files as the original data. The ground trajectory coordinates of the lane changing can be extracted from the software, which has been created by the research team (Liu et al., 2007).

Generally, lane-changing mode is divided into two patterns: left lane-changing and right lane-changing, according to the lane-changing direction compared with the previous driving direction. In our proposed models, N > 0 is taken, if the left lane-changing occurs, otherwise N < 0. Because of the limited text length, only right lane-changing is considered in the simulation experiment and thus n = -3.25 m is chosen. Figure 5 shows that the spline curves fit the actual trajectory very well, where square points are determined by the simulated data, and the curve, as well as the lane-changing virtual desire trajectory, is plotted through the simulation pattern in response to Eqs (1)~(11). The familiar results about left lane-changing could be easily concluded following the same experimental operation.

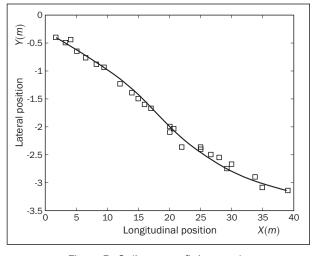


Figure 5 - Spline curve fitting results

Table 1 - Trajectory scatter coordinate

Number	X (m)	Y (m)	Number	X (m)	Y (m)
1	1.73	-0.40	14	20.70	-2.04
2	4.11	-0.45	15	20.08	-2.11
3	3.34	-0.51	16	22.31	-2.37
4	5.12	-0.65	17	25.13	-2.37
5	6.54	-0.77	18	25.04	-2.41
6	8.23	-0.89	19	26.71	-2.51
7	9.47	-0.94	20	28.32	-2.55
8	12.14	-1.23	21	30.21	-2.68
9	14.08	-1.40	22	29.25	-2.76
10	15.17	-1.52	23	33.84	-2.90
11	15.95	-1.60	24	35.17	-3.11
12	17.00	-1.67	25	39.24	-3.15
13	20.14	-2.00			

Table 2 - One sample parameters

	Test value = 0							
t	t-test	Column range	p of sig. (2- tailed)	ē	95% confi- dence interval of difference			
					Lower	Upper		
е	-0.037	24	0.971	-0.001	-0.032	0.031		

The simulation error should be checked through one-sample *t*-test in SPSS so as to determine whether there are significant differences between the overall average and the constituted test value from the sample data. Here we postulate the average error  $\bar{e} = 0$ . First, let us make a hypothesis  $H_0$ : there is no significant difference between the mean value and the test value, narrated:  $u = u_0 = 0$ . In this simulation experiment, we choose the confidence level  $\alpha = 0.05$ , as shown in *Table 2*.

The issue adopts double-tail test, and thus comparing  $\alpha/2$  and p/2 equals the same processing for  $\alpha$  and p. From *Table 1*, with p = 0.971 > 1 - 0.05 = 0.95, the average error is only 0.1%. Therefore, zero hypothesis should not be refused, so there is no significant difference between the mean and the test value. The 95% confidence interval means that 95% of the mean error lies between -0.032 and 0.031. The test value is included in the confidence interval and confirms the inference.

## 5. CONCLUSION

The paper introduces  $\beta$ -spline curves to simulate the lane-changing virtual desire trajectory and proves the validity of the model. The simulated trajectory curve, expressed by matrix form, has higher precision, compared with survey curve. In this paper, we consider the lane-changing behaviour occurring under free driving, with no interference by other vehicles. The control vertices and control polygon are obtained via the reverse algorithm which is calculated based on a few vital points. In actual operation, according to different traffic conditions, we can first determine the control polygon by avoiding collision. So the trajectory curves can be applied in different traffic situations. For future research it would be of interest to investigate the influence of speed on lane-changing trajectory construction as well as effective identification technique for dynamic condition so as to construct a practical safety monitoring system for traffic management system.

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#### 车道变换期望运行轨迹仿真

摘要:车道变换行为在实施过程中会产生交通冲突, 降低道路系统的运行效率,严重时将引发交通事故。根据 曲率的变化将车道变换过程划分为四个阶段,同时引入β 样条曲线,在给定边界条件的基础上,确定β样条曲线的 反求算法。在车辆转角、转角变化率及车辆的驾驶行为等 约束条件下,确立车道变换运行速度与车道变换长度的关 系,进而计算轨迹参数。以MATLAB仿真计算轨迹曲线,并 与实测数据对比,分析结果验证了模型的有效性,为车道 变换期望轨迹研究提供了一种新的思路。

#### 关键词

交通安全; 期望运行轨迹; β样条曲线; 车道变换

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