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A MODEL FOR DYNAMIC PARKING CHOICE OF DRIVERS

ABSTRACT

Taking time-varying parking demands and supplies into account, this paper develops a dynamic parking choice model to simulate drivers' searching for parking spaces between parking lots. In this study, each parking lot has one access passage and vehicles can move or wait for entering the park in the access passage. The flow propagation formula in the access passage of parking lots is established to obtain the number of vehicles entering the parking lot from the access passage within each time interval. This paper brings forward the instantaneous dynamic user-optimal condition of dynamic parking choice, constructs an equivalent variational inequality (VI) model and develops a diagonalization algorithm to solve the model. A numerical example demonstrates the rate of in-flow, the rate of out-flow, link flow, instantaneous travel time on each link or access passage of the parking lot, and the amount and routes of cruising traffic at each time point.

KEYWORDS

parking; dynamic; user-optimal; variational inequality (VI)

1. INTRODUCTION

In reality, drivers may need to search for a vacant parking space between parking lots in the vicinity of the destination. When the driver arrives at a park, they will park if vacant parking spaces are available at this parking lot; or else, they will assess the possible time necessary for waiting for a parking space to get vacated, and they will leave for another parking lot if the waiting time is too long. This paper models dynamic parking choice behaviours of drivers between parking lots, calculates the number of vehicles going into or out of each park at each time point, calculates link flows on the surrounding road networks, and finds out the routes of cruising traffic.

The dynamic traffic assignment focuses on route choice behaviours under time-varying traffic demands and time-varying traffic conditions. The models for dynamic traffic assignment can be classified into five categories according to research methods: Computer simulating [1, 2]; Mathematical programming [3, 4]; Optimal control theory [5, 6]; Variational inequality (VI) and non-linear complementarity [7, 8, 9, 10]; Fixed point theory [11]. The dynamic traffic assignment model based on computer simulating gives route choices of travellers by iteration, but it cannot guarantee the convergence and precision of solutions. The mathematical programming model also has many limitations because many dynamic traffic assignment problems cannot be converted into equivalent mathematical programming. The model based on optimal control theory has the advantage of good analysis, but in many cases the equivalent optimal control model to the user-optimal condition cannot be found. The models based on VI and non-linear complementarity have been developed in recent years; they have a simple form and can be easily solved. In order to more accurately simulate the dynamics characteristics of the network traffic flow, Lo and Szeto in [12] introduced the Cell Transmission Model (CTM) and constructed a dynamic traffic assignment model based on CTM using VI. Ramadurai and Ukkusuri in [13] proposed a dynamic user equilibrium model for combined activity-travel choices using activity-travel supernetwork representation, and a cell-based transmission model (CTM) is embedded to capture the traffic flow dynamics. Tian et al. in [14] presented a cumulative perceived value-based dynamic user equilibrium model by applying the prospect theory to formulate the travellers' risk evaluation on arrival time, and considered departure time and route choices simultaneously.

Although the dynamic assignment problems have been studied by a lot of researchers, these studies

did not take parking choice at the end of the trip into account in modelling the dynamic route choice behaviours of the drivers. This has encouraged the carrying out of this study. The dynamic parking choice model simulates drivers' searching for the parking spaces between parking lots, and considers not only road networks but also parking at the parking lots, so it extends the dynamic traffic assignment theory which has been studied by a lot of researchers [8, 9, 15].

2. NOTATIONS AND DEFINITIONS

The road network is denoted by $G=(N,A)$, where N is the set of all nodes, A is the set of all links; s denotes destination node; $a=(l,m)$ is the link with starting node l and end node m. E_l is the set of links pointing at node l; F_l is the set of links out of node l, see *Figure 1*.

Figure 2 shows the park and its access passage, and this paper supposes that each park has and only has one access passage, and that vehicles can move or wait for entering the park in its access passage.
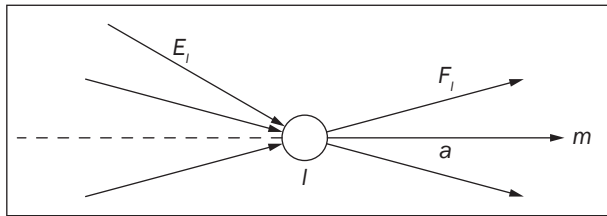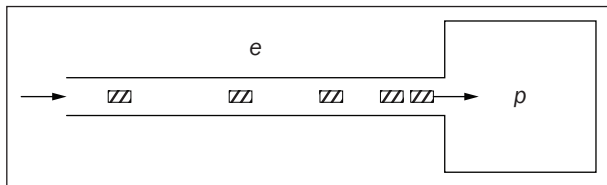

Figure 1 – Relation between links and nodes

Figure 2 – The park and its access passage

The time range is $[0, T]$, which should be long enough to make a traveller be able to complete their trip in this time interval. For $t \in [0, T]$, the following variables will be introduced.

$u_a(t)$ – Rate of in-flow of link a (i.e. the number of vehicles entering link a) at time point t, and $u_a^s(t)$ is the rate of in-flow of link a at time point t whose destination is s,
$$u_a(t) = \sum_s u_a^s(t).$$

$v_a(t)$ – Rate of out-flow of link a (i.e. the number of vehicles coming out of link a) at time point t, and $v_a^s(t)$ is the rate of out-flow of link a at time point t whose destination is s,
$$v_a(t) = \sum_s v_a^s(t).$$

$x_a(t)$ – Flow on link a (number of vehicles) at time point t, and $x_a^s(t)$ is the flow (number of vehicles) on link a at time point t whose destination is s, $x_a(t) = \sum_s x_a^s(t)$.

$g_l^s(t)$ – Flow produced at node l at time point t whose destination is s.

$c_a(t)$ – Instantaneous travel time or reactive travel time on link a at time point t, also has a form of $c_a[x_a(t), u_a(t), v_a(t)]$.

$\eta_l^s(t)$ – Minimal instantaneous path travel time from node l to destination s.

$\tau_a(t)$ – Actual travel time on link a of the vehicle entering the link at time point t, which is also influenced by the followed vehicles.

$\chi_p^s(t)$ – Parking duration of the vehicle entering park p at time point t whose destination is s.

Q_p – The number of parking spaces in park p.

$B_{p,s}$ – The time for walking egress trip from park p to destination s.

C_e – The maximal number of vehicles existing in passage e at the same time.

D_e – The free travel time of vehicles in passage e.

$\hat{x}_p(t)$ – The number of vehicles parking at park p at time point t. $\hat{x}_p^s(t)$ is the number of vehicles parking at park p at time point t whose destination is s, $\hat{x}_p(t) = \sum_s \hat{x}_p^s(t)$.

$\hat{u}_p(t)$ – Rate of in-flow of park p (i.e. the number of vehicles entering park p) at time point t, and $\hat{u}_p^s(t)$ is the rate of in-flow of park p at time point t whose destination is s, $\hat{u}_p(t) = \sum_s \hat{u}_p^s(t)$.

$\hat{v}_p(t)$ – Rate of out-flow of park p (i.e. the number of vehicles leaving park p) at time point t, and $\hat{v}_p^s(t)$ is the rate of out-flow of park p at time point t whose destination is s, $\hat{v}_p(t) = \sum_s \hat{v}_p^s(t)$.

3. FLOW PROPAGATION FORMULA AT PARKING LOTS

The time range $[0, T]$ is discretized into $K+1$ equally small time intervals 0, 1, 2, …, K. The starting point of time interval $k(0 \le k \le K)$ is also named as time point k, while the end point of time interval k is just the starting point of time interval $k+1$. The free-flow travel time on each link or passage should be larger than or equal to one small time interval and parking duration of each vehicle in the park should also be larger than or equal to one small time interval.

Figure 3 shows a simple road and park network. Node 1 is the origin; nodes 4 and 5 denote two parks; 2→4 is the access passage of park 4; 3→5 is the access passage of park 5; 1→2, 1→3, 2→3, 3→2 are road links; especially 2→3, 3→2 link park 4 with 5, and drivers can choose between parks 4 and 5 via links 2→3, 3→2.
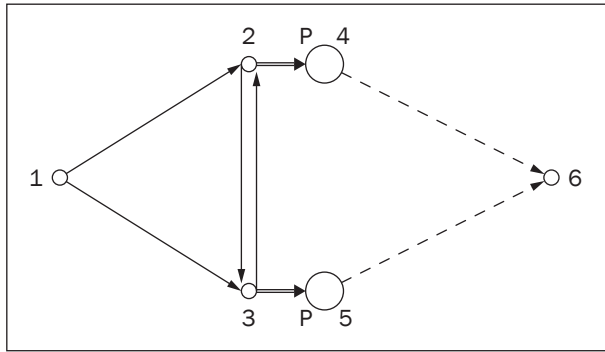
Figure 3 – Park and road network

Phase update formula of the number of vehicles parking at park p is

$$\hat{x}_p^s(k) = \hat{x}_p^s(k-1) + \hat{u}_p^s(k-1) - \hat{v}_p^s(k-1),$$ (1)
$$\forall \, p, \, s; \, k = 1, \, 2, \, \cdots, \, K$$

For park p, destination s and time interval k, $\hat{i}_{p,k}^s$ is a time point that satisfies $\hat{i}_{p,k}^s + \chi_p^s(\hat{i}_{p,k}^s) = k$. All the values of $\hat{i}_{p,k}^s$ construct the set $\hat{I}_{p,k}^s$. The flow propagation constraint of park p is

$$\sum_{\hat{i}_{p,k}^s \in \hat{I}_{p,k}^s} \hat{u}_p^s(\hat{i}_{p,k}^s) = \hat{v}_p^s(k), \quad \forall p, s, k$$ (2)

Phase update formula of the number of vehicles in passage e is

$$x_e^s(k) = x_e^s(k-1) + u_e^s(k-1) - v_e^s(k-1),$$ (3)
$$\forall \, e, \, s; \, k = 1, \, 2, \, \cdots, \, K$$

$v_e^s(k)$ is analysed based on the following suppositions (1)-(5).

1) Each park has and only has one access passage.

2) Overtaking is not permitted in the access passage of the park, i.e. it satisfies the first-in-first-out (FIFO) rule which denotes that the vehicle entering the passage first will be the first to enter the park.

3) Stop-and-go driving may occur because of vehicles queuing in the access passage of the park. Here, the starting acceleration time and braking deceleration time of vehicles in the passage will be ignored. And the speed of vehicles moving in the passage is regarded as fixed.

4) Vehicles in the passage only queue waiting to enter the parking lot not including the queuing from disturbance of traffic flows.

5) All queuing vehicles stop or move forward one by one at a known minimum safe distance when line-up appears in the passage.

The following analysis supposes that the park has enough vacant parking spaces. Figure 4 shows the park and its access passage.
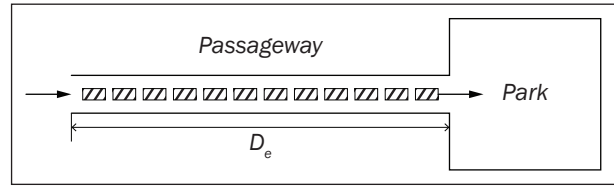

Figure 4 – The park and its access passage

Suppose that the free travel time (without queuing to wait) of vehicles in the passage is D_e time intervals. Since the passage length is known, the maximal number of queuing vehicles that the passage can accommodate at the same time is C_e according to supposition (5) that a fixed safe distance lies between immediate vehicles when they are queuing in the passage. The distance covered by the moving vehicle within one time interval is d_e, so the maximal number of vehicles existing in the space interval with length d_e at the same time is $\dfrac{C_e}{D_e}$ if they are queuing to wait.
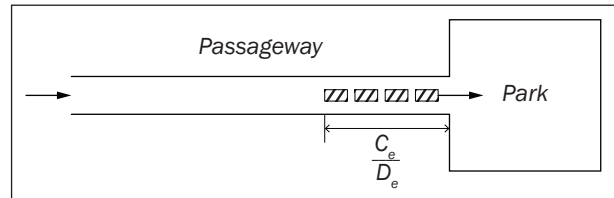

Figure 5 – The number of queuing vehicles in the space interval with length d_e

$x_e(k) = u_e(k-1) + u_e(k-2) + \ldots + u_e[k-(D_e-1)] + \ldots + u_e(0) - v_e(k-1) - v_e(k-2) - \ldots - v_e(0)$

At the start of time interval k, the farthest distance that vehicles $u_e(k-1)$ go in the passage is d_e, the farthest distance that vehicles $u_e(k-2)$ go in the passage is $2d_e$, ..., the farthest distance that vehicles $u_e[k-(D_e-1)]$ go in the passage is $(D_e-1)d_e$. At the end of time interval k, the farthest distance that vehicles $u_e[k-(D_e-1)]$ go in the passage is $D_e d_e$ (i.e. passage length), and the vehicle entering first the passage among vehicles $u_e[k-(D_e-1)]$ travels a space interval with length $D_e d_e$ in the passage (i.e. the whole passage length), and is about to leave but has not yet left the passage. So, according to first-in-first-out (FIFO) rule, vehicles $u_e(k-1)$, $u_e(k-2)$, ..., $u_e[k-(D_e-1)]$ have been staying in the passage by the end of time interval k. So, in time interval k, the vehicles which may leave the passage and then enter the park should be part of $u_e(k-D_e)$, $u_e[k-(D_e+1)]$, ..., $u_e(0)$, the remaining vehicles in the passage among which at the start of time interval k (most of which have entered the park before interval k) are $x_e(k) - \{u_e(k-1) + u_e(k-2) + \ldots + u_e[k-(D_e-1)]\}$.

Proposition 1:

After a vehicle starts queuing to wait (from moving to queuing) in the passage, it always stops or goes forward close to the nearest vehicle in the front with minimum safe interval until it enters the park.

Proof:

According to supposition (3) that the acceleration time and braking time of vehicles in the passage are ignored, after a vehicle starts queuing to wait from moving in the passage (the queuing vehicle keeps minimum safe distance from the nearest vehicle in the front), if the vehicle in the front moves, then the following queuing vehicle also moves at the same speed; if the vehicle in the front stops, then the following vehicle also stops to keep the minimum safe interval. Thus, it always stops or goes forward close to the nearest vehicle in the front with the minimum safe distance until it enters the park, and no bigger interval than the minimum safe distance between them occurs.

Proposition 2:

According to supposition (4) that vehicles in the passage only queue waiting to enter the parking lot without the presence of queuing from disturbance of traffic flows, if queuing occurs in the access passage of the park, then the queue starts from the gate (i.e. the link site of the park and its access passage) of the park and extends backwards along the passage.

Proof:

If a vehicle in the passage queues to wait while the vehicle in front of it moves instead of queuing to wait, then according to supposition (3) that acceleration time and braking time of vehicles in the passage are ignored, if a vehicle moves then the following vehicle also moves with the same speed instead of stopping. Here a contradiction occurs, so the vehicle in front of a queuing vehicle must be in the situation of queuing to wait, i.e. the queue starts from the gate (i.e. the link site of the park and its access passage) of the park and extends backwards along the passage.

Theorem 1

At time point *k*, if

$$x_e(k) - \{u_e(k-1) + u_e(k-2) + \cdots + u_e[k-(D_e-1)]\} > \frac{C_e}{D_e}$$

then the last vehicle entering the park among vehicles $u_e(k - D_e)$ must have queued (i.e. stopped) at one time point before. The number of vehicles leaving the passage and entering the park in one time interval is $\frac{C_e}{D_e}$ under ignoring acceleration time of vehicles and assuming the park has enough vacant parking spaces.
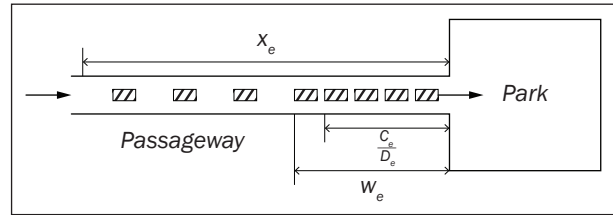

Figure 6 – The case when the number of queuing vehicles is larger than $\frac{C_e}{D_e}$ (w_e is the number of queuing vehicles, $w_e = x_e(k) - \{u_e(k-1) + u_e(k-2) + ... + u_e[k-(D_e-1)]\}$)

Proof:

Let us assume the opposite that, at time point *k*, the last vehicle entering the passage among $u_e(k - D_e)$ has never queued to wait for parking before, then the distance it travels is greater than or equal to $(D_e - 1)d_e$ and less than or equal to $D_e d_e$ at the start of time interval *k* or at the end of time interval *k* – 1. So, according to the FIFO condition, all the remaining vehicles in the passage among $u_e(k - D_e)$, $u_e[k - (D_e + 1)]$, ..., $u_e(0)$ lie in the space interval with length d_e from the gate of the park at time point *k*. However, the maximal number of vehicles the space interval with length d_e from the gate of the park in the passage can accommodate is $\frac{C_e}{D_e}$, while

$$x_e(k) - \{u_e(k-1) + u_e(k-2) + \cdots + u_e[k-(D_e-1)]\} > \frac{C_e}{D_e}$$

so there is a contradiction. Thus, the last vehicle into the park among $u_e(k - D_e)$, which is also the last vehicle of $x_e(k) - \{u_e(k-1) + u_e(k-2) + ... + u_e[k-(D_e-1)]\}$, must have queued to wait at one time point before. According to proposition 2, the queue starts from the gate of the park and extends backwards along the passage, i.e. all vehicles in front of it are also queuing; and according to proposition 1, a vehicle which has ever queued to wait for parking must afterwards stop or go forward close to the nearest vehicle in the front with minimum safe distance until it enters the park. So after this time point, all vehicles of $x_e(k) - \{u_e(k-1) + u_e(k-2) + ... + u_e[k-(D_e-1)]\}$ queue or move forward keeping a minimum safe distance from the vehicle in the front. It is obvious from *Figure 6* that the number of vehicles leaving the passage and entering the park in one time interval is just the number ($\frac{C_e}{D_e}$) of queuing vehicles in the space interval with length d_e which the vehicle travels in one time interval.

Theorem 2

At time point *k*, if

$$x_e(k) - \{u_e(k-1) + u_e(k-2) + \cdots + u_e[k-(D_e-1)]\} \leq \frac{C_e}{D_e}$$

then vehicles $u_e(k - D_e)$ all lie in the space interval with length d_e from the gate of the park. Vehicles $x_e(k) - \{u_e(k-1) + u_e(k-2) + ... + u_e[k-(D_e-1)]\}$ can all

go into the park in the time interval k under ignoring the acceleration time of the vehicle and assuming the park has enough vacant parking spaces.
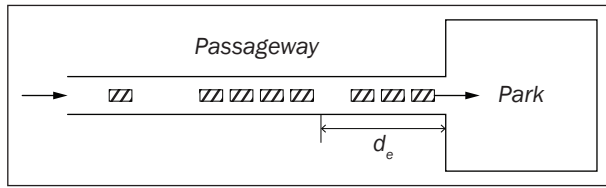

Figure 7 – The case when the number of queuing vehicles is less than or equal to $\frac{C_e}{D_e}$

Proof:

At time point k, if a vehicle of $u_e(k - D_e)$ (suppose it is the last vehicle going into the passage among $u_e(k - D_e)$) is outside the space interval with length d_e from the gate of the park, then it must have queued to wait at one time point before because if it has never queued for parking, it would have already been in the space interval with length d_e from the gate of the park at time point k. According to propositions 1 and 2, the queuing vehicle must afterwards stop or go forward close to the nearest vehicle in the front with minimum safe distance until it enters the park, and vehicles queuing or moving at intervals of minimum safe distance keeps from it to the gate of the park. However,

$$x_e(k) - \{u_e(k - 1) + u_e(k - 2) + \cdots + u_e[k - (D_e - 1)]\} \le \frac{C_e}{D_e}$$

which means that queuing vehicles are all in the space interval with length d_e from the gate of the park because the space interval with length d_e in the passage can accommodate $\frac{C_e}{D_e}$ queuing vehicles, which contradicts the supposition that the vehicle which has ever queued to wait for parking at one time point before or the last vehicle into the passage among $u_e(k - D_e)$ is outside the space interval with length d_e from the gate of the park. Thus, vehicles $u_e(k - D_e)$ should all lie in the space interval with length d_e from the gate of the park. According to the FIFO condition, the remaining vehicles in the passage among $u_e[k - (D_e + 1)]$, ..., $u_e(0)$ should also be in the space interval with length d_e from the gate of the park. Since all vehicles in the space interval with length d_e in the passage can go into the park within one time interval, so

vehicles $x_e(k) - \{u_e(k - 1) + u_e(k - 2) + ... + u_e[k - (D_e - 1)]\}$ can all enter the park in time interval k under ignoring acceleration time of vehicles and assuming the park has enough vacant parking spaces (See *Figure 7*).

So, synthetically from theorems 1 and 2, the number of vehicles entering the park from the passage in time interval k is

$$v_e(k) = \min(x_e(k) - \{u_e(k - 1) + u_e(k - 2) +$$
$$+ \cdots + u_e[k - (D_e - 1)]\}, \frac{C_e}{D_e}) \tag{4}$$

The above analysis is on the basis of the supposition that the park has enough vacant parking spaces. However, if the number of vacant parking spaces in park p connected with passage e is given, the restriction of the number of vacant parking spaces needs to be taken into account. The number of vacant parking spaces in time interval k is $Q_p - \hat{x}_p(k) + \hat{v}_p(k)$, so,

$$v_e(k) = \min(\min(x_e(k) - \{u_e(k - 1) + u_e(k - 2) +$$
$$+ \cdots + u_e[k - (D_e - 1)]\}, \frac{C_e}{D_e}), Q_p - \hat{x}_p(k) + \hat{v}_p(k)) \tag{5}$$

$v_e^s(k)$ may be determined after $v_e(k)$ is known.

As seen in *Figure 8*, according to the supposition that the free flow travel time on the passage is larger than or equal to one time interval, so any of $u_e(k)$ would not leave the passage in the time interval k. At time point k, the vehicles which have entered the passage are $u_e(k - 1)$, $u_e(k - 2)$, ..., $u_e(1)$, $u_e(0)$. At the start of time interval k,

$$\sum_{\lambda=0}^{k-1} v_e(\lambda) = \sum_{\lambda=0}^{k-1} v_p(\lambda) + x_p(k) = u_e(0) +$$
$$+ u_e(1) + \cdots + u_e(k_2 - 1) + \alpha u_e(k_2)$$

at the end of time interval k,

$$\sum_{\lambda=0}^{k} v_e(\lambda) = u_e(0) + u_e(1) + \cdots + u_e(k_1 - 1) + \beta u_e(k_1)$$

where k_1 and k_2 are the sequence numbers of time intervals between 0 and $k - 1$ and $k_1 \ge k_2$, $0 \le \alpha \le 1$, $0 \le \beta \le 1$.

$$v_e(k) = \sum_{\lambda=0}^{k} v_e(\lambda) - \sum_{\lambda=0}^{k-1} v_e(\lambda) = \beta u_e(k_1) +$$
$$+ u_e(k_1 + 1) + \cdots + (1 - \alpha)u_e(k_2)$$

Since vehicles $u_e^s(k - 1)$, ..., $u_e^s(1)$, $u_e^s(0)$ of $u_e(k - 1)$, $u_e(k - 2)$, ..., $u_e(1)$, $u_e(0)$ are all known, $v_e^s(k)$ of $v_e(k)$ can also be known.
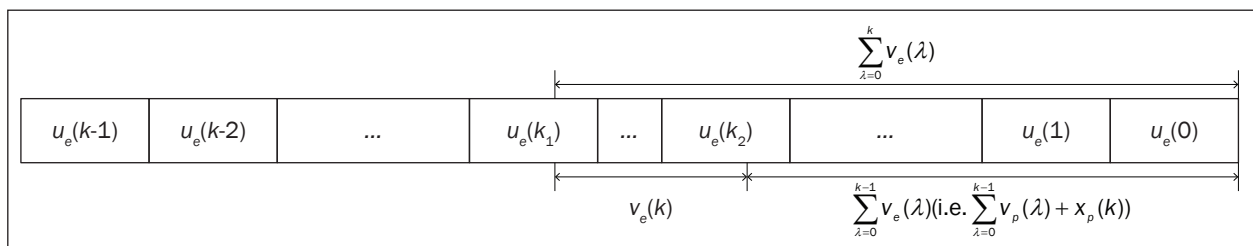

Figure 8 – Flow analysis in the passage

In the joint of park p and its access passage e,

$$\hat{u}_p^s(k) = v_e^s(k) \tag{6}$$

$u_e(k)$ is analysed as follows.

There is also a limit on the maximal number of vehicles which can enter the access passage of the park. If the passage has enough spaces, the maximal number of vehicles which can go into the passage within one time interval is $\dfrac{C_e}{D_e}$, since vehicles are supposed to move at a fixed speed in the passage and the maximal distance the vehicle can go within one time interval is d_e and the maximal number of queuing vehicles existing in the space interval with length d_e at the same time is $\dfrac{C_e}{D_e}$. If the restriction of spaces available in the passage is taken into account, the number of vehicles which can go into the passage should be less than or equal to the maximal number of vehicles $(C_e - x_e(k) + v_e(k))$ that the vacant spaces in the passage can accommodate. So, the number of vehicles which can go into the passage in time interval k must be less than or equal to min $(\dfrac{C_e}{D_e},\ C_e - x_e(k) + v_e(k))$.

4. VI MODEL FOR DYNAMIC PARKING CHOICE

4.1 Dynamic user-optimal (DUO) condition of dynamic parking choice based on instantaneous travel time

Instantaneous dynamic user-optimal condition of dynamic parking choice: during the time range $[0, T]$, at any time point and at any decision node (road network nodes or the joint of links and access passage of the park), if the instantaneous travel time on all the used paths are equal and equal to the minimum instantaneous travel time, while the instantaneous travel time on all unused paths is not less than the minimum instantaneous travel time, then the flows on the networks are subject to the DUO condition based on instantaneous travel time.

$u_a^s(t) > 0$ means that link $a = (l, m)$ is used by the traveller whose destination is s at time point t; $u_e^s(t) > 0$ means that the passage e and the corresponding park are chosen by the traveller whose destination is s at time point t. π^{ls} is a path from decision node l to destination node s, $\phi_{l,\pi^{ls}}^s(t)$ is the instantaneous travel time on path π^{ls} at time point t.

$$\phi_{l,\pi^{ls}}^s(t) = \sum_{a \in \pi^{ls}} c_a(t) + B_{p,s},\quad (p,s) \in \pi^{ls} \tag{7}$$

where (7) means that the travel time on a path from decision node l to destination node s includes travel time on all links and access passage of the park of

this path, and the walking time from the park to the destination, $\eta_l^s(t) = \min_{\pi^{ls}}\{\phi_{l,\pi^{ls}}^s(t)\}$.

The DUO condition based on instantaneous travel time can be written as (8)~(13). For road link a,

$$[\eta_m^s(t) + c_a(t) - \eta_l^s(t)]u_a^s(t) = 0,\quad \forall a, s, t \tag{8}$$

$$\eta_m^s(t) + c_a(t) \geq \eta_l^s(t),\quad \forall a, s, t \tag{9}$$

$$u_a^s(t) \geq 0,\quad \forall a, s, t \tag{10}$$

It is known from (8), (9), (10) that if $u_a^s(t) > 0$, then $\eta_m^s(t) + c_a(t) = \eta_l^s(t)$, which means that if road link a is chosen by the traveller whose destination is s, then road link a must be on the shortest path between nodes l and s; and if $u_a^s(t) = 0$, then $\eta_m^s(t) + c_a(t) \geq \eta_l^s(t)$.

For the passage $e = (l, m)$ (l is the start node of the passage, for example, nodes 2 and 3 in *Figure 3*; m is the park), the DUO condition with capacity constraints can be defined as (see literature [16])

$$\eta_m^s(t) + c_e(t) + \delta_e(t)\begin{cases} = \eta_l^s(t), & \text{if } u_e^s(t) > 0 \\ \geq \eta_l^s(t), & \text{if } u_e^s(t) = 0 \end{cases} \forall e, s, t \tag{11}$$

where $\delta_e(t)$ can be regarded as "virtual travel time" caused by the capacity constraint of passage e.

$$\delta_e(t)\begin{cases} \geq 0, & \text{if } \sum_s u_e^s(t) = u_{em}(t) \\ = 0, & \text{if } \sum_s u_e^s(t) < u_{em}(t) \end{cases} \forall e, t \tag{12}$$

where

$$u_{em}(t) = \min (\frac{C_e}{D_e},\ C_e - x_e(t) + v_e(t)) \tag{13}$$

$\eta_m^s(t)$ is the walking time from the chosen park to the destination, and a value independent of t. If $\sum_s u_e^s(t) < u_{em}(t)$ then $\delta_e(t) = 0$ and the capacity constraint does not work; if $\sum_s u_e^s(t) = u_{em}(t)$ and $\eta_m^s(t) + c_e(t) = \eta_l^s(t)$, then $\delta_e(t) = 0$;

and if $\sum_s u_e^s(t) = u_{em}(t)$ and $\eta_m^s(t) + c_e(t) < \eta_l^s(t)$, then $\delta_e(t) > 0$ and $\delta_e(t) = \eta_l^s(t) - \eta_m^s(t) - c_e(t)$.

The optimal solutions which are subject to (8)~(13) are denoted as $u_a^{s^*}(t)$, $u_e^{s^*}(t)$.

4.2 VI model equivalent to DUO condition based on instantaneous travel time

The time range $[0, T]$ is discretized into many equally small time intervals 0, 1, 2, …, K. For time interval k, actual travel time $\tau_a(k)$ on link a is temporally fixed and rounded to the nearest integer.

1) *Phase equations*

$$x_a^s(k) = x_a^s(k-1) + u_a^s(k-1) - v_a^s(k-1), \tag{14}$$
$$\forall a,\ s;\ k = 1, 2, \cdots, K$$

$$\hat{x}_p^s(k) = \hat{x}_p^s(k-1) + \hat{u}_p^s(k-1) - \hat{v}_p^s(k-1), \quad (15)$$
$$\forall\, p,\, s;\, k = 1,\, 2,\, \cdots,\, K$$

2) *Flow conservation constraints at nodes*

For node l in the road network, there is

$$\sum_{a \in E_l} v_a^s(k) + g_l^s(k) = \sum_{a \in F_l} u_a^s(k), \quad \forall l, s, k \quad (16)$$

At the joint of the park p and its access passage e, there is

$$\hat{u}_p^s(k) = v_e^s(k) \quad (17)$$

3) *Flow propagation constraints*

For road link a and time interval k, $I_{a,k}$ is a set whose elements $(i_{a,k})$ satisfy $i_{a,k} + \tau_a(i_{a,k}) = k$. There is

$$\sum_{i_{a,k} \in I_{a,k}} u_a^s(i_{a,k}) = v_a^s(k), \quad \forall a, s, k \quad (18)$$

For park p, there is

$$\sum_{\hat{i}_{p,k}^s \in \hat{I}_{p,k}^s} \hat{u}_p^s(\hat{i}_{p,k}^s) = \hat{v}_p^s(k), \quad \forall p, s, k \quad (19)$$

For passage e, the number of vehicles entering the park from the passage in time interval k is

$$v_e(k) = \min\;(\min\;(x_e(k) - \{u_e(k-1) + u_e(k-2) +$$
$$+ \cdots + u_e[k - (D_e - 1)]\},\; \frac{C_e}{D_e}),\; Q_p - \hat{x}_p(k) + \hat{v}_p(k)) \quad (20)$$

where e is the access passage of park p. $v_e^s(k)$ was analysed in *Figure 8*.

4) *Capacity constraint of the passage*

$$u_e(k) \le \min\;(\frac{C_e}{D_e},\; C_e - x_e(k) + v_e(k)), \quad \forall e, s, k \quad (21)$$

5) *Non-negative constraint*

$$x_a^s(k) \ge 0, u_a^s(k) \ge 0, v_a^s(k) \ge 0, \quad \forall a, s, k \quad (22)$$

$$\hat{x}_p^s(k) \ge 0, \hat{u}_p^s(k) \ge 0, \hat{v}_p^s(k) \ge 0, \quad \forall p, s, k \quad (23)$$

6) *Boundary (initial) condition*

$$x_a^s(0) = 0, v_a^s(0) = 0, \quad \forall a, s \quad (24)$$

$$\hat{x}_p^s(0) = 0, \hat{v}_p^s(0) = 0, \quad \forall p, s \quad (25)$$

The set of feasible dynamic flows satisfying (14)~(29) is denoted as Ω.

VI problem equivalent to DUO condition based on instantaneous travel time can be written as theorem 3.

Theorem 3

Dynamic feasible flow set Ω is under the instantaneous DUO condition if and only if they satisfy the VI problem (VIP1).

(VIP1)

$$\sum_s \sum_a [\eta_m^{s*}(t) + c_a^*(t)][u_a^s(t) - u_a^{s*}(t)] \ge 0, \quad (26)$$

$$\forall u_a^s(t) \in \Omega$$

The proof of equivalence of VI (26) and the DUO condition can be seen in Gao and Ren [16].

It is known from the constraints that, $x_a^s(k)$, $x_p^s(k)$ can be obtained from (14), (15) according to the data of time interval $k - 1$, and $v_a^s(k)$, $\hat{v}_p^s(k)$, $v_e^s(k)$ can be obtained from (18), (19), (20) according to the data of previous time intervals. So, in time interval k, $x_a^s(k)$, $\hat{x}_p^s(k)$, $v_a^s(k)$, $v_e^s(k)$ (i.e. $\hat{u}_p^s(k)$), $\hat{v}_p^s(k)$ are known and the only variables are $u_a^s(k)$ (including $u_e^s(k)$). After the time range $[0, T]$ is discretized into $K+1$ equally small time intervals 0, 1, 2, ..., K, (26) is converted into (VIP2).

(VIP2)

$$\sum_s \sum_a [\eta_m^{s*}(k) + c_a^*(k)][u_a^s(k) - u_a^{s*}(k)] \ge 0, \quad (27)$$

$$\forall k = 0,\, 1,\, 2,\, \cdots, K$$

subject to

$$\sum_{a \in E_l} \sum_{i_{a,k} \in I_{a,k}} v_a^s(i_{a,k}) + g_l^s(k) = \sum_{a \in F_l} u_a^s(k), \quad \forall l, s, k \quad (28)$$

$$u_e(k) \le \min\,(\frac{C_e}{D_e}, C_e - x_e(k) + v_e(k)), \quad \forall e, s, k \quad (29)$$

$$u_a^s(k) \ge 0, \quad \forall a, s, k \quad (30)$$

5. SOLUTION ALGORITHM

In Diagonalization (relaxation) algorithm, the solution of VI problem (VIP2) is decomposed into a series of solutions of sub-problems. In the solution of each sub-problem, $\eta_m^s(k)$ is temporarily fixed as $\bar{\eta}_m^s(k)$. The sub-problem is (VIP3).

(VIP3)

$$\sum_s \sum_a [\bar{\eta}_m^{s*}(k) + c_a^*(k)][u_a^s(k) - u_a^{s*}(k)] \ge 0, \quad (31)$$

$$\forall k = 0,\, 1,\, 2,\, \cdots, K$$

subject to (28)~(30).

Each sub-problem corresponds to an equivalent non-linear programming problem (NLP1).

(NLP1)

$$\min_{\mathbf{u}^s} Z[\mathbf{u}(k)] = \sum_a \int_0^{u_a(k)} c_a(\omega)d\omega + \sum_s \sum_a \bar{\eta}_m^s(k)u_a^s(k), \quad (32)$$

$$\forall k = 0,\, 1,\, 2,\, \ldots,\, K$$

s.t (28)~(30).

where $\mathbf{u}^s(k)$ is a vector whose elements are $u_a^s(k)$, $\mathbf{u}(k)$ is a vector whose elements are $u_a(k)$. To prove the equivalence of (NLP1) and (VIP3) one only needs to prove that the gradient of (32) equals the cost item of (31).

$$\frac{\partial Z(\mathbf{u})}{\partial u_a^s(k)} = c_a(k) + \bar{\eta}_m^s(k), \quad \forall a, s, k \quad (33)$$

The Frank-Wolfe algorithm [17] is applied to solve (NLP1). In each iteration, solve a linear programming sub-problem (34)-(37).

$$\min_{\mathbf{h}^s(k)} \hat{Z} = \nabla_{\mathbf{u}^s(k)} Z[\mathbf{u}(k)]\mathbf{h}^{s^T}(k), \quad \forall k \quad (34)$$

s.t

$$\sum_{a\in E_l}\sum_{i_{a,k}\in I_{a,k}} v_a^s(i_{a,k}) + g_l^s(k) = \sum_{a\in F_l} h_a^s(k), \quad \forall l \neq p, s; s, k \qquad (35)$$

$$h_e(k) \leq \min(\frac{C_e}{D_e}, C_e - x_e(k) + v_e(k)), \quad \forall e, s, k \qquad (36)$$

$$h_a^s(k) \geq 0, \quad \forall a, s, k \qquad (37)$$

where $\mathbf{h}^s(k)$ is the feasible solution of the sub-problem (34)-(37) corresponding to $\mathbf{u}^s(k)$.

In the shortest path algorithm to find a feasible solution of $\mathbf{h}^s(k)$, the travel time on the road link is

$$t_a^s(k) = \frac{\partial Z(u)}{\partial u_a^s(k)} = c_a[u_a(k)] + \overline{\eta}_m^s(k), \quad \forall a, s, k \qquad (38)$$

Let us analyse the influence of capacity constraint of the passage on flow assignment. In *Figure 9*, assume the flow at node 3 to enter park 5 has three destinations, namely 6, 7, 8. Links 3→2 and 3→5 both belong to F_l ($l=3$), and 3→5 is the access passage of the park.
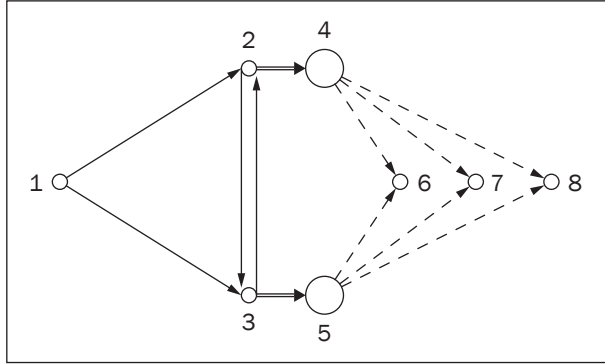

Figure 9 – Analysis under capacity constraint of the passage

In the Frank-Wolfe algorithm, all vehicles are assigned to the shortest path of (l, s) without capacity constraint of the passage. However, because of the capacity constraint of the passage, the shortest path to destination s not through $e \in F_l$ at node 3 needs to be found. Here, capacity constraints of road links are ignored. Suppose that at the gate ($l=3$) of the passage, the shortest paths to destinations 6, 7, 8 are L_1, L_2, L_3, and L_1, L_2, respectively, pass through the passage $e \in F_l$ but L_3 does not pass through $e \in F_l$. The shortest paths to destinations 6, 7 not through $e \in F_l$ are L_4, L_5, respectively. $h_a^s(k)$ can be obtained from solving (46)~(60).

$$\min(R_{L_1}y_1 + R_{L_4}y_2 + R_{L_2}y_3 + R_{L_5}y_4 + R_{L_3}y_5) \qquad (39)$$

s.t

$$\sum_{a\in F_l} h_a^s(k) = \sum_{a\in E_l}\sum_{i_{a,k}\in I_{a,k}} v_a^s(i_{a,k}) + g_l^s(k) = G_l^s(k), \qquad (40)$$

$$\forall l \neq p, s; s, k$$

$$y_1 + y_2 = G_3^6(k) \qquad (41)$$

$$y_3 + y_4 = G_3^7(k) \qquad (42)$$

$$y_5 = h_{3\to2}^8(k) = G_3^8(k) \qquad (43)$$

$$y_1 = h_{3\to5}^6(k) \qquad (44)$$

$$y_2 = h_{3\to2}^6(k) \qquad (45)$$

$$y_3 = h_{3\to5}^7(k) \qquad (46)$$

$$y_4 = h_{3\to2}^7(k) \qquad (47)$$

$$R_{L_1} = c_{3\to5}[u_{3\to5}(k)] + \overline{\eta}_5^6(k) \qquad (48)$$

$$R_{L_2} = c_{3\to5}[u_{3\to5}(k)] + \overline{\eta}_5^7(k) \qquad (49)$$

$$R_{L_3} = c_{3\to2}[u_{3\to2}(k)] + \overline{\eta}_2^8(k) \qquad (50)$$

$$R_{L_4} = c_{3\to2}[u_{3\to2}(k)] + \overline{\eta}_2^6(k) \qquad (51)$$

$$R_{L_5} = c_{3\to2}[u_{3\to2}(k)] + \overline{\eta}_2^7(k) \qquad (52)$$

and, the flows on paths L_1, L_2, which are through $e \in F_l$, satisfy

$$y_1 + y_3 \leq \min(\frac{C_{3\to5}}{D_{3\to5}}, C_{3\to5} - x_{3\to5}(k) + v_{3\to5}(k)) \qquad (53)$$

Solve (39)~(53) to calculate y_1, y_2, y_3, y_4, y_5 and then $h_a^s(k)$, $a \in F_3$.

Steps of Diagonalization (relaxation) algorithm are:

Step 0 – For time interval k, the actual travel time on link a ($\forall a$) is rounded to an integer ($\tau_a^0(k)$) and the value of $\tau_a^0(k)$ ($\forall a$) has been supposed. Calculate set $I_{a,k}^0$. Let $N=0$.

Step 1 – Initial conditions: $x_a^s(0) = 0$, $v_a^s(0) = 0$ ($\forall a, s$), $\hat{x}_p^s(0) = 0$, $\hat{v}_p^s(0) = 0$ ($\forall p, s$). Let $k=0$.

Step 2 – According to link-free flow travel time, carry out all-or-nothing assignment (if it is at the entrance node of access passage of the park, then calculate (39)~(53)) of flows $\sum_{a\in E_l}\sum_{i_{a,k}\in I_{a,k}^{s^N}} v_a^{s^N}(i_{a,k}) + g_l^s(k), \quad \forall l \neq p, s; s, k$

($g_l^s(0), \forall l \neq p, s; s, k$ if $k=0$) to obtain initial rate of inflow ($u_a^{(0)}(k)$) of links. Let $n=1$.

Step 3 – Calculate instantaneous travel time on each link $c_a^{(n-1)}(k) = c_a[x_a^N(k), u_a^{(n-1)}(k), v_a^N(k)] + \overline{\eta}_m^{s^{N-1}}(k)$,

where $\overline{\eta}_m^{s^{N-1}}(k) = \overline{\eta}_m^{s^{N-1}}[\mathbf{x}^N(k), \mathbf{u}^{N-1}(k), \mathbf{v}^N(k)]$ is a temporarily fixed value of the shortest instantaneous path travel time from m to s. Carry out all-or-nothing assignment (if it is at the entrance node of access passage of the park, then calculate (39)~(53)) of flows $\sum_{a\in E_l}\sum_{i_{a,k}\in I_{a,k}^N} v_a^{s^N}(i_{a,k}) + g_l^s(k), \quad \forall l \neq p, s; s, k$

($g_l^s(0), \forall l \neq p, s; s, k$ if $k=0$) to obtain feasible rate of in-flow of links $\{h_a^{s^{(n-1)}}(k)\}$ and then $\{h_a^{(n-1)}(k)\}$.

Step 4 – One-dimensional search. Solve

$$\min_{0 \le \gamma \le 1} Z = \sum_a \int_0^{u_a^{s(n-1)}(k)+\gamma[h_a^{s(n-1)}(k)-u_a^{s(n-1)}(k)]} c_a(\omega)d\omega +$$
$$+ \sum_a \sum_s \overline{\eta}_m^{s^{N-1}}(k)\{u_a^{s^{(n-1)}}(k) + \gamma[h_a^{s^{(n-1)}}(k) - u_a^{s^{(n-1)}}(k)]\}$$

to obtain step length $\gamma^{(n-1)}$.

Step 5 – Flows update: let

$$u_a^{s^{(n)}}(k) = u_a^{s^{(n-1)}}(k) + \gamma^{(n-1)}[h_a^{s^{(n-1)}}(k) - u_a^{s^{(n-1)}}(k)], \ (\forall a, s)$$

and then calculate $\{u_a^{(n)}(k)$.

Step 6 – If $n = n_m$ (n_m is the set maximal iterative number for the inner loop), then turn to step 7; while if $n < n_m$, then let $n = n + 1$ and turn to step 3.

Step 7 – If $k = K$, then turn to step 8; else, set $k = k + 1$ and turn to step 2.

Step 8 – Convergence check. Estimate actual travel time on links $\tau_a^N(k) (\forall k)$. If $\tau_a^N(k) \approx \tau_a^{N-1}(k)$, then $u_a(k) = u_a^{(n)^N}(k) (\forall a, k)$, where $u_a^{(n)^N}(k)$ is the value of $u_a^{(n)}(k)$ at the Nth iteration of outer loop, end; else, $u_a^N(k) = u_a^{(n)^N}(k)$, calculate $I_{a,k}^N$, let $N = N + 1$ and turn to step 1.

The number of iterations of the outer loop required relates to that of the inner layer. If the number of iterations of the outer loop is large, that of the inner loop can be small, meaning that Frank-Wolfe algorithm of the inner loop (steps 3~6) does not have to be convergent. This is called simplified diagonalization algorithm, and the whole convergence is good when the number of iterations of the inner loop takes 2~3 [18]. The algorithm (steps 0~8) ends when the actual travel time does not change or the change is small [8].

6. NUMERICAL EXAMPLE

In *Figure 3*, the total number of time intervals $K = 20$, and parking durations of vehicles entering parks (nodes 4 and 5) at each time point are shown in *Table 1*. O-D (origin-destination) traffic demands are in *Table 2*. The number of parking spaces in parks: $Q_4 = 10$, $Q_5 = 10$. The time for walking egress trip from parks to the destination: $B_{4,6} = 1$, $B_{5,6} = 1$. The maximal number of queuing vehicles that the access passage of parks can accommodate at the same time: $C_{2\to4} = 5$, $C_{3\to5} = 5$.

Instantaneous travel time on links:

$c_a(t) = 0.5 + c_{a,0} + 0.001(u_a(t))^2 + 0.001(v_a(t))^2 + 0.005(x_a(t))^2$, $\forall a, t$, where $c_{a,0}$ is the free flow travel time on link a, and $c_{1\to2,0} = 3$, $c_{1\to3,0} = 3.5$, $c_{2\to3,0} = 0.5$, $c_{3\to2,0} = 0.5$. The initial actual travel times on links are assumed as: $\tau_{1\to2}(k) = 2$, $\tau_{1\to3}(k) = 2$, $\tau_{2\to3}(k) = 2$, $\tau_{3\to2}(k) = 2$, $k = 0, 1, 2, ..., 19$. Here, suppose $\tau_a(t) = \text{round}(c_a(t))$. $D_{2\to4} = 1$, $D_{3\to5} = 1$, then (5) is simplified as $v_e(k) = \min(x_e(k), Q_p - x_p(k) + v_p(k))$. Assume instantaneous travel time on access passages of parks without parking information available:

If $v_{2\to4}(k) > 0$, then $c_{2\to4}(k) = \max(1, \text{round}(\frac{x_{2\to4}(k)}{v_{2\to4}(k)}))$;

If $v_{3\to5}(k) > 0$, then $c_{3\to5}(k) = 1 + \text{round}(\frac{x_{3\to5}(k)}{v_{3\to5}(k)})$;

and when vehicles queuing occurs in the passage but $v_e(k) = 0$, $c_e(k)$ takes a very large value, which is supposed to be infinity; while when $v_e(k) = 0$ but no vehicles queuing occurs in the passage, $c_e(k) = 0$.

The precision takes 0.01, and the solutions from diagonalization algorithm are in *Table 3*. When k is time intervals 0~4, 8, 15, 17, 18 or 19, $v_{2\to4}(k) = 0$, $x_{2\to4}(k) = 0$ which means no vehicles are queuing in the passage, so $c_{2\to4}(k) = 1$ according to the supposition. When k is time intervals 5, 6, 7, 9, 10, 12, 14 or 16,

$c_{2\to4}(k) = \max(1, \text{round}(\frac{x_{2\to4}(k)}{v_{2\to4}(k)})) = 1$. When k is time interval 11, $c_{2\to4}(k) = \max(1, \text{round}(\frac{x_{2\to4}(k)}{v_{2\to4}(k)})) = 2$. When k is time interval 13, $x_{2\to4}(k) = 1.5625$ but $v_{2\to4}(k) = 0$, so $c_{2\to4}(k)$ takes a large value. When k is time intervals 0~6, 10, 12, 14 or 16~19, $v_{3\to5}(k) = 0$, $x_{3\to5}(k) = 0$ which means no vehicles are queuing in the passage, so $c_{3\to5}(k) = 1$. When k is time intervals 7, 8, 9, 11, 13 or 15, $c_{3\to5}(k) = 1 + \text{round}(\frac{x_{3\to5}(k)}{v_{3\to5}(k)}) = 2$.

In time interval 5, 6 units of flows come out of link $1\to2$, meanwhile 4 units of flows just go from passage $2\to4$ to the park. There have been 4 units of flows in the passage $2\to4$ by the end of time interval 4, so 5 units of flows can go into the passage $2\to4$ in time interval 5, and the remaining 1 unit of flows go from node 2 to

Table 1 – Parking durations of vehicles entering parks at each time point

Parking durations	Time points (k)																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$\tau_4(k)$	3	4	4	5	3	2	2	4	4	3	4	4	2	3	2	3	1	1	4	1
$\tau_5(k)$	2	1	5	4	3	4	5	3	2	5	2	4	2	4	3	1	2	3	1	3

Table 2 – O-D traffic demands

O-D	Time intervals (k)																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$1\to6$	4	6	4	5	5	6	4	6	4	5	0	0	0	0	0	0	0	0	0	0

node 3 and then go into the passage 3→5 in one time interval and finally park at the park node 5 to walk to destination node 6. At time point 7, the amount of flows in the passage 3→5 is 3.0020, which is composed of 2.0020 coming out of link 1→3 and 1 coming out of link 2→3 in time interval 6. In time interval 9, 6 units of flows come out of link 1→2, meanwhile 3.4375 units of flows just go from passage 2→4 to the park. There have been 3.4375 units of flows in the passage 2→4 by the end of time interval 8, so 5 units of flows can go into the

passage 2→4 in time interval 9, and the remaining 1 unit of flows go from node 2 to node 3 and then go into the passage 3→5 in one time interval and finally park at the park node 5 to walk to destination node 6.

In time interval 11, rate of out-flow of link 1→2 is 6, in which 2.5625 units of flows go into the passage 2→4, and the remaining 3.4375 units of flows go from node 2 to node 3 and then arrive at node 3 in one time interval (namely in time interval 12). But at node 3, not all flows go to the park. The accumulated flows

Table 3 – Solutions of test network

Links	Time intervals	Rate of in-flow	Rate of out-flow	Flows	Instantaneous travel time on links	Passages or parks	Time intervals	Rate of in-flow	Rate of out flow	Flows	Instantaneous travel time on passages
1→2	0	4	0	0	3.5160	2→4	0	0	0	0	1
	1	6	0	4	3.6160		1	0	0	0	1
	2	1.9980	0	10	4.0040		2	0	0	0	1
	3	0	0	11.9980	4.2198		3	0	0	0	1
	4	3.4351	4	11.9980	4.2476		4	4	0	0	1
	5	6	6	11.4331	4.2256		5	5	4	4	1
	6	4.0000	1.9980	11.4331	4.1736		6	1.9980	5	5	1
	7	6.0000	0	13.4351	4.4385		7	0	1.9980	1.9980	1
	8	0	3.4351	19.4351	5.4004		8	3.4375	0	0	1
	9	5	6	16	4.8410		9	5	3.4375	3.4375	1
	10	0	4	15	4.6410		10	4	4.5645	5	1
	11	0	6	11	4.1410		11	2.5625	1.9980	4.4355	2
	12	0	0	5	3.6250		12	0	3.4375	5	1
	13	0	5	5	3.6500		13	0	0	1.5625	infinity
	14	0	0	0	3.5000		14	0	1.5625	1.5625	1
	15	0	0	0	3.5000		15	2.4375	0	0	1
	16	0	0	0	3.5000		16	0	2.4375	2.4375	1
	17	0	0	0	3.5000		17	0	0	0	1
	18	0	0	0	3.5000		18	0	0	0	1
	19	0	0	0	3.5000		19	0	0	0	1
1→3	0	0	0	0	4.0000	3→5	0	0	0	0	1
	1	0	0	0	4.0000		1	0	0	0	1
	2	2.0020	0	0	4.0040		2	0	0	0	1
	3	5	0	2.0020	4.0450		3	0	0	0	1
	4	1.5649	0	7.0020	4.2476		4	0	0	0	1
	5	0	0	8.5669	4.3670		5	0	0	0	1
	6	0	2.0020	8.5669	4.3710		6	3.0020	0	0	1
	7	0	5	6.5649	4.2405		7	4.9976	3.0020	3.0020	2
	8	4.0000	1.5649	1.5649	4.0307		8	1.5649	4.9976	4.9976	2
	9	0	0	4	4.0800		9	0	1.5649	1.5649	2
	10	0	0	4	4.0800		10	1	0	0	1
	11	0	0	4	4.0800		11	0	1	1	2
	12	0	4	4	4.0960		12	5	0	0	1
	13	0	0	0	4.0000		13	0	5	5	2
	14	0	0	0	4.0000		14	5	0	0	1
	15	0	0	0	4.0000		15	0	5	5	2
	16	0	0	0	4.0000		16	0	0	0	1
	17	0	0	0	4.0000		17	0	0	0	1
	18	0	0	0	4.0000		18	0	0	0	1
	19	0	0	0	4.0000		19	0	0	0	1

(7.4375) at node 3 in time interval 12 is composed of 4 from link 1→3 and 3.4375 from link 2→3, in which 5 units of flows go into the passage 3→5 and the remaining 2.4375 go back to node 2 (namely 3→2), so cruising traffic appears here. The accumulated flows (7.4375) at node 2 in time interval 13 is composed of back flows (2.4375) just from 3→2 and 5 units of flows coming out of link 1→2. Here, vehicles queue in the access passage of the park (park node 4) but rate of out-flow of the passage is 0, so $c_e(k)$ takes a large value according to the supposition, and then these 7.4375 units of flows go from node 2 to node 3 and arrive at node 3 in one time interval (namely in time interval 14). But at node 3, not all flows go to the park. In time interval 14, 5 of 7.4375 units of flows go into the passage and the remaining 2.4375 go back to node 2, so here cruising traffic appears. These 2.4375 units of flows arrive at node 2 in one time interval (namely in time interval 15) and all go into passage 2→4, and finally park at the park node 4 to walk to destination 6.

Table 3 (continued) – Solutions of test network

Links	Time intervals	Rate of in-flow	Rate of out-flow	Flows	Instantaneous travel time on links	Passages or parks	Time intervals	Rate of in-flow	Rate of out-flow	Flows	Instantaneous travel time on passages
2→3	0	0	0	0	1.0000	4	0	0	0	0	-
	1	0	0	0	1.0000		1	0	0	0	-
	2	0	0	0	1.0000		2	0	0	0	-
	3	0	0	0	1.0000		3	0	0	0	-
	4	0	0	0	1.0000		4	0	0	0	-
	5	1	0	0	1.0010		5	4	0	0	-
	6	0	1	1	1.0060		6	5	0	4	-
	7	0	0	0	1.0000		7	1.9980	4	9	-
	8	0	0	0	1.0000		8	0	5	6.9980	-
	9	1	0	0	1.0010		9	3.4375	0	1.9980	-
	10	0	1	1	1.0060		10	4.5645	0	5.4355	-
	11	3.4375	0	0	1.0118		11	1.9980	1.9980	10	-
	12	0	3.4375	3.4375	1.0709		12	3.4375	3.4375	10	-
	13	7.4375	0	0	1.0553		13	0	0	10	-
	14	0	7.4375	7.4375	1.3319		14	1.5625	8.0020	10	-
	15	0	0	0	1.0000		15	0	1.9980	3.5605	-
	16	0	0	0	1.0000		16	2.4375	1.5625	1.5625	-
	17	0	0	0	1.0000		17	0	2.4375	2.4375	-
	18	0	0	0	1.0000		18	0	0	0	-
	19	0	0	0	1.0000		19	0	0	0	-
3→2	0	0	0	0	1.0000	5	0	0	0	0	-
	1	0	0	0	1.0000		1	0	0	0	-
	2	0	0	0	1.0000		2	0	0	0	-
	3	0	0	0	1.0000		3	0	0	0	-
	4	0	0	0	1.0000		4	0	0	0	-
	5	0	0	0	1.0000		5	0	0	0	-
	6	0	0	0	1.0000		6	0	0	0	-
	7	0.0024	0	0	1.0000		7	3.0020	0	0	-
	8	0	0.0024	0.0024	1.0000		8	4.9976	0	3.0020	-
	9	0	0	0	1.0000		9	1.5649	0	7.9995	-
	10	0	0	0	1.0000		10	0	7.9995	9.5645	-
	11	0	0	0	1.0000		11	1	0	1.5649	-
	12	2.4375	0	0	1.0059		12	0	0	2.5649	-
	13	0	2.4375	2.4375	1.0356		13	5	0	2.5649	-
	14	2.4375	0	0	1.0059		14	0	1.5649	7.5649	-
	15	0	2.4375	2.4375	1.0356		15	5	1	6	-
	16	0	0	0	1.0000		16	0	5	10	-
	17	0	0	0	1.0000		17	0	5	5	-
	18	0	0	0	1.0000		18	0	0	0	-
	19	0	0	0	1.0000		19	0	0	0	-

7. CONCLUSIONS AND PROSPECTS

This paper has presented a dynamic parking choice model to simulate drivers' search for parking spaces between parking lots and then to obtain the data of vehicles going into or out of the park timely, traffic flows on the surrounding road network, and the amount and routes of cruising traffic owing to parking search. Modelling dynamic process of parking search helps evaluate whether the number and layout of parking lots is able to satisfy dynamic parking demands, so a new methodology is provided for parking planning. The model considers the parking choice at the end of the trip in modelling dynamic route choice behaviours of drivers, can give not only the number of vehicles going into or out of parking lots but also searching routes of vehicles for parking spaces, so it is suitable for evaluating the "ineffective traffic" and traffic congestion to surrounding networks caused by parking cruising. Therefore, changing generalized impedance involving time and expense of each route through changing parking charge of each park can change parking choices of vehicles, control cruising traffic and thus alleviate traffic congestion.

This paper intensively analyses flow propagation formula in the access passage of the parking lots. We believe this formula can be borrowed by flow propagation analysis for the congested road links. Traffic flows on the access passage of parking lots are like those on the congested links in road networks, so a similar analysis method can be applied there in the future work.

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驾驶员动态停车选择行为模型

摘要

考虑时变的停车需求及停车泊位供给，本文建立动态停车选择行为模型，模拟车辆驾驶员在各停车点的选择与搜索过程。在本研究中，每个停车场有且仅有一个进口通道，车辆通过进入口通道进入与其相连的停车场，车辆可在停车场的入口通道内行驶或排队等待进入停车场。建立了车辆在停车场（库）入口通道的车流流量传播方程，以计算每个时间间隔内从入口通道进入停车场（库）的车辆数。提出了动态停车选择的瞬时动态用户最优条件，并建立了等价的变分不等式模型，提出对角化算法对模型进行求解。通过一个算例对模型与算法进行了验证，可得到各个时刻各条路段及停车场入口通道的流入率、流出率、流量、瞬时阻抗以及巡游交通流流量与流向。

关键词

停车；动态；用户最有；变分不等式(VI)

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