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EXPRESS SERVICES MARKET ANALYSIS BASED ON THE LOTKA-VOLTERRA MODEL – CASE STUDY SERBIA

ABSTRACT

This paper provides a preview of the former stages through which the market of express postal services had gone and the possibilities of further development, both on the global and local level. The aim of this paper is to complete an estimation of the need for this type of express services using the competitive Lotka–Volterra model in Serbia. In order to reduce the complexity of the process, the division of competition was conducted in two segments: the public operator and the private segment (comprised of all private operators). The given model provides a description of a dynamic competition relationship by indicating the existence of the equilibrium point between the public and the private sectors, and the conditions of its stability. The obtained values indicate that the private sector affects the public operator. The existing predator-prey relationship gives preference to the private sector and can be described by the Lotka-Volterra model.

KEY WORDS

express services; Lotka–Volterra model; competition; equilibrium;

1. INTRODUCTION

The examination of the postal services market started in the 1970s. There was a prevailing attitude that the only task of a postal operator is to exercise its activities taking into account only the statutory obligations. A significant decline in performed postal services caused by the general economic recession which happened in that time, the development of telecommunications, as well as the emergence of a strong direct competition imposed a new task to public operators - profitable business. Basic types of institutional changes that all states accepted which indirectly encouraged the change in behaviour of the public postal operators are: separation of postal services from telecommunication services (United Kingdom in 1981, Germany in 1989, Poland in 1991, New Zealand in 1993, Serbia in 1997, the Netherlands in 1998, Austria in 1999, etc.) and the liberalization on the market of the postal services [1].

The growing needs of companies for the time-defined, guaranteed delivery which the public postal operators were unable to offer, led to the emergence of private express operators. The beginnings of express industry are linked to the end of the 1960s in the United States and the rapid growth that occurred in the late 1970s following the deregulation of air cargo transportation in the United States. Europe was first introduced with this type of services in the mid-1980s [2]. Express delivery industry evolved from the delivery of documents and packages to special items such as electronic components, fashion accessories, pharmaceuticals, etc. (*Figure 1*). The features of items which were delivered by express service are high value and low weight. The core essence of this type of express

industry is that it provides transportation from door to door, timed delivery, and other (tracking, overnight delivery, insurance, etc.). Express operators constitute an important component of international supply chains, and help improve the competitiveness of firms, especially in the light of the growing importance of just-in-time manufacturing, outsourcing, and online sales [3]. Express industry has also provided an opportunity for small and medium-sized enterprises to compete on the global market, allowing them access to the international distribution network.
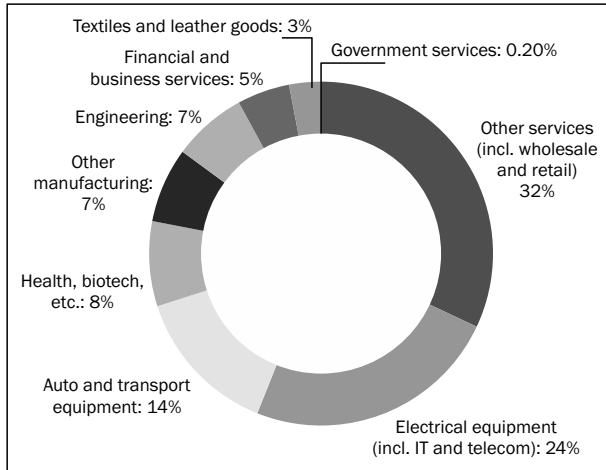

Figure 1 - Share of different sectors in the use of express services [4]

Due to the liberalization and ICT services, the decline in the volume of traditional postal services provided by public postal operators (*Figure 2*) led to the expansion of the range of offered services in order to improve their financial position. This consequently has extended the area of competition between postal and express operators. Many countries have adopted new developments on the postal market, and made such efforts in order for their postal operators to become more competitive through corporatization or privatization. Some countries, especially the members of the European Union (EU), decided to fully liberalize their postal markets by eliminating monopoly of national operators (reserved area) [5-7]. Thus, their market is made fully available to competitors in order to reduce the market impact of the previous monopolists and achieve their business more efficiently.
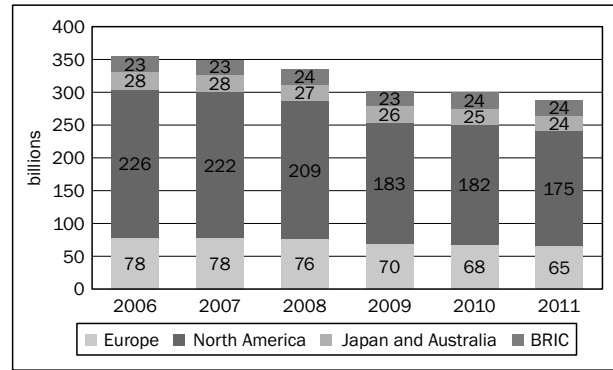

Figure 2 - Volume of letter post items in the period: 2006 - 2011 [8]

Nowadays, within the express delivery industry, operators acting on a global, regional and national level can be distinguished. Certainly, the biggest operators are DHL, FedEx, UPS and TNT, which are often referred to as "integrators" because they provide timed door-to-door services, while maintaining control through all the stages of the transfer process (e.g. the ability to change the destination in transit where the shipments are monitored at each step of delivery) [9]. According to WTO estimations [3], these four operators cover 55% of the global market of express items. It is estimated that in the EU these operators cover 41% of the market. In Europe, most postal operators provide their services on a national market of express services (*Table 1*). Industry of express items in the EU27 in 2010 generated revenues of 37.8 billion euros [4].

Express services industry can contribute to regional development, as it provides the possibility of placing production that is influenced by the rapid delivery in areas that are not geographically close to their markets, which is of great importance for the developing countries.

2. THE MARKET OF EXPRESS POSTAL SERVICES IN SERBIA

Market development in the field of express items in Serbia has started in 2002 when the national operator started to provide express services (emergence of Post Express), even though other operators were present, such as DHL (since 1989), TNT (1997), UPS (1998), and a number of local operators. Today's mar-

Table 1 - Markets of national postal operators [10]

Regional scale	European scale	Worldwide scale
Austrian post (Trans-o-flex)	La Poste (DPD/Geopost)	Deutsche Post (DHL)
PostNord AB (Scandinavian region)	Royal Mail (GLS)	TNT
Posten Norge (Scandinavian region)		
Itella (Scandinavian and Baltic region)		
CTT Correios (Tourline Express, Spain)		

ket consists of 48 operators, which are divided into three groups [11]:

- forty-three registered operators, exclusively for collection, processing, transportation and delivery of express items, including the global private operators FedEx (representative-Flying cargo YU), TNT (representative Tim kop-TNT) and Express courier - UPS (within the group 16 operators conduct their business on a national level, and 6 operators operate at international level as well);
- two operators registered for the service of collection, processing, transportation and delivery, both of express postal items and packages weighing more than 10 kg (New Balkan Trans operates on a national level, and the City delivery covers the cities of Belgrade, Novi Sad and Bačka Palanka)
- three operators that in addition to the previously mentioned registered services also provide the collection, processing and transportation of international packages from Serbia, weighing more than 10 kg, as well as the service of delivery of international packages weighing more than 20 kg on arrival (DHL International, YU PD EXPRESS-D Express and Post Serbia (Public Enterprise Post Serbia)).

In terms of ownership, the operators providing express services in Serbia can be divided into public (Post Serbia) and private (other) ones. The public postal operator has achieved tremendous growth in this segment of services as the number of express items in the period from 2003 - 2012 increased more than 20 times (*Figure 3*). The share of these services in the total number of services provided by the public operators ranged from 0.6% to 0.8%, while the share in total income is from 3.5% to 4.5%. (*Figure 4*) The scope of services of the private sector has been available since 2010, when a regulatory body for postal services market was established (Republic Agency for Postal Services - RAPUS). The participation of private operators in the express shipments market from the perspective of provided service ranged from 71.8% in 2010, 67.4% in 2011 and 68.3% in 2012. In terms of revenues, it is estimated that the private sector accounts for about 80% of the market, whose value in 2012 was estimated at 46.6 million euros.
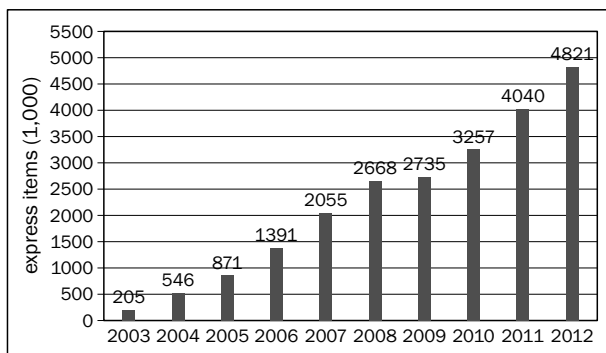

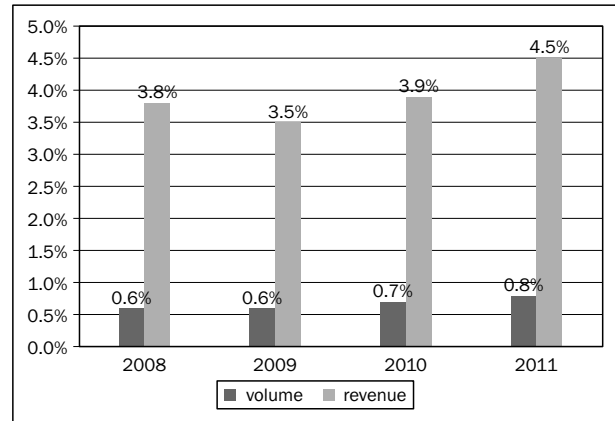*Figure 3 - Express items of public operator*

Figure 4 - Share of express services in the revenue and volume of the public operator

The aim of this paper is to estimate the demand for express services in Serbia by addressing the mutual influence between the public and the private operators. The demand function was estimated using the Lotka-Volterra model with populations in the competition. This model has been chosen because it enables the best dynamics description of the predator-prey relationship. The population dynamics observed in biological systems has been an inspiration in the works for the modelling of social, economic and technological changes [12-15]. Lotka-Volterra model enables the analysis of the equilibrium level in the distribution of a particular resource (in this case "a particular resource" refers to the user through the implementation of services), as well as analysing the changes which arrive with the new redistribution of resources among market participants. The model is suitable to describe economic principles such as the effect of market competition and law of increasing returns, takes into account the effect of a group and individual behaviour by modelling internal and external influences on different types of population. Furthermore, this approach provides the ability to detect interactions or unexpected market behaviour such as symbiosis which can affect the strategic behaviour [18]. The model predicts dynamic scenarios, where the equilibrium point exists as a possible asymptotic state.

3. DESCRIPTION OF THE LOTKA-VOLTERRA MODEL

Interaction between two competitors in the Lotka-Volterra model can be represented by two-differential equations by applying the Murray notation [16]:

$$\frac{dX}{dt} = a_1 X \left(1 - \frac{X}{K_1} - m_1 \frac{Y}{K_1} \right) \tag{1}$$

$$\frac{dY}{dt} = a_2 Y \left(1 - \frac{Y}{K_2} - m_2 \frac{X}{K_2} \right) \tag{2}$$

where X and Y represent the two types of population in the competition at time t, where a_i is linear birth

rate of the i-th population, K_i is the carrying capacity ($K_i \neq 0$) of the i-th population, and m_1 and m_2 measure the impact of competition Y on X, or, X on Y, respectively. Equations (1) and (2) describe the dynamics of the community, which consists of two populations (X, Y). The populations are interdependent, that is, each population affects the competitors' growth rate by reducing its capacity. K_i can be defined as the maximum sustainable amount of economic activity in a given environment [12].The parameter a_i represents the population growth and reproduction rates of equivalent organisms. The greatest value of this parameter is when an organization enters into an empty market niche. Until the population size reaches threshold value K_i, a_i reduces itself (a_i equals zero when the population size reaches the value of K_i). If the population size exceeds the value of K_i, a_i gets a negative value seeking to return to value of K_i. Values of coefficients m_1 and m_2 indicate the strength and direction of mutual interaction between populations X and Y, and vice versa, and how a member of one population affects the limit capacity of other populations.

In order to apply data in discrete time it is necessary to transform the continuous Lotka-Volterra model into discrete form [17], where the Equations (1) and (2) take the following form:

$$X(t+1) = \frac{\alpha_1 X(t)}{1 + \beta_1 X(t) + \gamma_1 Y(t)} \quad (3)$$

$$Y(t+1) = \frac{\alpha_2 Y(t)}{1 + \beta_2 Y(t) + \gamma_2 X(t)} \quad (4)$$

Parameters α_i and β_i are the logistic parameters for the i-th population when it exists on its own, while coefficients γ_i indicate the intensity of the impact that one population has on the growth of other population. The connection between the coefficients Equations (1), (2) and Equations (3), (4) is given as follows:

$$a_i = \ln \alpha_i \quad (5)$$

$$K_i = \frac{\alpha_i - 1}{\beta_i} \quad (6)$$

$$m_i = \frac{\gamma_i K_i}{\alpha_i - 1} = \frac{\gamma_i}{\beta_i} \quad (7)$$

The sign of the competition coefficient - m_i defines the role of the competitor, that is, interaction between

two competitors is described based on the values of these coefficients.

As shown in *Table 2*, the relationship between competitors in the market can range from perfect competition (+, +) to neutral relationship between them. In the case of perfect competition, niche competitors overlap. In such conditions, the population growth is limited by the main capacity of a competitor and causes a reduction in the number of its population. Conversely, when competition coefficient m_i equals 0, the interaction between competitors does not exist.

4. PARAMETER ANALYSIS

Express services market in the Republic of Serbia is an area where competitors operate. Potential users of express services were seen as resource around which the competitors compete. In this paper, in addition to the fact that there are a number of express operators (over 40), the attention is focused on the competition between the public and the private sectors. In fact, one segment of the market represents express services of public operators, while variables related to the private segment represent the sum of all private operators' services.

Determining the value of parameters in Equations (3) and (4) is necessary for the Lotka-Volterra model to be applied successfully for the assessment of market demand functions for express services in Serbia. In accordance with Equations(3) and (4) the monthly number of express services of a public operator is denoted by X, while the number of express services of a private sector is marked with a Y. Monthly service number of listed market segments in the period from January 2010 to December of 2012 was used to estimate the defined parameters. Parameter estimation was carried out by a method of nonlinear least squares using the software *Statistica 10* (StatSoft software package). Nonlinear least squares method uses an iterative procedure called the Levenberg-Marquardt algorithm [19]. Iterative procedure is controlled by the maximum number of iterations, as well as with a defined convergence criterion. Convergence criterion is considered to

Table 2 – Types of interactions in relation to values of mi [18]

Sign m_i		Type	Explanation
+	+	Pure competition	Occurs when both species suffer from each other's existence
+	-	Predator-prey	Occurs when one of them serves as direct food to the other
-	-	Mutualism	Occurs in case of symbiosis or a win-win situation
-	0	Commensalism	Occurs in a parasitic type of relationship in which one benefits
			from the existence of the other, who nevertheless remains unaffected
+	0	Amensalism	Occurs when one suffers from the existence of the other, who
			is impervious to what is happening
0	0	Neutralism	Occurs if there is no interaction whatsoever

Table 3 - Parameter estimation for Lotka-Volterra model

Public operator		Private sector	
Parameters	Estimates	Parameters	Estimates
α_1	1.336200298	α_2	1.708964221
β_1	5.59054E-07	β_2	8.75165E-07
γ_1	1.50012E-07	γ_2	-1.35066E-08

be a change in the value of the parameters that lead to stopping the iterative procedure, or if the change is less than the defined parameters of the convergence criteria, the algorithm ends. Convergence criterion, in this case is a value of 0.001, or the procedure is stopped if the change percentage in the coefficient is less than 0.1% [13, 20]. The estimated value of the parameters for Equations (3) and (4) is given in *Table 3*.

The number of the completed express services based on the estimated values of the parameters from *Table 3* is shown in Figures 5 and 6. The figures show that the estimated values correspond quite well to the real data.

Along with the estimated demand function for express services of public and private operators, the condition of actual data is also given. How this proposed model is convenient to describe the express services market has been estimated based on the mean absolute error (MAE) and mean absolute percentage error (MAPE). *Table 4* shows the sizes of the listed errors on the basis of which the given model is being estimated.

Certainly, smaller values of these values indicate better adjustment of this model to real conditions that this proposed model describes.

Table 4 - Estimated errors of this model

		Values
Public	MAPE	8.01%
	MAE	27,334
Private	MAPE	10.78%
	MAE	73,978

The interpretation of MAPE values proposed by Lewis [21] is reflected in the following: less than 10% is highly accurate forecasting, 10% to 20% is good forecasting, 20% to 50% is reasonable forecasting, and 50% or more is inaccurate forecasting. From *Table 4* we can see that MAPE value for the public sector is less than 10% which indicates highly accurate forecasting, while the private sector is just above 10% which is quite a satisfying value.
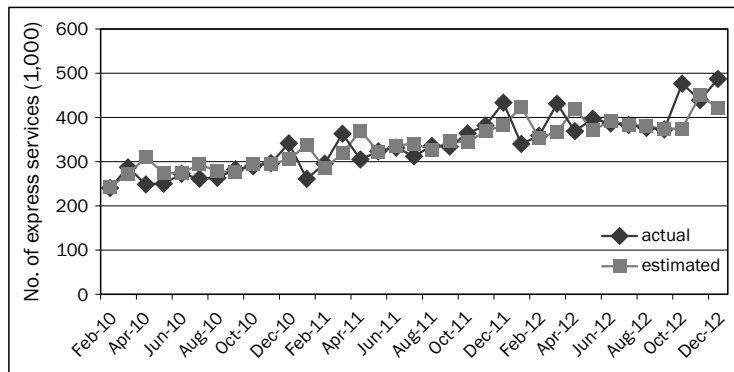

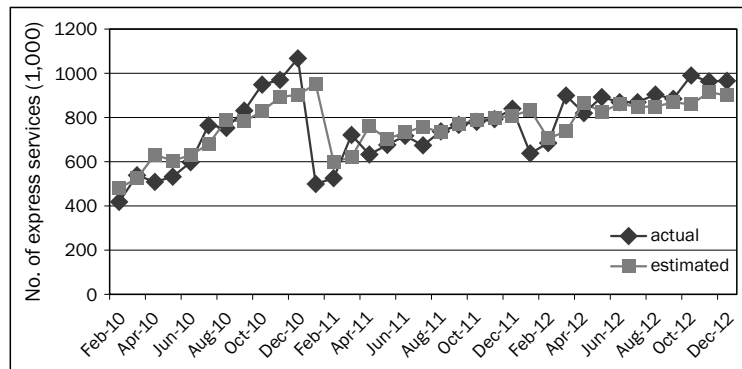*Figure 5 - Estimated and actual number of express services of the public sector*

Figure 6 - Estimated and actual number of express services of private sector

Based on the relations between the coefficients between the differential and difference equation proposed by Leslie (1958), one can also determine the value of parameters a_i, K_i and m_i. The values of these parameters are given in *Table 5*.

Table 5 - Parameter values of differential equations

Public operator		Private operators	
a_1	0.28983	a_2	0.535887
K_1	601,374	K_2	810,092
m_1	0.268332	m_2	-0.01543

Based on the interaction coefficient values of mi, according to *Table 2*, the express services market in Serbia is in a predator-prey relationship in favour of the private sector. Namely, the activities of the public operators that lead to an increase in the volume of implemented services will also lead to an increase in the private sector services. In contrast, private sector activities aimed at increasing the volume of services will lead to a decrease in the volume of public operators. Therefore, the current state of the market goes in favour of the private sector because in this predator-pray relationship the public sector services serve as direct incentive for a more successful operation of the private sector.

5. DISCUSSION AND VALUES

Analysis of the competitive Lotka–Volterra model can provide information such as the state of equilibrium, and the trajectory of change over time [22]. When analysing the equilibrium Equations (1) and (2) are equal to 0 because under that condition nothing changes over time for the competitor, so the population equilibrium can be observed:

$$\frac{dX}{dt} = 0 \text{ and } \frac{dY}{dt} = 0 \tag{8}$$

Taking into account the condition (8) for Equations (1) and (2) we get:

$$a_1 X\left(1 - \frac{X}{K_1} - m_1\frac{Y}{K_1}\right) = 0 \tag{9}$$

$$a_2 Y\left(1 - \frac{Y}{K_2} - m_2\frac{X}{K_2}\right) = 0 \tag{10}$$

By solving the Equations (9) and (10) we obtain:

$$X = K_1 - m_1 Y \text{ and } Y = K_2 - m_2 X \tag{11}$$

In the case when $X < (K_1 - m_1 Y)$ in Equation (11) it follows that $dX/dt > 0$ or the number of public operators service will grow. Conversely, when $X > (K_1 - m_1 Y)$ this leads to the fact that $dX/dt < 0$, i.e. the number of express services of the public operator declines. In the event that the lines defined by Equation (11) intersect in the first quadrant, we can conclude that there is an equilibrium point in the market.

If there is equilibrium, its stability depends on the coefficients which describe the Lotka-Volterra model.
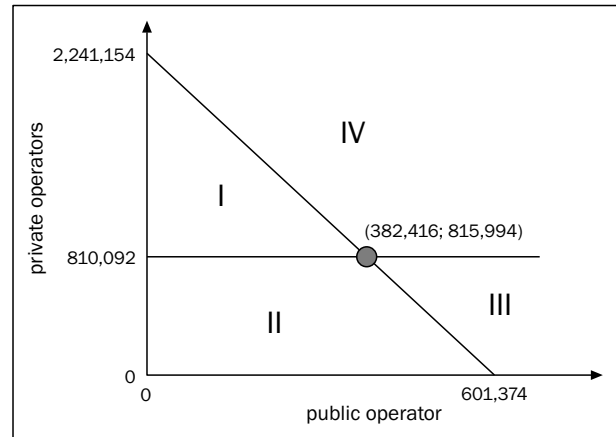

Figure 7 - Equilibrium point in the market of express items.

In *Figure 7*, we can see that the lines $dX/dt = 0$ and $dY/dt = 0$ intersect indicating the existence of the equilibrium point, and that these two market segments can exist without any dynamic changes based on the empirical results. The stability of the equilibrium point can be defined whether the changes lead to a return to a point of equilibrium or not. If we start from the equilibrium point to area I where the number of private operators' services is more than 815,994, it leads to the area where the number of public operators' services will increase while the number of private operators' services will decline, which consequently returns to the state of equilibrium. If we find ourselves in area I where the number of private operators' services ranges between 810,092 and 815,994 it will lead to a situation where the number of public operators' services has a tendency to increase, while the number of private operators' services has a tendency to decrease, which will lead us to area II. In case we are in area II, where the number of public operators' services is less than 382,416, services of both the public and the private operator tend to grow, which results in returning to the point of equilibrium. When the market is positioned in area II where the number of public operators' services is more than 382,416, then the services of both private and public sector will have a tendency of growth which will lead us to area III. If the starting point is located in III, where the number of services of the private sector is less than 815,994, the public operators' services will decrease while the private sector services will increase until reaching the equilibrium. When the number of private sector services is more than 815,994 in area III, the number of public operators services has a tendency to drop while the number of private sector services increases, which leads to area IV. In case of area IV, where the number of public operators' services is more than 382,416, the private sector and the public operator have a tendency to decrease, leading to the equilibrium. In the latter case, if we are in area IV, where the number of express services of the public operator is less than

382,416, both sectors tend to decrease the number of services leading to area I.

Based on the above, it can be noted that in the Serbian market of express services there is an equilibrium which, depending on the changes in the market shows a partial stability. The latest available data are related to the year 2012, when the private sector performed 965,686 services, while the public operator performed 487,000 express services. The specified value is in the area where both sectors have a tendency to drop in the number of services, that is, the movement of the market could be expected to reach the defined equilibrium point.

6. CONCLUSION

Express services market in Serbia has a tendency of continuous growth. The number of operators that provide such services confirms the attractiveness of this segment of the postal market. This paper gives an explanation for the situation on the market of express services from the competitors' point of view.

The obtained results show that the interaction between the market segments (in order to simplify it, both public and private sector have been observed) can successfully be described using the Lotka-Volterra model. The parameter values of this model indicate that there is a competitive relationship between these segments; in other words, that the private sector has the benefit of the presence of a public operator, while the public operator is in danger from the private sector (despite the fact that the public operator is in advantage since the implementation of its express services is freed from VAT collection, and partially uses the network resources provided for the implementation of universal service). It also points to the existence of a partially stable equilibrium point.

The liberalization or removal of anti-competitive barriers leads to increased customer satisfaction and reduced cost which leads to increased efficiency and more innovative operators. Changes that are taking place on this market have to be analysed in order for the operators to ensure their existence. The proposed method of observing the express services market while conducting their business can be used by market participants in order to review their position and to take appropriate actions to improve their own position. Furthermore, the model can be applied to other services provided by postal operators, in order to examine the influence of both the direct competition (those who perform services of the same type), and the effect of indirect competition, that is, the technologies that contribute to the substitution of traditional postal services. On the other hand, this methodology can also consider access to the network of public operator if the capacity of the public operator is viewed as a re-

source that is available to a number of competitors in the private sector.

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ABSTRAKT

ANALIZA TRŽIŠTA EKSPRES USLUGA NA OSNOVU LOTKA-VOLTERA MODELA – STUDIJA SLUČAJA SRBIJA

Rad pruža pregled prethodnih faza kroz koje je prošlo tržište poštanskih ekspres usluga i mogućnosti daljeg razvoja, kako na globalnom tako i na lokalnom nivou. Cilj rada je da izvrši procenu potreba za ekspres uslugama u Srbiji primenom konkurentnog Lotka-Voltera modela. U pravcu smanjenja kompleksnosti procesa, podela konkurencije sprovedena je na dva segmenta: javnog operatora i privatni segment (obuhvata sve privatne operatore). Dati model omogućava opisivanje dinamičnog konkurentnog odnosa i ukazuje na postojanje ravnotežne tačke između javnog i privatnog sektora, kao i uslove njene stabilnosti. Dobijene vrednosti ukazuju da privatni sektor utiče na javnog operatora. Postojeći predator-žrtva odnos daje prednost privatnom sektoru i može se opisati Lotka-Voltera modelom.

KLJUČNE REČI

ekspres usluge; Lotka-Voltera model; konkurencija; ekvilibrijum;

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