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MATHEMATICAL MODEL AND STOCHASTIC MULTI-CRITERIA ACCEPTABILITY ANALYSIS FOR FACILITY LOCATION PROBLEM

ABSTRACT

This paper studies a real-life public sector facility location problem. The problem fundamentally originated from the idea of downsizing the number of service centres. However, opening of new facilities is also considered in case the current facilities fail to fulfil general management demands. Two operation research methodologies are used to solve the problem and the obtained results are compared. First, a mathematical programming model is introduced to determine where the new facilities will be located, and which districts get service from which facilities, as if there were currently no existing facilities. Second, the Stochastic Multi-criteria Acceptability Analysis-TRI (SMAA-TRI) method is used to select the best suitable places for service centres among the existing facilities. It is noted that the application of mathematical programming model and SMAA-TRI integration approach on facility location problem is the first study in literature. Compression of outcomes shows that mixed integer linear programming (MILP) model tries to open facilities in districts which are favoured by SMAA-TRI solution.

KEY WORDS

case study; facility location problem; mixed integer linear programming; Stochastic Multi-criteria Acceptability Analysis; public sector;

1. INTRODUCTION

Location-allocation problems seek for locating facilities at candidate sites and assign customers to opened facilities. In public location-allocation problems, all public service demand has to be fulfilled with limited funds. The aim should provide public access to these facilities efficiently and effectively. The access can be measured by travel distance or time to reach these facilities.

A real-life public sector facility location problem is considered in this paper. The problem fundamentally originated from the idea of downsizing the number of service centres. However, opening of new facilities is also considered in case the current facilities fail to fulfil general management demands. The problem analysed in the study takes place in the public sector. The study also sought to find the number of facilities to be located and the facilities that have service capacities. It is a static, single-stage, and a single-product problem that has deterministic input parameters. Given the information, the problem overlaps with the Capacitated Facility Location Problem (CFLP) in literature [15]. The CFLP consists of deciding which facilities to open from a given set of potential facility locations and how to allocate customers to those facilities [16]. If an arbitrary number of customers can be connected to a facility, the problem is called uncapacitated facility location problem (UFLP). If each facility has a limit on the number of customers it can serve, it becomes a capacitated facility location problem [17]. The significance of this work is the utilization of two operation research methodologies to solve a real-life problem. The study can be separated in two parts. In the first part a mathematical programming model is introduced to determine where the new facilities will be located, and which districts get service from which facilities as if there were currently no existing facilities. In the second part, the Stochastic Multi-criteria Acceptability Analysis-TRI (SMAA-TRI) is used to decide which facilities should stay open and which facilities should be closed. The compression of outcomes shows that MILP model tries to open the facilities in districts which are favoured by SMAA-TRI solution.

The rest of the paper is organized as follows. In the next section, an overview and a summary of the existing literature of facility location problem and solution approaches is provided. The third section presents the description of the two aforementioned operation research methodologies; Section 4 presents their applications and analyses of the results, and the last section comprises the conclusions about the study.

2. LITERATURE REVIEW

This section first presents a brief review on the most relevant and recent literature on facility location problems; followed by the same for SMAA-TRI method and integrated approaches.

2.1 Literature on Facility Location Problems

There has been an increasing number of studies about the facility location problems since the 17th century. But, it was Weber [1] who identified the first facility location problem. In this problem, he located a single facility to serve a finite set of demand points with the objective of minimizing the sum of the weighted distances from facility to demand points. Four

Table 1 – Categorization of facility location problems

Categorization Subject	Categorization Types	Explanation
Topology	Continuous	In continuous models, demands are distributed continuously across a service region and facilities can be located anywhere in that region.
	Network	In network models, there is a network composed of links and nodes. Demands on nodes and facilities can be located on nodes or links.
	Discrete	In discrete location models, there are demands arising on nodes and facilities can be located only on a set of candidate nodes.
Objective function	Minsum	Minsum models have the aim to minimize average/total distances.
	Minmax	Minmax models have the aim to minimize the maximum distances.
Solution method	Exact algorithms	Algorithms that try to find the optimal solution.
	Heuristics	Algorithms that search for an approximate solution.
Demand type	Single-product	The problem includes only one type of product.
	Multiple-product	There can be different product types.
Supply chain type	Single-stage	Single-stage models focus on service distribution systems with only one stage.
	Multi-stage	Multi-stage models consider the flow of service through several hierarchical levels.
Time horizon	Static	Static models optimize the problem deciding all variables simultaneously.
	Dynamic	Dynamic models consider different time periods with data variation across these periods, and give solutions for each time period adapting to the different conditions.
Input parameters	Deterministic	In deterministic models, the parameters are forecast with specific values and thus the problems are simplified for easy and quick solutions.
	Stochastic	Stochastic/probabilistic location models capture the complexity inherent in real-world problems through probability distributions of random variables or considering a set of possible future scenarios for the uncertain parameters.
Number of facilities	One	The purpose of the problem is locating only one facility.
	Certain	Number of facilities to be located is a certain number.
	Uncertain	Number of facilities to be located is uncertain. Problem also searches for the number of facilities.
Facility type	Desirable	Closeness of facility (such as hospital) to demand center is better.
	Undesirable	People want these facilities (such as nuclear reactor) far from demand centers.
Sector type	Private	It seeks for maximizing profit while locating facilities.
	Public	Optimization of the population's access is the priority.

components play the role of characterizing the location problems. They are customers with demands, facilities to be located, a space in which customers and facilities are located, and finally metric that indicates distances or times between customers and facilities [2]. Common types of location problems in literature are median, covering, centre, and hub-location problems. The p-median problem was first introduced by Hakimi [3]. His goal was to find the minimum weighed distance location of p facilities on a demand of n nodes. He also proved that in a network at least one node is optimal to locate a facility. He called the property 'node optimality'. By the help of property, search for the optimal solution set is highly reduced. It is proven by Kariv and Hakimi [4] that the general p-median problem is NP-Hard. There were several solution approaches in literature. Some of these solution approaches were introduced in a solution survey for p-median problems [5]. Lim et al. [6] introduced some improvements for these solution approaches. Starting from interchange algorithm by Teitz and Bart [7], in a recent paper, the solution methods were mentioned and a new one was introduced [8]. Hakimi [9] again introduced the covering problem to allocate police stations to the nodes. In covering the problem each customer can be served by each facility in the coverage distance or time. The first MILP model in covering problems was developed by Toregas et al. [10]. Later, Church and ReVelle [11] formulated the maximum covering problem. Several types of covering problems take place in literature (set covering, partial covering, etc.). The p-centre problem consists of locating p facilities and assigning clients to them in order to minimize the maximum distance between a client and the facility to which they are allocated [12]. Hub location problems aim to locate hubs between arriving and departure points of people, service, information, etc. These hubs are applied to decrease the number of transportation links between origin and destination nodes [13]. Facility location models can be categorized broadly by their topography, objectives, demand allocation types, number of product types, solution methods, features of facilities, supply chain type, time horizon, and input parameters [14]. *Table 1* shows categorization of facility location problems based on different aspects.

2.2 Literature on SMAA-TRI Method

After the introduction of SMAA by Lahdelma et al. [47], some extensions of SMAA are developed. SMAA-3 was introduced and an application was demonstrated by Hokkanen et al. [48]. Instead of choosing the best alternative, a ranking method was introduced with SMAA-2 by Lahdelma and Salminen [49]. Several applications of facility location handled with the method. SMAA-O extends SMAA-2 to consider ordinal criteria measurements [50]. Ref-SMAA or SMAA-A generates random reference points from the reference point space and evaluates the decision alternatives based on an achievement function [51]. SMAA-TRI was developed for the parameter stability analysis of ELECTRE TRI and extends ELECTRE TRI to allow ignorance on the parameter values [52]. SMAA-D applies efficiency measures of data envelopment analysis, instead of the utility function [53]. One can find some SMAA methods used in location problems. In their paper Hokkanen et al. [54] used SMAA method to choose a location for Helsinki cargo harbour. They had 11 criteria for the problem and most of them were related to nature protection. Lahdelma et al. [55] considered to locate a waste treatment facility by using SMAA-O method. Seventeen criteria with ordinal measurement values took place in their study. Menou et al. [56] used SMAA-O method for centralizing cargo at a Moroccan airport hub with six criteria.

2.3 Literature on solution approaches for CFLP

Most research on the CFLP has focused on the development of efficient solution algorithms [18]. Kuehn and Hamburger [19] present one of the earliest models and a heuristic procedure for the CFLP. Davis and Ray [20] used Benders' decomposition to solve CFLP. Akinc and Khumawala [21] developed a branch and bound algorithm by using linear programming relaxation. Jacobsen [22] introduced a variant of CFLP and some heuristics for the problem. The cross-decomposition algorithm of Van Roy [23] and the Lagrangean-based approach of Beasley [24] are among the most effective techniques that were subsequently devised for solving the CFLP. Magnanti and Wong [25] provide an overview of solution methods for CFLP. Delmaire et al. [26] deal with the capacitated single-source plant location problem and describe four heuristic algorithms including GRASP and a taboo search for the problem. Agar and Salhi [27] represented performance of Lagrangean heuristics applied to a variety of CFLPs. Hindi and Pienkosz [28] discuss a large-scale and a single-source CFLP in their paper. Hinojosa et al. [29] consider a two-echelon problem in which plants, warehouses, and customers form a system. To minimize the inventory holding and production set-up costs, Alfieri et al. [30] investigated the capacitated lot-sizing problem. Baldacci et al. [31] formulated the capacitated p-median problem as a set-partitioning problem. Diaz and Fernandez [32] consider a version of CFLP, in which the objective function is to minimize construction and transportation costs. In a study, Ghiani et al. [33] consider a CFLP where they can locate multiple facilities at the same site. Recently a taboo search [34], a multi-exchange heuristic [35], a scatter search algorithm [36], and a hybrid algorithm combining Lagrangean heuristic and ant colony system [37] were proposed to solve the single-source CFLP.

One can find a hybrid algorithm to solve CFLP and a comparison with Benders' decomposition algorithm in Lai et al. [38]. Multi-criteria decision analysis (MCDA) methods have also been used to solve location problems. These problems deal with the selection of candidate sites for locating certain facilities so that they optimally fulfil the needs of the users, taking into account some criteria. There are several applications of the MCDA for facility location problems in literature [39, 40, 41, 42]. A comparison of MCDA methodologies for a location problem takes place in a study [43]. One can see / observe several applications that use a single MILP model or an MCDA method in literature but it is the first time SMAA and CFLP are used together in the same facility location problem.

3. METHODOLOGIES

Two operation research methodologies are used to solve a real life facility location problem in this study. These methods are briefly mentioned in the following sub-sections.

3.1 Capacitated Facility Location Problem

CFLP can be illustrated starting from *p*-median problem. *P*-median problem is concerned with allocation of *p* facilities to candidate sites. The objective is the minimization of the average travel distance of customers. For this purpose, we can introduce the following variables and parameters:

I – set of nodes;

J – set of candidate sites to locate facilities;

w_i – demand of node i;

d_{ij} – distance from node i to candidate site j;

x_j – if a facility is located at candidate site j, it is 1 and otherwise 0;

y_{ij} – if demand node i gets service from facility located at j, it is 1 and otherwise 0.

With the notation above the *p*-median problem which is an MILP model is formulated as follows [44]:

$$\min \sum_{j\in J} \sum_{j\in J} w_i d_{ij} y_{ij} \quad (1)$$

$$\text{s.t.} \sum_{j\in J} Y_{ij} = 1 \qquad \forall i \in I \quad (2)$$

$$y_{ij} - x_j \leq 0 \qquad \forall i \in I, \forall j \in J \quad (3)$$

$$\sum_{j\in J} x_j = p \quad (4)$$

$$x_j \in \{0,1\} \qquad \forall j \in J \quad (5)$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \quad (6)$$

Objective Function 1 minimizes the demand-weighed total distance. *Constraint 2* ensures that each demand is assigned to a facility. *Constraint 3* demand node is assigned to an open facility. *Constraint 4* allows *p* facilities to be opened. *Constraints 5* and *6* are integrity constraints on variables. The proposed model assumes that the costs of all the facility locations at each candidate site are the same. Also the number of facilities is constant. Accordingly, UFLP emerged to overcome the mentioned assumption and make it a more realistic problem. Erlenkotter [45] presents the following formulation of UFLP:

$$\min \sum_{j\in J} f_j x_j + \alpha \sum_{j\in J} w_i d_{ij} y_{ij} \quad (7)$$

s.t.: *Constraints 2, 3, 5, 6*.

In the formulation, f_i stands for the fixed costs of opening facilities. *Objective Function 7* minimizes the total fixed and variable costs. Again, *Constraint 2* ensures that all the demand is assigned, whereas they are assigned to opened facilities as required by *Constraint 3*. UFLP assumes that the facilities have limitless service capacity unlike real world situations. With a capacity constraint the problem seems to be more realistic. This type of UFLP is called the CFLP. Let a_i, $i \in I$, be the customer's demands and b_j the facility's capacity. Then the CFLP is formulated as:

$$\min \sum_{j\in J} f_j x_j + \sum_{j\in J} \sum_{i\in I} C_{ij} Y_{ij} \quad (8)$$

s.t.: 2, 5, 6 and Constraint 9.

$$\sum_{i\in I} a_i y_{ij} - b_j x_j \leq 0 \qquad \forall i \in I, \forall j \in J \quad (9)$$

Constraint 9 provides the opened facilities service at most their capacity. It is noted that the models expressed from *1* to *9* refer to MILP, since two kinds of variables are involved, integer based, and real number based (costs, distances).

3.2 Stochastic Multi-criteria Acceptability Analysis (SMAA)

A variety of MCDA methods need criterion measurements from decision-makers. It is not always possible to get all these information, especially in public political problems. Besides, they require preference information for describing the relative importance of different criteria. One of the advantages of SMAA over most other MCDA methods is that it can be used without any preference information [46]. SMAA has been proposed for problems which criteria values and weights are not correctly known. Stochastic variables ξ_{ij} with accepted or estimated joint probability distribution and density function $f(\xi)$ in space X show uncertain or guessed criteria values. In addition, decision-makers' unknown or partly known preferences are symbolized by a weight distribution with density function $f(w)$ in the set of feasible weights W defined as

$$W = \left\{ w \in R^n : w \geq 0 \; and \sum\nolimits_j w_j = 1 \right\} \quad (10)$$

The utility function is then used to map stochastic criteria and weight distributions into utility distributions ($u(\xi_i)$,w). The SMAA method concludes for each alternative the set of *favourable weights* $W_i(\xi)$ described as

$$W_i(\xi) = \{ w \in W : u(\xi_i, w) \geq u(\xi_{k,w}), \forall_k \} \quad (11)$$

The properties of these sets direct all additional analyses. SMAA uses three main measurements [49]:

- The first measure is the *acceptability index* a_i which is described as the expected (n-1) dimensional volume of the approvable weights. It is figured out as a multidimensional integral over the criteria distributions ξ and approvable weight space as

$$a_i = \int_X f(\xi) \int_{W_i(\xi)} f(w) dw d\xi \quad (12)$$

The acceptability index can be used for labelling the alternatives into more or less acceptable ones (a>0), and those which are not acceptable (a_i zero or near zero). For the case with two criteria, deterministic values and a linear utility function, the approvable weights and acceptability indices can be clarified graphically as in *Figure 1*.

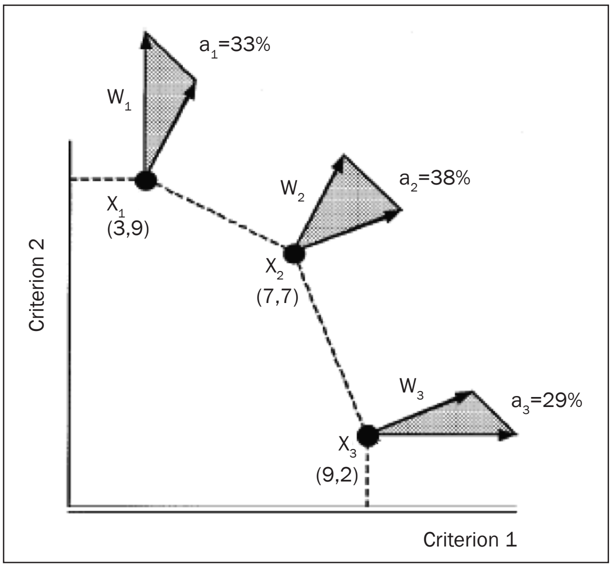

Figure 1 – Approvable weights and acceptability indices

- The central weight vector w_i^c is described as the expected centre of gravity of the approvable weight space. The central weight vector is figured out as an integral of the weight vector over the criteria and weight distributions by

$$w_i^c = \int_X f(\xi) \int_{W_i(\xi)} f(w) w dw d\xi / a_i \quad (13)$$

DMs can find out how different weights correspond to different choices by using the central weights.

- The *confidence factor* P_i^c is described as the probability for the alternative to be the picked one if the central weight vector is elected. The confidence factor is figured out as an integral over the criteria distributions ξ by

$$p_i^c = \int_{\xi : u(\xi i, w_i^c) \geq u(\xi_{k}, w_i^c)} f(\xi) d\xi \quad (14)$$

The confidence factor measures whether the criteria data are correct enough to determine the alternatives when the central weight vector is used. More detailed information about the SMAA can be easily obtained from the papers [47, 49]. The decision makers in public political decision situation prefer methods that do not require them to express their preferences explicitly, but rather describe their consequences in an appropriate form, in order to allow the final decision to be made by themselves and SMAA has been developed to support decision makers in public environmental multicriteria decision-making [47]. There are three main MCDA problem statements: choosing, ranking and sorting [57]. *Table 2* shows the decision tree to choose the SMAA method for a specific problem. SMAA-TRI is the first SMAA method for the sorting problem statement. Dividing alternatives into groups are among favourite research fields in various disciplines. The SMAA-TRI is one of the useful methods for sorting, because it does not need weight elicitation [58]. The method is an improved form of ELECTRE-TRI to use stochastic values.

4. APPLICATIONS

The topic of the study is a real life facility location problem in public sector institution. Because of the restriction of the institution, city and region names are not mentioned, instead they are coded by D_1,...D_{55}. The institution currently has 24 service facilities in a region. That region has 55 districts (D_1-D_{55}) and 6 of them are central districts (D_1, D_{12}, D_{22}, D_{32}, D_{42}, and D_{55}). The cities which are coded by "D_1, D_{12}, D_{22}, D_{32}, D_{42}, D_{52}" are central districts as mentioned in the study.

A district which has not got a facility gets service from another facility in that region. General management desires to reduce the number of facilities. If necessary, additional facilities can be opened but the first aim is to close some facilities. Also, after the new system is established, they want to make the new allocations of districts to the facilities. The facilities in the system are of five different types with different capacities. The management demands a new system, because the existing system no longer provides an adequate and cost-effective service. The reasons for this change can be listed as follows:

- Change of population within the districts due to migration. Some facilities have enormous work to do, while others have residual capacities.
- Change in the transportation technology and transportation ways.

Table 2 – Decision-tree to choose the SMAA variant [46]

Problem Statement?	Ranking			Sorting
Reference model?	Weight of scaling factor		Reference points	
Aggregating procedure?	Utility or value function	Outranking procedure	Achievement function	
Method	SMAA-2	SMAA-3	Ref SMAA	SMAA TRI
Descriptive measures	- Cross confidence factors - Rank acceptability indices - Acceptability indices - Central weight vectors	- Acceptability indices - Central weight vectors	- Central reference points - Reference acceptability indices	Category acceptability indices
Aggregate measures		- Holistic acceptability indices - kbr acceptability indices		

- Downsizing lets the management use the remaining workers in other works.
- Closed facility buildings will be used by other public institutions.
- Savings from the reduced facility expenditures and worker fees.

Two operation research methodologies are used to solve the problem. First, a mathematical programming model is introduced to determine where the new facilities will be located, and which districts get service from which facilities as if there are currently no existing facilities. Second, the SMAA-TRI method is used to determine which facilities should stay open and which facilities should be closed.

4.1 MILP model application

The work uses the CFLP model in literature with some problem-specific constraints. General management has some demands for establishing the new system. These demands can be listed as follows:

- There must be facilities in central districts.
- The location of the facility should have enough security and development level.
- For the service policy, citizens should not travel more than 90 km to reach a facility.

To solve the location-allocation problem proposed integer-programming model is illustrated below.

Sets:

N - set of districts to get service and set of sites to locate facilities;

N_1 - set of districts that has facility;

N_2 - set of districts that does not have facility;

N_c - set of central districts (D_1, D_{12}, D_{22}, D_{32}, D_{42}, D_{52});

N_s - set of districts which has required security level (the districts other than "D3, D6, D25, D44");

N_d - set of districts which has required development level (the districts other than "D_2, D_7, D_{31}, D_{41}, D_{54}");

T - set of facility types (facilities are grouped into different classes based on the number of workers. While there are 25-35 workers in a type A facility, 3-5 workers serve in a type E facility. The other numbers for each type of facility are: B=18; C=10-12; D=5-8)

Input parameters:

W_i - population of district i;

C_k - capacity of k type facility;

M_k- annual operating cost of k type facility;

F_k - fixed cost of opening k type facility;

d_{ij} - distance from district i to site j in km.

Constants:

c - travel cost per km;

δ - specific distance in accordance with the request.

Decision variables:

$$x_{ij} = \begin{cases} 1 \; if\ district\ i\ gets\ service\ from\ facility\ j, \\ 0 \; otherwise \end{cases}$$

$$y_{ij} = \begin{cases} 1 \; if\ k\ type\ of\ facility\ opened\ at\ node\ j, \\ 0 \; otherwise \end{cases}$$

Objective function:

$$\min = \sum_{i \in N} \sum_{j \in N} W_i d_{ij} x_{ij} c + \\ + \sum_{j \in N_2} \sum_{k \in T} F_k y_{jk} + \sum_{j \in N} \sum_{k \in T} M_k y_{jk} \quad (15)$$

Objective function minimizes the weighted travel distance of customers to facilities, fixed facility opening costs, and operating costs of facilities for a certain period. Fixed facility opening costs for currently existing facilities are taken as zero in calculation.

Constraints:

s.t.

$$\sum_{j \in N} x_{ij} = 1 \qquad \forall i \qquad (16)$$

$$\sum_{k \in T} y_{ik} \leq 1 \qquad \forall j \qquad (17)$$

$$\sum_{i \in N} x_{ij} \leq n \sum_{k \in T} y_{jk} \qquad \forall j \qquad (18)$$

$$\sum_{k \in T} y_{jk} c_k \geq \sum_{i \in N} x_{ij} w_i \qquad \forall j \qquad (19)$$

$$x_{ij} d_{ij} \leq \delta \qquad \forall i,j \qquad (20)$$

$$\sum_{j \in N_c} \sum_{k \in T} Y_{jk} \geq n \qquad (21)$$

$$\sum_{k \in T} y_{jk} = 0 \qquad \forall j \notin N_s \qquad (22)$$

$$\sum_{k \in T} y_{jk} = 0 \qquad \forall j \notin N_d \qquad (23)$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in N, \forall j \in N \qquad (24)$$

$$y_{jk} \in \{0,1\} \qquad \forall j \in N, \forall k \in T \qquad (25)$$

We guarantee all districts to be assigned a facility with *Constraint 16*, and there will be only one kind of facility, if opened in the candidate site by *Constraint 17*. To be in accordance with first request, *Constraint 21* is added to the model. This constraint ensures the locating at least one kind of facility in those central districts. General management divided districts into 5 security level based on management's subjective assessment. There is no desire to locate a facility in a district which is in the lowest security level. *Constraint 22* is added to the model in order to avoid opening facilities at districts which has not enough security level. General management's requests about district development level are reflected to the model with the data provided by Ministry of Development [60]. Ministry of Development calculated all the districts' development indexes and put them into 6 development group. A part of the work is shown in *Table 3*. As it can be seen from the *Table 3*, D1 is more developed district than D2 with the smaller development order and higher development index. Under development boundaries are represented by the minus numbers in *Table 3*.

Constraint 23 provides opening facilities at districts which have sufficient development level. *Constraint 20* ensures the third demand of general management. A sample of distance matrix which shows distance between districts is shown in *Table 4*.

Constraint 18 links x_{ij} and y_{ij} variables. They state that if there is an open facility it can serve districts; otherwise, no districts can be assigned to it. The capacity *Constraint 19* prevents facilities from serving population more than they can. The capacities are found by calculating a worker's daily capacity of service and former experiences. Finally, *Constraints 24* and *25* are integrity constraints on variables. The proposed model with 55-node locations has 3,300 discrete variables and 3,190 constraints. GAMS IDE 2.0.34.19 program is used to solve the problem and find an exact solution. The comparison of CFLP solution and the existing facility settlement is shown in *Table 5*. The CFLP solution indicates that 11 currently existing facilities should be closed (D_2, D_7, D_8, D_{10}, D_{11}, D_{24}, D_{38}, D_{41}, D_{49}, D_{50}, D_{51}) and 3 new facilities should be opened (D_{14}, D_{18}, D_{55}). According to the results, the management can reduce the number of facilities by 8.

Table 3 – District development matrix

District	Development order	Development group	Development index
D_1	145	2	0.75922
D_2	827	6	-1.17754
...	...	...	...
D_{54}	821	6	-1.15538
D_{55}	763	6	-0.83619

Table 4 – Distance matrix between districts

District	D_1	D_2	D_3	...	D_{53}	D_{54}	D_{55}
D_1	-	61	36	...	201	241	215
D_2		-	64	...	152	189	209
D_3			-	...	251	240	220
...				-	...	...	...
D_{53}					-	181	61
D_{54}						-	121
D_{55}							-

4.2 SMAA-TRI Application

In this section SMAA-TRI is used to decide which facilities should stay open and which facilities should be closed based on the criteria determined by the general management. The reasons for choosing SMAA-TRI as a solution method are as follows:

- Some criteria measurements are imprecise.
- As it is a public political decision there are several decision-makers. So, it is difficult to get exact preference information from them.
- The weights of criteria are imprecise. Only the hierarchy of criteria importance is provided by decision-makers.
- This is a sorting process of facilities into two groups (stay open or closed).

General management demands that each central district has a facility. Currently, 6 of 24 facilities stand in the central districts (D_1, D_{12}, D_{22}, D_{32}, D_{42}, and D_{52}). So, the selection problem can be reduced to decide which of the 18 non-central facilities should stay open. The following criteria are specified by the general management (*Table 6*).

General management provides the order of importance of these criteria, but not preference informa-

Table 5 – Comparison of CFLP solution and existing facility settlement

Existing facilities	Facility Types					Model proposal	Facility Types				
	A	B	C	D	E		A	B	C	D	E
D_1	x					D_1			x		
D_2				x		D_9					x
D_7			x			D_{12}		x			
D_8				x		D_{14}					x
D_9		x				D_{18}					x
D_{10}				x		D_{21}				x	
D_{11}			x			D_{22}				x	
D_{12}	x					D_{23}					x
D_{21}		x				D_{31}					x
D_{22}		x				D_{32}				x	
D_{23}			x			D_{39}					x
D_{24}			x			D_{40}				x	
D_{31}				x		D_{42}		x			
D_{32}		x				D_{43}				x	
D_{38}			x			D_{52}				x	
D_{39}			x			D_{55}					x
D_{40}		x				Total	0	2	1	6	7
D_{41}			x								
D_{42}	x										
D_{43}		x									
D_{49}			x								
D_{50}				x							
D_{51}			x								
D_{52}			x								
Total	3	6	10	5	0						

Table 6 – Criteria for SMAA-TRI

Criteria	Explanation
C1-Population of district (Number)	The criterion shows us the number of people getting service from that facility. Of course, districts, which have dense population, have more chance to stay open.
C2-Distance to central district (Distance)	Town centers are sure to be opened. So the criterion reflects the distance between facility location and central district. The desire of general management is to close facilities which are near to the central districts.
C3-Operating cost (Cost)	The criterion shows the annual operating cost of the facility.
C4-Development level (Development)	The criterion shows the development level of districts. For general management, the development level of facilities which will remain open needs to be sufficiently higher.
C5-Security level (Security)	The security levels of all districts in the region are evaluated by the general management according to their experiences. General management desires that the facilities which will remain open should be secure enough.
C6-Building condition (Building)	The criterion shows the condition of buildings.

tion. Without the precise weight information of criteria ordinal hierarchy for them is provided by the general management. The order of criteria is shown as follows: $C_1 \geq C_2 \geq C_6 \geq C_5 \geq C_4 \geq C_3$. With these criteria the facilities are categorized into two groups. The first group shows the facilities which will remain open, while the second one forms the group of facilities to be closed. Tervonen [59] provided JSMAA program which is an open source implementation in Java for the solution of SMAA-2 and SMAA-TRI problems. In this study, JSMAA program is used for the SMAA-TRI application. In the study, eighteen alternatives, six criteria and two categories are described, respectively. Preferences can be chosen as missing, ordinal or cardinal. "Ordinal" has been chosen and criteria are ranked 1 to 6 by DMs. Measurement values for alternatives could be described as exact, interval, Gaussian, lognormal, logitnormal, beta and discrete. Researchers can find more information for the JSMAA at the website (http://smaa.fi/jsmaa/). JSMAA is used to group alternative as suitable enough to remain open or not. The output of the program reveals the category acceptability indices of all alternatives. These indices show how much an alternative fits that category. *Table 7* shows the category acceptability indices of alternatives. The method proposes to close eight facilities. Those are D_2, D_7, D_8, D_{10}, D_{11}, D_{24}, D_{31}, and D_{41}. According to category acceptability indices in *Table 7*; D_9, D_{21}, D_{23}, D_{38}, D_{39}, D_{40}, D_{43}, D_{49}, D_{50} and D_{51} should stay open.

Table 7 – Results of SMAA-TRI

	Category acceptability indices	
	CLOSE	REMAIN OPEN
D_2	0.55	0.45
D_7	0.98	0.02
D_8	0.58	0.42
D_9	0	1
D_{10}	0.84	0.16
D_{11}	0.54	0.46
D_{21}	0	1
D_{23}	0.14	0.86
D_{24}	0.54	0.46
D_{31}	0.98	0.02
D_{38}	0	1
D_{39}	0.08	0.92
D_{40}	0	1
D_{41}	0.9	0.1
D_{43}	0	1
D_{49}	0.13	0.87
D_{50}	0.13	0.87
D_{51}	0.03	0.97

Table 8 illustrates the results of two methodologies. MILP model proposes the closing of eleven existing facilities and opening of three new facilities. SMAA-TRI

Table 8 – Models comparison

MILP MODEL			SMAA-TRI		BOTH
MILP model proposes to open	*MILP* model proposes to close	*MILP* model proposes to remain open	*SMAA-TRI* proposes to close	*SMAA-TRI* proposes to remain open	Both models propose to close
D_{14}	D_2	D_1	D_2	D_1	D_2
D_{18}	D_7	D_9	D_7	D_9	D_7
D_{55}	D_8	D_{12}	D_8	D_{12}	D_8
	D_{10}	D_{21}	D_{10}	D_{21}	D_{10}
	D_{11}	D_{22}	D_{11}	D_{22}	D_{11}
	D_{24}	D_{23}	D_{24}	D_{23}	D_{24}
	D_{38}	D_{31}	D_{31}	D_{32}	D_{41}
	D_{41}	D_{32}	D_{41}	D_{38}	
	D_{49}	D_{39}		D_{39}	
	D_{50}	D_{40}		D_{40}	
	D_{51}	D_{42}		D_{42}	
		D_{43}		D_{43}	
		D_{52}		D_{49}	
				D_{50}	
				D_{51}	
				D_{52}	

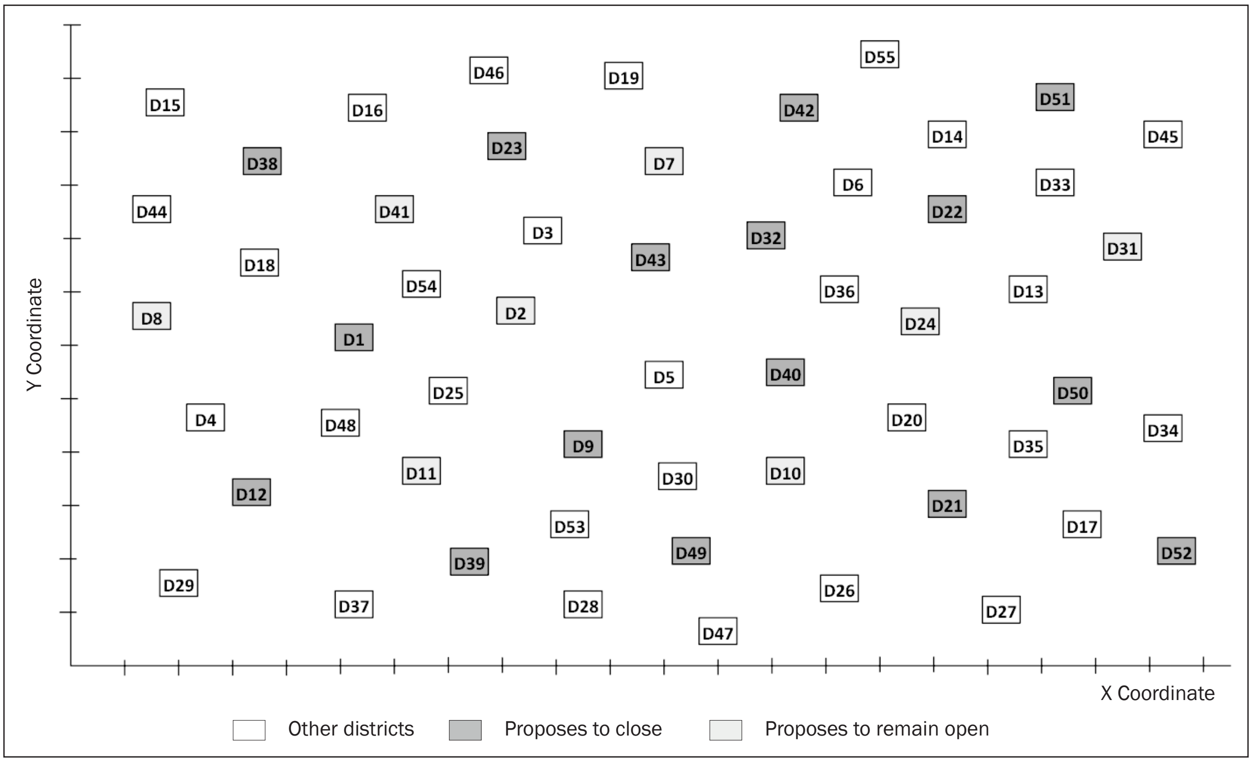

Figure 2 – Results of SMAA-TRI

solution offers to close eight existing facilities. The results of two applications agree on closing seven same facilities. Comparison of outcomes shows that SMAA-TRI yields very close solution to optima (*Figure 2*).

In the first part of the application, a mathematical model with 55-node locations has 3,300 discrete variables and 3,190 constraints are built for the problem and GAMS IDE 2.0.34.19 program is used to solve it. The MILP model cannot represent imprecise criteria measurements and weights. So, it is not capable to reflect subjective assessments. In this study SMAA-TRI method is used to overcome the challenge of imprecise evaluation.

The main contribution of this study is proposing two operation research methodologies, namely MILP and SMAA-TRI to eliminate the weaknesses of each other while solving a real life problem. It is considered that in this way more reliable solution alternatives can be proposed to decision makers. SMAA-TRI model which yields very similar results to optima can be easily built by other researchers. In addition, a hybrid MILP-SMAA-TRI model is the first built solving facility location problem.

5. CONCLUSION

Facility location is an important problem due to high cost of unfavourable site selection. This problem is especially troublesome in public sector because of political and several other reasons. Including many kinds of uncertainties in the process it is really difficult to make an unfailing decision. In these circumstances, there is always a feeling of some deficiency if the solution is based on a single approach. In order to overcome this, more than one methodology should be used to solve real-life public sector facility location problems.

In this study two eligible approaches have been used to solve a real-life public sector facility location problem. First, a mathematical programming model is introduced to determine where the new facilities will be located, and which districts get service from which facilities as if there are currently no existing facilities. Second, the SMAA-TRI method is used to select the best suitable places for service centres among the existing facilities. Compression of outcomes shows that MILP model tries to open facilities in districts which are favoured by the SMAA-TRI solution. The use of an SMAA-TRI approach allows for the explicit incorporation of uncertainty parameters in the model. An appealing characteristic of the outranking model applied in SMAA-TRI is that it allows the veto effect to be modelled, meaning that a poor performance in one criterion cannot be compensated for by good performance in other criteria.

To the best knowledge of the authors, this paper is the first study which applies mathematical programming model and SMAA-TRI integration on facility location problem. For future studies, (i) application of ELECTRE-TRI method, and (ii) meta-heuristic approaches for MILP model should be suggested.

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MATEMATIKSEL MODEL VE STOKASTIK ÇOK KRITERLI KABUL EDILEBILIRLIK ANALIZI ILE BIR KAMU KURUMU IÇIN TESIS YERI SEÇIMI

ÖZET

Bu çalışmada bir kamu kurumuna ait gerçek bir tesis yeri seçimi problemi ele alınmıştır. İlk olarak sistemde il merkezlerindeki tesisler dışında hiç bir tesis yokmuş gibi yeni tesislerin nerelere yerleştirilmesi gerektiği ve hangi ilçelerin hangi tesislerden hizmet alacağı bir matematiksel programlama modeli ile belirlenmeye çalışılmıştır. Daha sonra mevcut tesisler arasında, kamu kurumunun sahip olmasını istediği niteliklerden en fazlasına sahip olanlar Stokastik Çok Kriterli Kabul Edilebilirlik Analizi-TRI (SMAA-TRI) metodu ile tespit edilmiştir. Elde edilen sonuçlar karşılaştırıldığında matematiksel modelin açılmasını önerdiği tesislerin SMAA-TRI çözümünde de tercih edilen tesisler olduğu görülmüştür.

ANAHTAR KELIMELER

Vaka Analizi; Tesis Yerleşim Problemi; Karma Tamsayılı Doğrusal Programlama; Stokastik Çok Kriterli Kabul Edilebilirlik Analizi; Kamu Sektörü;

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