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AN MPCC FORMULATION AND ITS SMOOTH SOLUTION ALGORITHM FOR CONTINUOUS NETWORK DESIGN PROBLEM

ABSTRACT

Continuous network design problem (CNDP) is searching for a transportation network configuration to minimize the sum of the total system travel time and the investment cost of link capacity expansions by considering that the travellers follow a traditional Wardrop user equilibrium (UE) to choose their routes. In this paper, the CNDP model can be formulated as mathematical programs with complementarity constraints (MPCC) by describing UE as a non-linear complementarity problem (NCP). To address the difficulty resulting from complementarity constraints in MPCC, they are substituted by the Fischer-Burmeister (FB) function, which can be smoothed by the introduction of the smoothing parameter. Therefore, the MPCC can be transformed into a well-behaved non-linear program (NLP) by replacing the complementarity constraints with a smooth equation. Consequently, the solver such as LINDOGLOBAL in GAMS can be used to solve the smooth approximate NLP to obtain the solution to MPCC for modelling CNDP. The numerical experiments on the example from the literature demonstrate that the proposed algorithm is feasible.

KEY WORDS

urban transportation network; continuous network design problem (CNDP); mathematical programs with complementarity constraints (MPCC); non-linear complementarity problem (NCP); user equilibrium (UE);

1. INTRODUCTION

The network design problem (NDP) is seeking of a transportation network configuration that minimizes some objective functions, subject to a traditional Wardrop user equilibrium (UE) as the constraints. The continuous NDP (CNDP) has become one of the most computationally intensive problems in the transportation field [1]. The CNDP is to determine how to expand the link capacity to minimize the total system travel cost while the users follow the Wardrop's first principle of traffic equilibrium to choose their routes, i.e. no user can decrease their travel time by a unilateral change of route at equilibrium [2]. The measurement of the system performance can be described as the sum of total system travel time and the investment cost to expand the link capacity. Due to different objectives for formulating CNDP, it has been modelled as a bi-level programming problem with the upper level (a non-linear programming problem to minimize the total system cost or maximize the social surplus) and the lower level (a UE problem to account for the users' route choice behaviour). Allsop [3] firstly proposed the algorithm for addressing CNDP, and subsequently CNDP has been continuously studied by many researchers during the last five decades. Lots of related publications have grown over time including reviews by Yang and Bell [1] and Farahani et al. [4]. Various algorithms have been proposed to solve CNDP (see *Table 1* for details).

In this paper, the CNDP's objective is to minimize the sum of the total system travel time and the investment cost by expanding the link capacity, while route choice behaviour of travellers follows UE, which is described by non-linear complementarity problem (NCP). Thus, the CNDP model can be formulated as mathematical programs with complementarity constraints (MPCC). However, solving MPCC is a hard task because the Mangasarian Fromovitz constraint qualification (MFCQ) does not hold at any feasible point [29, 30]. To circumvent these problems, some algorithmic approaches have focused on avoiding this formulation. Subsequently, researchers have developed special-purpose algorithms for MPCC such as the branch-and-bound method [31], the implicit non-smooth approach [32], the piece-wise sequential quadratic programming (SQP) method [33], and the perturbation and penalization approach [34] analysed in Ref. [35]. Recently, some exciting new developments have

Table 1 – Some algorithms for solving CNDP

Abbreviation	Name of the algorithm	Sources
IOA	Iterative optimization-assignment algorithm	Allsop [3]
HJ	Hooke–Jeeves algorithm	Abdulaal and LeBlanc [5]
EDO	Equilibrium decomposed optimization	Suwansirikul et al. [6]
MINOS	Modular in-core nonlinear system	Suwansirikul et al. [6]
BDA	Bilevel descent algorithm	Kim and Suh [7]
SA	Simulated annealing algorithm	Friesz et al. [8]
SAB	Sensitivity analysis-based algorithm	Yang and Yagar [9]
BLABG	Bileve linear approximation based on gradient	Yang [10]; Gao et al. [11]
AL	Augmented Lagrangian algorithm	Meng et al. [12]
GP	Gradient Projection method	Chiou [13]
CG	Conjugate gradient projection method	Chiou [13]
QNEW	Quasi-Newton projection method	Chiou [13]
PT	PARATAN version of gradient projection method	Chiou [13]
RELAX	Relaxation method	Ban et al.[14]; Wang et al. [15, 16]
PSO	Particle swarm optimization	Gao et al. [17]
GA	Genetic algorithm	Sumalee [18]
CSP	Conjugate Subgradient Projection	Chiou [19]
PMILP	Path based mixed-integer linear program	Wang and Lo [20]
LMILP	Link based mixed-integer linear program	Luathep et al.[21]
GOM	A global optimization method	Li et al. [22]; Liu and Wang [23]
HS	Harmony search algorithm	Baskan [24]
MODE	Modified Differential Evolution Algorithm	Baskan and Ceylan [25]
CCA	cutting constraint algorithm	Wang et al. [26]
DDIA	Dimension-Down Iterative Algorithm	Liu and Chen [27]
AFW	Genetic algorithm and Frank-Wolfe (FW) algorithm	Sun [28]

demonstrated that the gloomy prognosis about the use of transforming MPCC to a well-behaved NLP may have been premature. Several algorithms are proposed for solving MPCC by transforming it to a well-behaved NLP. In particular, Fukushima and Pang [36] considered a smoothing continuation method for the mathematical programming with equilibrium constraints (MPEC). And, under MPEC-linear independence constraint qualification and the asymptotic weak non-degeneracy, they proved that an accumulation point of KKT points is a B-stationary point of the original problem when it satisfies the second-order necessary conditions for the perturbed problems. Subsequently, similar schemes were presented by Scholtes [35] and Lin and Fukushima [37, 38]. However, these methods need to solve an infinite sequence of non-linear programs. Lin and Fukushima [39] have developed a hybrid approach with active set identification to compute a solution or a point with some kind of stationarity by solving a finite number of non-linear programs. For the success of NLP solvers, Leyffer [40] relaxed equivalent condition to replace the usual complementarity condition. Through modelling CNDP as MPCC, Ban et al. [14] relaxed the strict complementarity condition by a relaxation parameter. The relaxed NLP was solved by the existing NLP solvers when this parameter was progressively reduced. Moreover, the relaxation scheme proposed in Ref. [14] can guarantee to solve the original MPCC successfully under certain conditions [41, 42]. In addition, Fletcher and Leyffer [43] considered solving MPCC as NLP using standard NLP solvers. They demonstrated the numerical experience on a large collection of MPCC test problems, called by MacMPEC, to indicate the suitability of SQP methods for solving MPCC and its out-performance over interior-point solvers regarding speed and reliability. In this paper, the complementarity constraint in MPCC is substituted by a non-smooth equation $\Phi(\mathbf{v},\mathbf{x}^w,\mathbf{y},\rho^w)=0$ using Fischer-Burmeister (FB) function φ_{FB} proposed in Ref. [44]. Then, the Φ is smoothed by introducing a parameterization $\Psi(\mathbf{v},\mathbf{x}^w,\mathbf{y},\rho^w,\theta)$ that is differentiable if the scalar θ is non-zero but coincides with Φ when $\theta=0$ [45]. Consequently, the standard NLP solvers such as LINDOGLOBAL in GAMS [46] can solve the problem reliably and efficiently. Finally, the numerical experiments on the example from the references demonstrate the feasibility of our model and algorithm.

This paper is organized as follows. An MPCC formulation describing CNDP model is presented in Section 2. Section 3 discusses the algorithm for the proposed MPCC model. Section 4 provides numerical experiments on the example from the references to show the feasibility of the MPCC model with the algorithm. Finally, conclusion is given in Section 5.

2. PROBLEM FORMULATION

2.1 Notations

In this section, the notations are stated:

$G=(N,A)$ - transportation network with nodes and links

N - set of nodes in G, where $N=\{1,2,\ldots,n\}$ and n denotes the node

A - set of links in G, where $(i,j)\in A$ denotes the link, $i,j\in N$

W - set of OD pairs, and $w\in W$ denotes the OD pair

y_{ij} - incremental capacity on expanded link $(i,j)\in A$, and $\mathbf{y}=(y_{ij}),(i,j)\in A$ denotes the incremental capacity vector

l_{ij}, u_{ij} - lower and upper bounds for capacity expansion of link $(i,j)\in A$, respectively. and $\mathbf{l}=(l_{ij})$, $\mathbf{u}=(u_{ij}),(i,j)\in A$

$g_{ij}(y_{ij})$ - cost of incremental capacity, y_{ij}, on expanded link $(i,j)\in A$, and $\mathbf{g}=(g_{ij}),(i,j)\in A$

d_w - travel demand between the OD pair $w\in W$, and the OD demands are given and fixed in this paper

x_{ij}^w - flow of link $(i,j)\in A$ on the OD pair $w\in W$, and $\mathbf{x}^w=(x_{ij}^w),(i,j)\in A$, $w\in W$

v_{ij} - aggregate flow on link $(i,j)\in A$, $\mathbf{v}=(v_{ij})$, $(i,j)\in A$

$t_{ij}(v_{ij},y_i)$ - link travel time function on link $(i,j)\in A$

$\wedge$ - node-link incidence matrix associated with the network, where

$\wedge=\begin{bmatrix}\delta_1^1\ldots\delta_a^1\\ \vdots\ \ddots\ \vdots\\ \delta_1^n\ldots\delta_a^n\end{bmatrix}$ with $\delta_a^n=1$ if node lies at the exit of link $a\in A$, δ_a^n=-1 if node lies at the entrance of link $a\in A$, and δ_a^n=0 otherwise. To ensure feasibility, assume that there exists at least one directed path for every OD pair in the network

$\mathbf{E}^w$ - vector indicating the origin and the destination for OD pair $w\in W$, and has exactly two non-zero components: one has value 1 in the component corresponding the origin node of OD pair w, and the other has value of -1 in the component for the destination

β - relative weight of the investment cost on the link capacity expansion and total system travel time in the objective function, or the dual variable for the budget constraint $\sum_{(i,j)\in A} g_{ij}(y_{ij})\le B$, where B is the value of the predetermined budget for network capacity expansion.

Next, some assumptions used in this paper are presented as follows [6]:

1) The link travel time function $t_{ij}(v_{ij},y_i),(i,j)\in A$ is strictly increasing and continuously differentiable with respect to the link flow $v_{ij},(i,j)\in A$, for any fixed link capacity expansion $y_{ij},(i,j)\in A$.
2) The link travel time function $t_{ij}(v_{ij},y_i),(i,j)\in A$ and $\frac{\partial t_{ij}(v_{ij},y_{ij})}{\partial y_{ij}},\ (i,j)\in A$ are all continuous with respect to (v_{ij},y_{ij}).
3) The capacity expansion cost function $g_{ij}(y_{ij}),(i,j)\in A$ is continuously differentiable with respect to y_{ij}.

2.2 Reformulation of traffic assignment problem

Using the above notations, the set containing all feasible flow distributions for the network, Ω, in terms of link-flow can be described as follows:

$$\Omega=\Big\{v=(v_{ij})\big|\, v_{ij}=\sum_{w\in W}x_{ij}^w,\wedge x^w=E^w d_w,\ x^w=(x_{ij}^w)\ge 0,\forall(i,j)\in A,\forall w\in W\Big\} \tag{1}$$

Ω is a bounded polyhedron because it is comprised of a set of linear equality constraints [21]. Here, travellers are assumed to use the Wardrop's UE to describe their route choice behaviour in the transportation network [47]. Within each OD pair, a traveller chooses a route to minimize their travel cost. Here, the NCP is used to represent this UE condition. In the link-flow feasible region, Ω, the NCP problem describing UE is to find v and ρ^w ("node potential" in Ref. [48]) such that

$$0\le\left[t_{ij}(v_{ij},y_{ij})-(\rho_i^w-\rho_j^w)\right]\perp x_{ij}^w\ge 0,\ \forall(i,j)\in A,\forall w\in W \tag{2}$$

where symbol "⊥" is the "perp" operator such that $a \perp b \Longleftrightarrow a^T b$. The complementarity constraint [30] requires a product of two non-negative variables to be zero, consequently making their values complementary, i.e. when one variable is positive, the other must be zero. It is also possible for both variables to be zero, a case in which the complementarity condition does not hold strictly [49].

2.3 The MPCC formulation for the continuous network design problem

In this research, the objective of the CNDP is to minimize the sum of the total system travel time and the investment cost to expand the link capacity, while route choice behaviour of travellers follows UE described by NCP. The CNDP model can be formulated as the following MPCC:

$$\begin{aligned}
&\min_{(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^{\mathbf{w}})} f(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) = \\
&= \sum_{(i,j)\in A} v_{ij} t_{ij}(v_{ij},y_{ij}) + \beta \sum_{(i,j)\in A} g_{ij}(y_{ij}) \\
&\text{subject to} \\
&l_{ij} \le y_{ij} \le u_{ij}, \forall (i,j)\in A \\
&0 \le [t_{ij}(v_{ij},y_{ij}) - (\rho_i^w - \rho_j^w)] \perp x_{ij}^w \ge 0, \\
&\forall (i,j)\in A, \forall w \in W \\
&\mathbf{v}\in\Omega
\end{aligned} \tag{3}$$

For simplicity, the following notations are introduced.

$$\begin{aligned}
&G_{ij}(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) = [t_{ij}(v_{ij},y_{ij}) - (\rho_i^w - \rho_j^w)], \\
&H_{ij}(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) = x_{ij}^w, \forall (i,j)\in A, \forall w \in W \\
&\mathbf{G}(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) = (G_{ij}), \mathbf{H}(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) = (H_{ij}), (i,j)\in A
\end{aligned} \tag{4}$$

Then, *Problem 3* can be simplified into the following form:

$$\begin{aligned}
&\min_{(v,x^w,y,\rho^w)} f(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) = \\
&= \sum_{(i,j)\in A} v_{ij} t_{ij}(v_{ij},y_{ij}) + \beta \sum_{(i,j)\in A} g_{ij}(y_{ij}) \\
&\text{subject to} \\
&\mathbf{l} \le \mathbf{y} \le \mathbf{u}, \\
&0 = \mathbf{G}(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) \perp \mathbf{H}(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^w) \ge 0 \\
&\mathbf{v}\in\Omega
\end{aligned} \tag{5}$$

Let us rewrite *Problem 5* in the following form:

$$\begin{aligned}
&\min_{(z,r,s)} f(\mathbf{v},\mathbf{x}^{\mathbf{w}},\mathbf{y},\rho^{\mathbf{w}}) = \\
&= \sum_{(i,j)\in A} v_{ij} t_{ij}(v_{ij},y_{ij}) + \beta \sum_{(i,j)\in A} g_{ij}(y_{ij}) \\
&\text{subject to} \\
&\mathbf{l} \le \mathbf{y} \le \mathbf{u}, \\
&0 \le r \perp s \ge 0 \\
&\mathbf{G}(z) - r = 0 \\
&\mathbf{H}(z) - s \\
&\mathbf{v}\in\Omega
\end{aligned} \tag{6}$$

where $\mathbf{z}=(\mathbf{v},\mathbf{x}^{\mathrm{w}},\mathbf{y},\rho^{\mathrm{w}})$

Now, let us focus on solving the MPCC *Problem 6*.

3. SOLUTION ALGORITHM

Here, function $\varphi:\mathrm{R}^2\rightarrow\mathrm{R}$ is called an NCP-function if $\varphi(a,b)=0 \Longleftrightarrow ab=0, a\ge 0, b\ge 0$. One popular choice of an NCP-function is the FB function [44]: $\varphi_{FB}(a,b) = a + b - \sqrt{a^2+b^2}$. The FB function has many interesting properties that φ_{FB} is a convex NCP-function and differentiable on R^2 expect the origin, and φ_{FB}^2 is continuously differentiable on R^2 [50]. FB function has been used to solve the traffic equilibrium problem [51, 52]. By using φ_{FB}, the complementarity constraint $0\le r\perp s\ge 0$ can be equivalently transformed into the following non-smooth equation: $\varphi_{FB}(r,s) = r + s - \sqrt{r^2+s^2}$.

Then, using the smooth function, $\varphi_{FB}(a,b,\theta) = a + b - \sqrt{a^2+b^2+\theta^2}$, for FB function φ_{FB}, the $\varphi_{FB}(r,s)$ is smoothed by introducing a parameterization $\Psi(r,s,\theta)$ that is differentiable if the scalar θ is non-zero but coincides with Φ when $\theta=0$ [45]. The introduction of the smoothing parameter θ has three consequences [53]: non-smooth problems are transformed into smooth problems, except when $\theta=0$; well-posedness can be improved in the sense that feasibility and constraint qualifications, hence stability, are often more likely to be satisfied for all values of θ; and the solvability of quadratic approximation problems is improved. Therefore, the smooth and approximate equation for the complementarity constraint is obtained as follows: $\Psi(r,s,\theta) = r + s - \sqrt{r^2+s^2+\theta^2}$.

Hence, *Model 3* for CNDP formulated as MPCC can be transformed into the following smooth and approximate problem:

$$\begin{aligned}
&P_\theta \min_{(z,r,s)} f(\mathbf{z}) \\
&\text{subject to} \\
&\quad \mathbf{l} \le \mathbf{y} \le \mathbf{u}, \\
&\quad \Psi(r,s,\theta) = 0 \\
&\quad \mathbf{G}(z) - r = 0 \\
&\quad \mathbf{H}(z) - s = 0 \\
&\quad \mathbf{v}\in\Omega
\end{aligned} \tag{7}$$

We define $\Omega(\theta)=\{(r,s)\,|\,\Psi(r,s,\theta)=0\}$. We analyse the convergence of the smoothing perturbation-based approach by demonstrating the convergence of $\Omega(\theta)$ to $\Omega(0)$ as $\theta\rightarrow 0$.

Theorem: For $\Omega(\theta)=\{(r,s,)\,|\,\Psi(r,s,\theta)=0\}$, we have $\lim_{\theta\rightarrow 0}\Omega(\theta) = \Omega(0)$

Proof: For any $(r,s)\in \limsup_{\theta\rightarrow 0}\Omega(\theta)$, then there exist $\theta^k\rightarrow 0$ and $(r^k,s^k)\in\Omega(\theta^k)$ such that $(r^k,s^k)\rightarrow (r,s)$.

$(r^k,s^k)\in\Omega(\theta^k)$ implies

$\Psi(r^k,s^k,\theta^k) = r^k + s^k - \sqrt{(r^k)^2+(s^k)^2+(\theta^k)^2} = 0.$

Thus, we have

$\varphi_{FB}(r,s) = \Psi(r,s,0) = r + s - \sqrt{r^2+s^2} = 0$

from $\theta \to 0$ and $(r_k,s_k) \to (r,s)$. That is, $(r,s) \in \Omega(0)$. Therefore we have $\limsup_{\theta \to 0} \Omega(\theta) \subset \Omega(0)$

For any $(r,s) \in \Omega(0)$, let $I_+=\{i|r_iz>\}$, $J_+=\{i|s_i>0\}$, $I_0=\{1,2,...,|A|\}\backslash(I_+ \cup J_+)$, where $|A|$ is the number of links in the transportation network G.

For any $\theta>0$, $(r_i(\theta), s_i(\theta))$ is defined by

$$(r_i(\theta), s_i(\theta)) = \begin{cases} \left(r_i, \dfrac{\theta^2}{2r_i}\right), & if\ i \in I_+ \\ \left(\dfrac{\theta^2}{2s_i}, s_i\right), & if\ i \in J_+ \\ \left(\dfrac{\theta}{\sqrt{2}}, \dfrac{\theta}{\sqrt{2}}\right), & if\ i \in I_0 \end{cases}$$

Then $\Psi(r_i,s_i,\theta) = r_i + s_i - \sqrt{r_i^2 + s_i^2 + \theta^2} = 0$ for $i=1,2,...,|A|$ and equivalently $\Psi(r(\theta),s(\theta),\theta) = 0$ or $(r(\theta),s(\theta)) \in \Omega(\theta)$.

Obviously $((r(\theta),s(\theta)) \to (r,s)$ and this implies that $\liminf_{\theta \to 0} \Omega(\theta) \supset \Omega(0)$. Therefore $\Omega(\theta) \to \Omega(0)$ as $\theta \to 0$.

Hence, the solution to the problem (7) converges to the solution the model (3) as $\theta \to 0$.

Consequently, the standard NLP solvers to obtain the global solution such as BARON, COIN-OR, LINDO-GLOBAL, LGO, MSNLP and OQNLP in GAMS [46] can be used to deal with *Problem 7* reliably and efficiently. The steps of algorithm to solve *Model 3* are as follows:

Step 1: initialization

The parameters are set as follows: $\theta^0>\theta,\varepsilon_1,\varepsilon_2>0$ the iteration limit M, $\lambda \in (0,1)$.

Step 2: Major Iteration

Solve the current relaxed problem (7) for $\theta=\theta^k$. The obtained solution is $\mathbf{z}^k$.

Step 3: Stop Condition

If $k \geq M$, stop and go to Step 4. Otherwise, compute $\dfrac{\|z^{k+1}-z^k\|}{\|z^k\|}$ and $\left|\dfrac{f(z^{k+1})-(z^k)}{f(z^k)}\right|$. If $\dfrac{\|z^{k+1}-z^k\|}{\|z^k\|} \leq \varepsilon_1$ or $\left|\dfrac{f(z^{k+1})-(z^k)}{f(z^k)}\right| \leq \varepsilon_2$, stop and go to Step 4.

Otherwise, set $\varepsilon^{k+1}=\lambda\, \varepsilon^k$ and $k=k+1$, then go to Step 2.

Step 4: Solution Report

The optimal solution is $(\mathbf{v}^*,\mathbf{x}^w,\mathbf{y}^*,\rho^{w^*})=z^*$ and the objective function value is $f^*=f(z^k)$.

4. NUMERICAL EXAMPLES

4.1 The 16-link network example

In this subsection, the 16-link network, which was first presented by Harker and Friesz [54], is chosen as the numerical example. The network has been extensively tested in the CNDP literatures such as Ref. [6, 8, 12, 13, 14, 20, 21]. As shown in *Figure 1*, the 16-link network consists of two OD pairs, six nodes and links. All input information by Suwansirikul et al. [6] for the test network is presented in *Table 2*. The travel demand of (1,6) is assumed to be d, which is half of that for (6,1).

In this paper, the numerical experiments consist of three parts. In the first part, we compare the optimal solutions and the objective function values by our proposed algorithm with those by other algorithms in literature. In the second part, we present the results for the test network with different OD demand to make further test on our model and algorithm. In the last part, we consider the total investment cost for link capacity

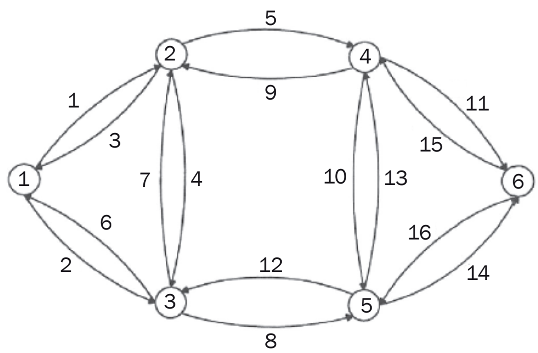

Figure 1 – The 16-link network

Table 2 – Parameters in the 16-link network

Link *(i,j)*	A_{ij}	B_{ij}	K_{ij}	d_{ij}	Link *(i,j)*	A_{ij}	B_{ij}	K_{ij}	d_{ij}
1	1.0	10.0	3.03.0	2.0	9	2.0	8.0	45.0	2.0
2	2.0	5.0	10.0	3.0	10	3.0	3.0	3.0	5.0
3	3.0	3.0	9.0	5.0	11	9.0	2.0	2.0	6.0
4	4.0	20.0	4.0	4.0	12	4.0	10.0	6.0	8.0
5	5.0	50.0	3.0	9.0	13	4.0	25.0	44.0	5.0
6	2.0	20.0	2.0	1.0	14	2.0	33.0	20.0	3.0
7	1.0	10.0	1.0	4.0	15	5.0	5.0	1.0	6.0
8	1.0	1.0	10.0	3.0	16	6.0	1.0	4.5	1.0
$t_{ij} = A_{ij} + B_{ij}\left(\dfrac{v_{ij}}{K_{ij}+y_{ij}}\right)^4$					$f = \sum_{(i,j)\in A}(t_{ij} + d_{ij} \cdot y_{ij})$				

expansion as a constraint by removing it from the objective function to be a constraint function. Then, the model is solved with different values of predetermined budget for total investment on link capacity expansion. In these experiments, a personal computer with Intel Core i5, 3.20 GHz CPU, 4.00 GB RAM, and Windows XP operating system was used for all numerical tests. In addition, GAMS23.5.2 and our familiar LINDOGLOBAL solver was used for solving the smooth approximate problems [46]. In numerical experiments, the initial parameters are chosen as follows: θ^0=1, ε_1=1e-4, ε_2=1e-6, M=15, λ=0.2.

In the first part of the experiments, we compare our proposed algorithm with those in the references. A low travel demand level with d=5 and a high travel demand level with d=10 are considered for the tests, which are the same as those in references. *Table 3* presents the results of the link capacity expansions under the low and high travel demand cases, respectively obtained by our proposed algorithm and those in the previous studies. Notice that the upper bound of capacity expansion for each link in these two cases is different. The links with zero capacity expansions under all algorithms are not presented in tables for the purpose of space saving.

For the case with low travel demand, the objective function value 199.625 achieved by our algorithm is higher than that by SA, CG, QNEW and LMILP. Among all the previous models, SA, which is regarded to produce the global optimal solution [54], achieves the lowest objective function value 198.10378 . From the value, we can see that the objective function value obtained by our algorithm is very close to that obtained by the SA method. The difference in the objective function values between our algorithm and SA is less than 8%. For the case with high travel demand, the objective function value 522.723 by our algorithm is only higher than that by RELAX. The objective function value by our algorithm nearly equals that of RELAX. The difference in the objective function values between our algorithm and RELAX is less than 0.02%.

In the second part of the experiments, the model is solved under different OD demand cases. The link capacity expansion with different d of CNDP is shown in *Table 4*. The upper bound of capacity expansion for each link under these cases are also different.

It is obvious that the objective function values increase with the increase of d. From *Table 4*, it can be seen that the links y_6 and y_{16} are the most necessary ones, whose link capacity should be enhanced. And, y_6 reaches the largest link capacity expansion when d=25,30,35,40,45,50. The capacity improvement on link y_{16} reaches the largest link capacity expansion when d=10,15,30.

In the third part of the experiment, the objective function only considers the total travel times, i.e. $T=\sum_{(i,j)\in A} v_{ij}t_{ij}(v_{ij},y_{ij})$, while the total investment cost to expand the link capacity is considered as a constraint, i.e. $\sum_{(i,j)\in A} g_{ij}(y_{ij})\le B$, where B is the value of predetermined budget for total investment on link capacity expansions. Therefore, the model is solved with different values of B. *Table 5* presents the link capacity expansions for the different B of CNDP. The upper bound of capacity expansion for each link under these cases is identical.

It is obvious that the total travel times decrease as the value of predetermined budget for the total investment cost of link capacity expansions increases. While links y_6 and y_{16} are also the targets to enhance link capacity expansions for decreasing the total travel times.

However, links y_6 and y_{16} cannot be selected to enhance the link capacity expansions when the value of predetermined budget for total investment cost of link capacity expansions is large enough, such as when B=350,400,450,500.

Table 4 – The results with different OD demand for test network

Link	d=5	d=10	d=15	d=20	d=25	d=30	d=35	d=40	d=45	d=50
y_2		4.600	10.950	19.114	26.478	33.608	41.035	48.429	55.736	63.065
y_3		9.911	16.744	20.146	22.740	25.952	35.873	42.424	48.986	55.546
y_6	5.195	7.379	17.842	34.839	50.000	60.000	70.000	80.000	90.000	100.000
y_8		0.584	5.353	11.120	16.449	21.647	27.009	32.353	37.649	42.957
y_9					1.398	6.069	20.602	30.159	39.749	49.331
y_{12}			5.243	15.135	25.902	35.492	38.154	44.301	50.432	56.567
y_{14}		1.313	11.832	22.614	33.280	43.911	54.587	65.257	75.914	86.575
y_{15}			1.499	30.131	32.895	9.603	46.919	53.916	60.924	67.929
y_{16}	7.596	20.000	30.000	15.722	26.015	60.000	37.738	43.608	49.471	55.337
f	199.625	522.723	937.702	1,378.497	1,797.937	2,216.156	2,643.823	3,066.105	3,488.883	3,911.537

Note: The lower bound of **y** *is set as 0. And, the upper bound of* **y** *is set as 2d with different d.*

Table 3 – Comparison of algorithms under different demand for test network

Link	IOA	HJ	EDO	MINOS	SA	SAB	AL	GP	CG	QNEW	PT	PMILP	LMILP	RELAX	This paper
y_3		1.20	0.13				0.0062								
y_6	6.95	3.00	6.26	6.58	3.1639	5.8352	5.2631	5.8302	6.1989	6.0021	5.9502	5.19	5.24	5.19458	5.195
y_7							0.0032								
y_{12}							0.0064								
y_{15}	5.66	3.00	0.13	7.01		0.9739	0.7171	0.8700	0.0849	0.1846	0.5798		0.002		
y_{16}	1.79	2.80	6.26	0.22	6.7240	6.1792	6.7561	6.1090	7.5888	7.5438	7.1064	7.50	7.585	7.596208	7.596
f	210.86	215.08	201.84	211.25	198.10378	204.7	202.991	202.24	199.27	199.68	200.60	199.781	199.622	199.6253	199.625
y_1						0.0189		0.1013	0.1022	0.0916	0.1010				
y_2	4.55	5.40	4.88	4.61		2.2246	4.6153	2.1818	2.1796	2.1521	2.1801	4.41	2.722	4.614426	4.600
y_3	10.65	8.18	8.59	9.86	10.1740	9.3394	9.8804	9.3423	9.3425	9.1408	9.3339	10.00	9.246	9.910446	9.911
y_6	6.43	8.10	7.48	7.71	5.7769	9.0466	7.5995	9.0443	9.0441	8.8503	9.0361	7.42	8.538	7.373796	7.379
y_7			0.26				0.0016								
y_8	0.59	0.90	0.85	0.59		0.0175	0.6001	0.0080	0.0074	0.0114	0.0079	0.54		0.592238	0.584
y_9							0.0010								
y_{12}						0.0816	0.1130	0.0375	0.0358	0.0377					
y_{14}	1.32	3.90	1.54	1.32		0.0198	1.3184	0.0089	0.0083	0.0129	0.0089	1.18		1.315255	1.313
y_{15}	19.36	8.10	0.26	19.14		2.1429	2.7265	1.9433	1.9483	1.9706	1.9429				
y_{16}	0.78	8.40	12.52	0.85	17.2786	18.9835	17.5774	18.9859	18.986	18.575	18.9687	19.50	20.000	20.000	20.000
f	556.61	557.22	540.74	557.14	528.497	536.084	532.710	534.017	534.109	534.08	534.02	523.627	526.488	522.6439	522.723

Note: The lower bound of **y** *is set as 0, and the upper bounds of* **y** *are set as 10 and 20 for the case of d=5 and d=10, respectively.*

Table 5 – The results with different values of predetermined budget for test network

Link	*B*=50	*B*=100	*B*=150	*B*=200	*B*=250	*B*=300	*B*=350	*B*=400	*B*=450	*B*=500
y_2		4.752	0.669	4.828	6.830	9.228	8.510	10.083	11.602	13.121
y_3	5.559	10.122	13.512	10.308	13.102	15.569	21.176	23.742	26.219	28.696
y_6	6.661	7.425	12.340	7.796	7.437	6.779				
y_8		0.694		0.765	2.199	3.936	3.416	4.556	5.657	6.758
y_9								2.851	6.471	10.091
y_{12}			2.236							
y_{14}		1.533	7.398	1.760	4.556	8.045	6.998	9.291	11.507	13.722
y_{15}				19.623	22.603	25.237	31.225	33.966	36.611	39.256
y_{16}	15.542	21.029	28.013	0.867	0.682	0.322				
T	503.131	422.732	412.131	358.165	323.783	305.510	296.214	283.724	275.193	269.196

Note: The lower bound of **y** *is set as 0; and the upper bound of* **y** *is set as 40*

4.2 Sioux Falls network example

To further test the performance of our proposed algorithm, we have chosen the Sioux Falls network (shown in *Figure 2*) as another example. This network consists of 24 nodes, 76 links and 552 OD pairs. A subset containing 10 links (16, 17, 19, 20, 25, 26, 29, 39, 48, 74) was chosen for capacity improvement. The link cost function with the corresponding parameters and the OD demand are presented in *Tables 6* and *7* [6]. The experimental environment and the parameters are adopted as the same as those in Subsection 4.1. We have computed the Sioux Falls network by our proposed algorithm and compared the obtained solution with the corresponding objective function values with those by other algorithms in literature, which are presented in *Table 8*. Notice that the upper bound of capacity expansion for each link is 25.0. The links with zero capacity expansions under all algorithms are not presented in tables for space saving.

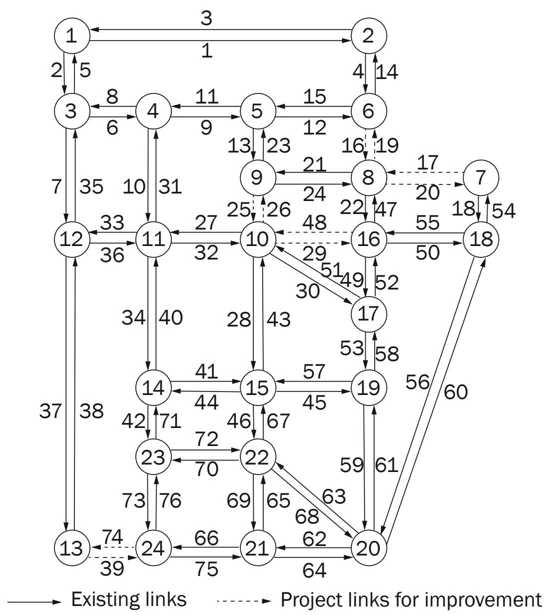

Figure 2 – Sioux Falls network

Table 6 – The link cost function and parameters in Sioux Falls network

Link *(i,j)*	T_{ij}	b_{ij}	τ_{ij}	Link *(i,j)*	T_{ij}	b_{ij}	τ_{ij}
1 and 3	0.06	25.9002		33 and 36	0.06	4.9088	
2 and 5	0.04	23.4035		34 and 40	0.04	4.8765	
4 and 14	0.05	4.9582		37 and 38	0.03	25.9002	
6 and 8	0.04	17.1105		39 and 74	0.04	5.0913	34.00
7 and 35	0.04	23.4035		41 and 44	0.05	5.1275	
9 and 11	0.02	17.7828		42 and 71	0.04	4.9248	
10 and 31	0.06	4.9088		45 and 57	0.04	15.6508	
12 and 15	0.04	4.9480		46 and 67	0.04	10.3150	
13 and 23	0.05	10.0000		49 and 52	0.02	5.2299	
16 and 19	0.02	4.8986	26.00	50 and 55	0.03	19.6799	
17 and 20	0.03	7.8418	40.00	53 and 58	0.02	4.8240	
18 and 54	0.02	23.4035		56 and 60	0.04	23.4035	
21 and 24	0.10	5.0502		59 and 61	0.04	5.0026	
22 and 47	0.05	5.0458		62 and 64	0.06	5.0599	
25 and 26	0.03	13.9158	25.00	63 and 68	0.05	5.0757	
27 and 32	0.05	10.0000		65 and 69	0.02	5.2299	
28 and 43	0.06	13.5120		66 and 75	0.03	4.8854	
29 and 48	0.05	5.1335	48.00	70 and 72	0.04	5.0000	
30 and 51	0.08	4.9935		73 and 76	0.02	5.0785	
$t_{ij}(v_{ij},y_{ij}) = T_{ij}\left[1+0.15\left(\frac{v_{ij}}{b_{ij}+y_{ij}}\right)^4\right]$				$f=\sum_{(i,j)\in A} v_{ij}t_{ij}(v_{ij},y_{ij}), g_{ij}(y_{ij})=\tau_{ij}y_{ij}^2$			

Table 7 – The OD demand in Sioux Falls network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1		0.11	0.11	0.55	0.22	0.33	0.55	0.88	0.55	1.43	0.55	0.22	0.55	0.33	0.55	0.55	0.44	0.11	0.33	0.33	0.11	0.44	0.33	0.11
2	0.11		0.11	0.22	0.11	0.44	0.22	0.44	0.22	0.66	0.22	0.11	0.33	0.11	0.11	0.44	0.22	0.00	0.11	0.11	0.00	0.11	0.00	0.00
3	0.11	0.11		0.22	0.11	0.33	0.11	0.22	0.11	0.33	0.33	0.22	0.11	0.11	0.11	0.22	0.11	0.00	0.00	0.00	0.00	0.11	0.11	0.00
4	0.55	0.22	0.22		0.55	0.44	0.44	0.77	0.77	1.32	1.54	0.66	0.66	0.55	0.55	0.88	0.55	0.11	0.22	0.33	0.22	0.44	0.55	0.22
5	0.22	0.11	0.11	0.55		0.22	0.22	0.55	0.88	1.10	0.55	0.22	0.22	0.11	0.22	0.55	0.22	0.00	0.11	0.11	0.11	0.22	0.11	0.00
6	0.33	0.44	0.33	0.44	0.22		0.44	0.88	0.44	0.88	0.44	0.22	0.22	0.11	0.22	0.99	0.55	0.11	0.22	0.33	0.11	0.22	0.11	0.11
7	0.55	0.22	0.11	0.44	0.22	0.44		1.1	0.66	2.09	0.55	0.77	0.44	0.22	0.55	1.54	1.10	0.22	0.44	0.55	0.22	0.55	0.22	0.11
8	0.88	0.44	0.22	0.77	0.55	0.88	1.1		0.88	1.76	0.88	0.66	0.66	0.44	0.66	2.42	1.54	0.33	0.77	0.99	0.44	0.55	0.33	0.22
9	0.55	0.22	0.11	0.77	0.88	0.44	0.66	0.88		3.08	1.54	0.66	0.66	0.66	0.99	1.54	0.99	0.22	0.44	0.66	0.33	0.77	0.55	0.22
10	1.43	0.66	0.33	1.32	1.10	0.88	2.09	1.76	3.08		4.40	2.2	2.09	2.31	4.40	4.84	4.29	0.77	1.98	2.75	1.32	2.86	1.98	0.88
11	0.55	0.22	0.33	1.54	0.55	0.44	0.55	0.88	1.54	4.4		1.54	1.1	1.76	1.54	1.54	1.10	0.11	0.44	0.66	0.44	1.21	1.43	0.66
12	0.22	0.11	0.22	0.66	0.22	0.22	0.77	0.66	0.66	2.2	1.54		1.43	0.77	0.77	0.77	0.66	0.22	0.33	0.44	0.33	0.77	0.77	0.55
13	0.55	0.33	0.11	0.66	0.22	0.22	0.44	0.66	0.66	2.09	1.1	1.43		0.66	0.77	0.66	0.55	0.11	0.33	0.66	0.66	1.43	0.88	0.88
14	0.33	0.11	0.11	0.55	0.11	0.11	0.22	0.44	0.66	2.31	1.76	0.77	0.66		1.43	0.77	0.77	0.11	0.33	0.55	0.44	1.32	1.21	0.44
15	0.55	0.11	0.11	0.55	0.22	0.22	0.55	0.66	0.99	4.4	1.54	0.77	0.77	1.43		1.32	1.65	0.22	0.88	1.21	0.88	2.86	1.10	0.44
16	0.55	0.44	0.22	0.88	0.55	0.99	1.54	2.42	1.54	4.84	1.54	0.77	0.66	0.77	1.32		3.08	0.55	1.43	1.76	0.66	1.32	0.55	0.33
17	0.44	0.22	0.11	0.55	0.22	0.55	1.10	1.54	0.99	4.29	1.10	0.66	0.55	0.77	1.65	3.08		0.66	1.87	1.87	0.66	1.87	0.66	0.33
18	0.11	0.00	0.00	0.11	0.00	0.11	0.22	0.33	0.22	0.77	0.11	0.22	0.11	0.11	0.22	0.55	0.66		0.33	0.44	0.11	0.33	0.11	0.00
19	0.33	0.11	0.00	0.22	0.11	0.22	0.44	0.77	0.44	1.98	0.44	0.33	0.33	0.33	0.88	1.43	1.87	0.33		1.32	0.44	1.32	0.33	0.11
20	0.33	0.11	0.00	0.33	0.11	0.33	0.55	0.99	0.66	2.75	0.66	0.44	0.66	0.55	1.21	1.76	1.87	0.44	1.32		1.32	2.64	0.77	0.44
21	0.11	0.00	0.00	0.22	0.11	0.11	0.22	0.44	0.33	1.32	0.44	0.33	0.66	0.44	0.88	0.66	0.66	0.11	0.44	1.32		1.98	0.77	0.55
22	0.44	0.11	0.11	0.44	0.22	0.22	0.55	0.55	0.77	2.86	1.21	0.77	1.43	1.32	2.86	1.32	1.87	0.33	1.32	2.64	1.98		2.31	1.21
23	0.33	0.00	0.11	0.55	0.11	0.11	0.22	0.33	0.55	1.98	1.43	0.77	0.88	1.21	1.1	0.55	0.66	0.11	0.33	0.77	0.77	2.31		0.77
24	0.11	0.00	0.00	0.22	0.00	0.11	0.11	0.22	0.22	0.88	0.66	0.55	0.88	0.44	0.44	0.33	0.33	0.00	0.11	0.44	0.55	1.21	0.77	

Table 8 – The obtained results by different algorithms

Link	IOA	HJ	EDO	SA	SAB	AL	GP	CG	QNEW	PT	LMILP	This paper
y_{16}	4.6875	4.8	4.59	5.38	5.7392	5.5728	5.4277	4.7691	5.3052	5.0237	5.362	5.906
y_{17}	3.9063	1.2	1.52	2.26	5.7182	1.6343	5.3235	4.8605	5.0541	5.2158	2.057	2.502
y_{19}	1.2695	4.8	5.45	5.50	4.9591	5.6228	1.6825	3.0706	2.4415	1.8298	5.486	5.906
y_{20}	1.6599	0.8	2.33	2.01	4.9612	1.6443	1.6761	2.6836	2.5442	1.5747	1.895	2.502
y_{25}	2.3331	2.0	1.27	2.64	5.5066	3.1437	2.8361	2.8397	3.9328	2.7947	2.556	2.940
y_{26}	2.3438	2.6	2.33	2.47	5.5199	3.2837	2.7288	2.9754	4.0927	2.6639	2.618	2.940
y_{29}	5.5651	4.8	0.41	4.54	5.8024	7.6519	5.7501	5.6823	4.3454	6.1879	3.741	3.360
y_{39}	4.6862	4.4	4.59	4.45	5.5902	3.8035	4.9992	4.2726	5.2427	4.9624	4.551	4.955
y_{48}	5.4688	4.8	2.71	4.21	5.8439	7.3820	4.4308	4.4026	4.7686	4.0674	3.741	3.360
y_{74}	6.2500	4.4	2.71	4.67	5.8662	3.6935	4.3081	5.5183	4.0239	3.9199	4.489	4.955
T	77.516	76.324	80.068	75.632	73.401	74.623	77.536	76.051	76.534	77.748	75.973	75.030
B'	6.604	5.079	3.132	5.487	10.796	8.743	6.483	6.629	6.456	6.295	4.910	5.500
f	84.121	81.402	83.200	81.119	84.197	83.366	84.019	82.679	82.990	84.043	80.883	80.530
f'	87.34	81.25	83.47	80.87	84.21	81.752	82.57	82.53	83.07	82.57	-	-

Note: The lower bound of **y** *is set as 0; and, the upper bound of* **y** *is set as 25. f is the objective function values from the Ref. [21]. f' is the objective function value from reference. B' is the value of the investment on adding link capacity.*

5. CONCLUSION

In this paper, MPCC has been modelled to describe the CNDP by describing UE as NCP. We have substituted the complementarity constraint in MPCC by a smoothed equation by introducing parameterization Ψ. Therefore, MPCC is transformed into a well-behaved non-linear program (NLP), which can be solved by LINDOGLOBAL in GAMS because Ψ is differentiable if the scalar θ is non-zero but coincides with Φ when θ=0. The numerical experiments on the 16-link network show that our proposed algorithm can obtain good solutions under the lower and high-level demand cases compared to other algorithms. We have also demonstrated the results about the link capacity enhancement under different OD demand cases and the value of predetermined budget for total investment on link capacity expansions. The Sioux Falls network is also presented to further demonstrate the feasibility of our proposed algorithm.

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城市交通连续均衡网络设计问题的MPCC模型及其平滑化算法

摘要

城市交通连续均衡网络问题是在考虑道路出行者服从传统Wardrop用户均衡选择路径的基础上最小化交通网络的总出行时间和道路扩充成本的和。本文应用非线性互补问题描述用户平衡准则从而建立带有互补约束的数学规划模型(MPCC)描述城市道路交通连续均衡网络设计问题。MPCC中的互补约束使得标准非线性规划的约束规格在该问题的可行域上不再成立，所以求解MPCC难度很大。本文首先应用Fischer-Burmeister (FB)函数构造非平滑方程组以代替MPCC中的互补约束条件。然后引入参数来平滑化该FB 函数，从而得到的平滑的FB 函数。这样，MPCC 就可以转化为平滑的非线性规划问题。因而，我们采用GAMS中标准的非线性求解器LINDO-GLOBAL等求解松弛的非线性规划问题，得到原MPCC模型的最优解。最后我们用两个网络验证了本文的模型和算法。

关键词
城市交通网络； 连续交通网络设计问题； 带有互补约束的数学规划问题； 非线性互补问题； 用户均衡

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