DUŠAN TEICHMANN, Ph.D.
E-mail: dusan.teichmann@vsb.cz
MICHAL DORDA, Ph.D.
E-mail: michal.dorda@vsb.cz
Faculty of Mechanical Engineering, Institute of Transport,
Technical University of Ostrava
17. listopadu 15/2172, 708 33 Ostrava - Poruba,
Czech Republic
HELENA BÍNOVÁ, Ph.D.
E-mail: binova@fd.cvut.cz
Faculty of Transportation Sciences,
Department of Logistics and Management of Transport,
Czech Technical University in Prague
Konviktská 20, 110 00 Praha,
Czech Republic
MARTIN LUDVÍK
E-mail: ludvik@hsfsystem.cz
Faculty of Mechanical Engineering, Institute of Transport,
Technical University of Ostrava
17. listopadu 15/2172, 708 33 Ostrava - Poruba,
Czech Republic

Traffic in the Cities
Preliminary Communication
Submitted: July 3, 2014
Approved: Feb. 3, 2015
http://dx.doi.org/10.7307/ptt.v27i1.1593

OPTIMIZATION OF PARKING PUBLIC TRANSPORT VEHICLES IN OSTRAVA

ABSTRACT

A typical trait of public transport is a spatially scattered demand. A route net that is operated by a carrier (or several carriers) has to be adapted to the demand. Public transport vehicles that are not used during a period of a day are usually parked in defined parking lots that have a given capacity. When the vehicle goes from the place where its schedule ends (usually a terminus of the last connection served by the vehicle) to the place where the vehicle should be parked, a non-productive journey occurs. The same occurs at the beginning of the vehicle schedule as well. The main goal of the paper is to present a mathematical model that enables minimization of the total length of all the non-productive journeys. Functionality of the proposed mathematical model was tested in the conditions of a real bus public transport network.

KEY WORDS

optimization; mathematical model; linear programming; public transport;

1. INTRODUCTION – WHY IS IT IMPORTANT TO SOLVE THE PROBLEM?

In 2005 an essential operating change occurred in the conditions of bus public transport in the city of Ostrava. Garages and parking places for vehicles located in the central urban district Moravska Ostrava were closed down. This fact resulted in the need for a decision how the vehicles that had been parked in this locality should be assigned to other parking lots.

However, the problem of vehicle parking optimization in public transport is not only the problem arising from such changes. Public transport operation is a dynamical process that is determined by national deadlines enabling changes in transport organization and operation. In these deadlines some minor or major changes can be done in public transport organization. The most frequent changes are in the route net, in times of individual connections or in vehicle scheduling. These are the operational factors that can influence the total length of all the unproductive journeys. It is obvious that the high sum of the unproductive journey lengths (deadhead kilometres) influences negatively the public transport effectiveness and increases the total amount of necessary subsidies. If public transport is financed so that a provider of subsidy contracts the number of kilometres being subsidized per year with a carrier, the high amount of the non-productive journeys has an impact on the productive journeys because the carrier has fewer kilometres for the productive journeys.

The presented paper lays stress on a presentation of a possible method how to solve the problem of assigning the vehicles to the parking lots under the condition that the total amount of non-productive (dead-head) kilometres is maximally reduced. To solve the problem mathematical programming is used, that is suitable for solving such type of the planning tasks. The optimization software Xpress-IVE [20] that is very universal is used.

It should be noted that such situations that have a trivial solution will not be solved. An example can be mentioned, if the carrier utilizes only a single parking lot. Another trivial example is if there is only a single place where the vehicles start their daily schedule and their schedule ends in the same place.

2. STATE OF THE ART

Mathematical programming offers very effective methods that were employed for solving a lot of practical problems in transport in the past. In the area of public transport, mathematical programming was most often used for planning a route net [1–6] or vehicle scheduling planning [14–17]. Mathematical programming was also used for example to solve a problem of assigning the vehicles to individual routes depending on round-trip times [18] or with acceptation of a request on approximate homogeneity from the point of view of the vehicle capacity [19]. Mathematical programming was also successfully used for time coordination of public transport connections [12] or designing zone tariffs in integrated transport systems [11]. The methods of mathematical programming are often combined with heuristic methods for solving the problems of public transport; for example sources [1], [2] or [10] can be mentioned. Thanks to the fact that a lot of universal computationally high-performance tools (such as Xpress-IVE) are available, calculation times are not a limiting factor when solving extensive practical problems and therefore mathematical programming has a big potential for solving the practical problems of public transport.

The problem of assigning the vehicles to the defined parking lots can be considered to be a partial part of a more general task about vehicle scheduling [14–17]. Such type of the task was successfully programmed in the past (the program KASTOR [14]) and repeatedly applied in suburban public transport [4]. It resulted in substantial savings in the running costs of bus carriers that were caused by minimizing the number of vehicles and the value of non-productive kilometres. But in practice there can be some situations when it is not possible to essentially decrease the number of vehicles by planning their journeys among the routes (the vehicles do not go from a connection of a route to a connection of a different route). In such cases the main advantage of the optimization process that the system KASTOR utilized is lost. The main reasons are obvious – the number of the vehicles cannot be decreased and the total sum of the non-productive kilometres increases. An exact border which would say when the program KASTOR can bring substantial savings is not known to the authors of the paper. The value of potential savings probably depends on several operating parameters of the routes that are run (headways, distances between the termini and so on). The authors estimate that the significance of the effects got by the program KASTOR strongly decreases in the situations when the headway in public transport does not exceed 30 minutes.

In such situation it makes sense to search for an alternative approach to optimization of savings when planning vehicle scheduling. The savings can be found when arrivals of vehicles in the beginning of their daily schedule and their returns to a depot are planned. It is not difficult to find out that such alternative approach does exist. It is known that in our case places from which the vehicles have to go to serve the first connection (the parking lots) and places where the vehicle starts its first productive journey (usually the bus terminus where the vehicle starts its daily schedule) are given. On the other hand the places where the vehicles finish their daily schedule (usually the bus terminus of the last connection the vehicle has to serve) and the places where the vehicles are parked are also known. Let us assume that the lengths of both types of non-productive journeys are given. It is more than obvious that this task resembles the transportation problem that was formulated in [8] and for its solving we know all the necessary data. The parking lots from which the vehicles depart to serve the first connection of their daily schedule and the bus termini where the vehicles finish their last productive journey according to their schedule represent sources in the transportation problem. On the other hand, the bus termini where the vehicles start their first productive journey (serving the first connection) and the places where the vehicles are parked after serving the last connection represent destinations of transportation problem. The lengths of the non-productive journeys correspond to the unit costs that are used in the transportation problem. A decision should be made about the number of the non-productive journeys among the individual places so that the total length of all the non-productive journeys is minimal.

Due to the fact that the vehicles run non-productively in two phases (before the first connection and after the last connection), the original mathematical model of the transportation problem needs not be sufficient for solving the task and therefore it is necessary to modify the original model for the conditions of the concrete carrier.

The paper is limited only to optimization of non-productive journeys before the first connection and after the last connection of the planned vehicle schedule during working days. This means that the vehicle schedule between the first connection and the last one is not changed. Moreover, we also optimize parking of only one kind of vehicles (trams, buses, trolleybuses ...). This is because it is not usually possible (or it is very complicated) to use parking lots intended for one kind of the vehicles for parking other kinds of the vehicles. For example, the tram parking lots cannot be used for parking the buses.

3. MATHEMATICAL MODEL

It is possible to take into consideration two modifications of the transportation problem for solving our task – the balanced transportation problem and the unbalanced transportation problem with the excess of the capacities over the demand. However, our task is more complicated. The complications follow from the operational conditions of the concrete carrier and they are the following (please note that the following constraints are the constraints for the Ostrava Transport, joint-stock co.):

1. The carrier uses different vehicles but they are the same kind (for example only buses).
2. The concrete vehicle is assigned to the single parking lot (it is due to record keeping, for example all the technical documentation of the concrete vehicle is available in this parking lot) – the vehicle departs from the parking lot to serve the first connection and after serving the last connection the vehicle returns back.
3. Sometimes it can happen that the vehicle starts to serve the first connection and finishes the last connection at different termini.
4. For some kinds of the vehicles it is necessary to build up specialized facilities, therefore it is requested to park all of the vehicles in the same parking lot. Let us consider the situation that the carrier owns more kinds of vehicles that must be parked in the same parking lot. For solving such problem two variants exist. The first variant enables parking of all vehicles in the same parking lot. The second variant admits that parking of the vehicles can be in several parking lots.

Let us pay attention to the transportation problem and its modification that is necessary for solving the problem. At first, let us characterize what destinations (or customers) of the original transportation problem and their demands represent our task. In our case, the destinations correspond to the places where non-productive journeys are terminated. It results from the previous text that we have two types of destinations. The first type consists of the bus termini where the individual vehicles start their daily schedules; the second type corresponds to the parking lots in which the vehicles are parked after finishing their daily schedules. Note that the places need not be the same. Thus it is necessary to use a different way of how to formulate the destinations and their demands (or requests). At first, let us define the term – cumulated place. The cumulated place has its importance in the cases when the bus terminus where the vehicle starts the schedule differs from the bus terminus of the last connection. In such cases the fictional cumulated place represents both of the termini described in the previous sentence. The request of each cumulated place is given by the number of vehicles for which their schedule starts or terminates at the bus terminus represented by this cumulated place. Establishing the fictional cumulated places is very important. Using them we are able to model various non-productive journeys of the vehicles and the original information about the real value of non-productive covered distances is kept.

Let us explain the cumulated places on an example. Let us have two bus termini A and B. If the vehicle starts its daily schedule at the bus terminus A and its schedule ends at the same bus terminus A, then the cumulated place for this vehicle is the original bus terminus A. The non-productive covered distance corresponds to a sum of the distance from the parking lot to the bus terminus A and back in this case. If the vehicle starts at the bus terminus A and finishes at the bus terminus B, then the cumulated place for this vehicle is a new fictional place A-B. The non-productive covered distance is the distance from the parking lot to the bus terminus A plus the distance from the bus terminus B to the parking lot.

Now let us discuss what the sources and their capacities of the original transportation problem represent in our case. The sources correspond to the places at which the non-productive journeys begin. From the previous text it is known that there are two types of the sources. The first type is represented by the parking lots from which the vehicles depart to serve their first connections in the frame of the planned schedule. The second type comprises the bus termini of the last connections according to the schedules. Thanks to the definition of the cumulated place, that also includes information about the second type of the sources; we consider only the parking lots to be the sources of our modified transportation problem.

Finally, we can define the meaning of the last group of entry data – the rates that influence the value of the objective function. In the original transportation problem the rates represent how much a unit carriage from the source to the destination costs. In our case the rates will correspond to the distances among the sources (the parking lots) and the destinations (the cumulated places). Let us presume that the vehicle begins to serve the first connection in the frame of its schedule at the same bus terminus where the vehicle finishes the service of the last scheduled connections. In this case the rate for the individual parking lot is calculated as a sum of the distance from the parking lot to the bus terminus and back. Now let us consider that the vehicle begins to serve the first connection at the bus terminus that differs from the bus terminus at which the vehicle finishes the last scheduled connection. In this case the rate for the individual parking place is calculated as a sum of the distance from the parking lot to the first bus terminus of the vehicle schedule and the distance from the last bus terminus of vehicle schedule to the parking lot.

Formulation of the mathematical model:

Let a set of the vehicle types I be given for the solved kind of the vehicles. For each type of solved kind of the vehicles $i \in I$ its parking norm p_i is defined and it is given if all the vehicles of the individual type have to be parked together in the same parking lot or not. The parking norm represents how many parking places the individual vehicle needs for parking. Using the parking norm in the model enables us to model parking of vehicles with different needs of parking places if it is necessary. Therefore, the vehicle set I is divided into two subsets denoted I_1 and I_2. Let us assume that subset I_1 contains all the vehicle types for which it is requested that all the vehicles from the individual vehicle type have to be parked together in a single parking lot. Subset I_2 includes such vehicle types for which the carrier does not request that all of them have to be parked in the same parking lot. For the sets I, I_1 and I_2 it holds that $I_1 \cup I_2 = I$ and $I_1 \cap I_2 = \emptyset$.

For each vehicle type a set of the possible parking lots J and a set of cumulated places K are given. For each parking lot $j \in J$ its capacity a_j is known. For each cumulated place $k \in K$ its request b_{ik} for the vehicles of the type $i \in I$ is defined. For each journey between the parking lot $j \in J$ and the cumulated place $k \in K$ the distance c_{jk} in kilometres is known (the distance does not depend on the vehicle type). Our task is to decide about how many vehicles run between the individual parking lots and the individual cumulated places so that the total non-productive covered distance is minimal.

In order to write down the mathematical model it is necessary to establish two groups of variables:

- x_{ijk} – the number of vehicles of type $i \in I$ running between the parking lot $j \in J$ and the cumulated place $k \in K$,
- y_{ij} – an auxiliary bivalent variable that models the decision about parking the vehicles of the type $i \in I$ in the parking lot $j \in J$. If $y_{ij} = 1$ then the vehicles of type $i \in I$ will be parked in the parking lot $j \in J$. If $y_{ij} = 0$ then the vehicles of type $i \in I$ will not be parked in the parking lot $j \in J$.

The mathematical model has the following form:

$$\min f(x,y) = \sum_{j \in J} \sum_{k \in K} c_{jk} \sum_{i \in I_1 \cup I_2} x_{ijk} \qquad (1)$$

subject to:

$$\sum_{i \in I_1 \cup I_2} p_i \sum_{k \in K} x_{ijk} \leq a_j \ \text{ for } j \in J, \qquad (2)$$

$$\sum_{j \in J} x_{ijk} = b_{ik} \ \text{ for } i \in I_1 \cup I_2 \text{ and } k \in K, \qquad (3)$$

$$\sum_{k \in K} x_{ijk} \leq y_{ij} T \ \text{ for } i \in I_1 \text{ and } j \in J, \qquad (4)$$

$$\sum_{j \in J} y_{ij} = 1 \ \text{ for } i \in I_1, \qquad (5)$$

$$x_{ijk} \geq 0 \ \text{ for } i \in I_1 \cup I_2,\ j \in J \text{ and } k \in K, \qquad (6)$$

$$y_{ij} \in \{0,1\} \ \text{ for } i \in I_1 \text{ and } j \in J. \qquad (7)$$

It is supposed in the presented model that the parking lots have their capacities so that all the vehicles can be parked in the parking lots.

Function (1) expresses the total distance that is covered non-productively (deadhead kilometres). Let us briefly discuss why the non-productive covered distance is used as the objective criterion. It is clear that it is possible to consider also other criteria such as fuel consumption or emission of pollutants when deadheading (please note that using a different optimization criterion can change the results of the model). However, the city of Ostrava, which is the founder and the subsidizer, evaluates traffic performance of Ostrava Transport, joint-stock co. according to the total covered distance. Currently in the Czech Republic, it is usual that the amount of subsidy is lower than it was in the past. Ostrava Transport, joint-stock co. is therefore forced to reduce the total covered distance due to the decreasing subsidy. Reducing the non-productive covered distance is the most natural way how to reduce the total covered distance with no influence on the productive covered distance.

The group of constraints (2) ensures that the given capacity of each parking lot will not be exceeded. The group of constraints (3) assures that all the planned vehicle schedules will be served. The groups of constraints (4), (5) and (7) are active only when subset I_1 is not empty – $I_1 \neq \emptyset$. The group of constraints (4) ensures logical links among the groups of the variables x_{ijk} and y_{ij}. The group of constraints (5) assures that the request that all the vehicles of type $i \in I$ must be parked in the same parking lot will be satisfied. The groups of constraints (6) and (7) define domains of definition for all the variables of the model.

Symbol T represents a suitably chosen prohibitive constant. To assess its value before the beginning of the optimization calculation for example formula (8) can be applied:

$$T = \max_{i \in I_1, k \in K} \{b_{ik}\} \qquad (8)$$

4. GENERAL DESCRIPTION OF OPTIMIZATION ALGORITHM

The optimization algorithm consists of several steps:

Step 1 – create a list of the parking places (set J). The list of the parking places forms a list of the sources in the presented transportation problem modification.

Step 2 – define capacity a_j for each parking place $j \in J$ (how many vehicles can be assigned to the parking place).

Step 3 – for each vehicle find out the bus terminus where the vehicle starts its daily schedule (the initial bus terminus) and the bus terminus where the vehicle ends its daily schedule (the final bus terminus).

Step 4 – on the basis of existing combinations of the initial and final bus termini create a list of the cumulated places (set K). The list of the cumulated places forms the list of the customers in the presented transportation problem modification.

Step 5 – for each cumulated place $k \in K$ define its request b_{ik} – the request represents how many schedules that are served by the vehicle type $i \in I$ starts at the initial bus terminus or ends at the final bus terminus that form the cumulated place k.

Step 6 – calculate the non-productive covered distances between the individual parking lots and the individual cumulated places – the values represent the coefficients of the objective function.

Step 7 – create the mathematical model and implement it in suitable software on the basis of input data you have got in the previous steps. After successful implementation solve the mathematical model.

Step 8 – calculate the savings in the non-productive covered distance (NCDS) according to formula (9):

$$NCDS = \frac{NCD_b - NCD_a}{NCD_b} \cdot 100 , \tag{9}$$

where:

- $NCDS$ – the savings in the non-productive covered distance,
- NCD_b – the non-productive covered distance before the optimization experiment,
- NCD_a – the non-productive covered distance after the optimization experiment.

5. NUMERICAL CALCULATION

The presented mathematical model was tested using an example from the practice. The model was applied to plan the non-productive journeys of the buses in the conditions of Ostrava Transport, joint-stock co.

The total number of the buses that were included in the calculation experiment is equal to 104; their types are as follows: ten buses are Fiat or Mercedes, seven buses are Renault City Bus and the other vehicles are of other types. For the buses Fiat and Mercedes there is a request that they have to be parked together in the same parking lot; the same must be ensured for all the buses Renault City Bus as well. According to the notation we established in the previous section it holds that $|I| = 3$, $|I_1| = 2$ and $|I_2| = 1$. For the experiments, the applied parking norm was equal to $p_i = 1$ for $i = 1,\ldots,3$ because all the vehicle types have the same need for parking places. In this case the original model can be simplified and the parking norm p_i can be omitted in formula (2).

The vehicles can be parked in three parking lots that are situated in different urban districts, which means $|J| = 3$. For the purposes of the experiment the parking lots were named as follows – Slezska Ostrava, Martinov and Poruba. The total capacity of the parking lots is 160 parking places; please note that in this case parking for 104 vehicles is planned. The capacity of the parking lots is listed in *Table 1*. Due to the fact that the number of vehicles is less than the total capacity of all the parking lots, it is necessary to use the modified mathematical model that has the group of the constraints in form (2).

For each vehicle schedule it was necessary to find two important bus termini of the vehicle schedule – the initial bus terminus of the first connection and the final bus terminus of the last connection. In total there were 53 different bus termini. From these bus termini 73 cumulated places were created in order to apply the presented mathematical model; that means $|K| = 73$. Instructions how to get the cumulated places are explained by way of a simple example in Section 3. The requests of the individual cumulated places for the individual vehicle types were in the range from 1 to 6.

For each combination of the parking lot and the cumulated place the corresponding distance was computed. Consequently, we created a matrix of objective function coefficients that had $3 \times 73 = 219$ elements.

Detailed information about the creation of the cumulated places and the values of distances covered non-productively can be found in publication [13].

The mathematical model has the following parameters: Objective function (1) has 219 terms. The set of the constraints consists of 84 constraints in total. The group of constraints (2) includes 3 constraints; the group of constraints (3) contains 73 constraints. The group of constraints (4) is formed by 6 constraints and the group of constraints (5) includes 2 constraints. The total number of the non-negative variables y_{ij} for $i \in I_1$ and $j \in J$ is 6. The model has 79 variables in total. Numerical experiments were performed on PC equipped with the processor Intel® Core™2 Duo E8400 and 3.25 GB of RAM (hardware configuration is important regarding calculation times). The results obtained by the experiment are summarized in Tables 1 and 2.

Table 1 - The capacity of the individual parking lots and results of the optimization experiment - the number of vehicles being parked in the parking lot before and after the experiment

Parking lot	Parking lot capacity	Vehicles being parked before optimization	Vehicles being parked after optimization
Slezska Ostrava	102	58	49
Martinov	25	13	22
Poruba	33	33	33
In total	160	104	104

Table 2 - The summary of the results – the number of the vehicles assigned to the individual parking lots before and after the optimization process

Vehicle type	Situation	Slezska Ostrava	Martinov	Poruba	In total
Fiat and Mercedes	before optimization	0	10	0	10
	after optimization	0	10	0	10
Renault City Bus	before optimization	7	0	0	7
	after optimization	7	0	0	7
Other vehicles	before optimization	51	3	33	87
	after optimization	42	12	33	87
Vehicles being parked before optimization		58	13	33	104
Vehicles being parked after optimization		49	22	33	104
Difference		-9	+9	0	0

Applying formula (9) the savings in the non-productive covered distance were calculated after the optimization experiment. The rounded decrease of the total non-productive covered distance expressed as a percentage was 4% after the optimization experiment. The non-productive covered distance before the optimization experiment was 1,166.09 km, the non-productive covered distance after the optimization experiment is 1,120.67 km. The difference seems to be small but if we realize that a year has approximately 250 working days we get the savings of about 11,355.00 km.

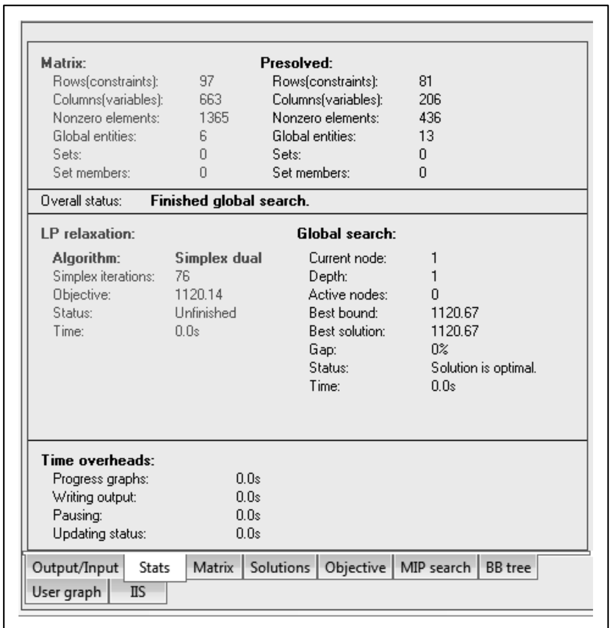

Figure 1 – Information about searching the optimal solution

The information about searching the optimal solution is depicted in *Figure 1*. The important pieces of information in *Figure 1* are: information about the value of the optimization criterion (the total non-productive covered distance) and the time of the optimization calculation. As you can see in *Figure 1*, the calculation time is insignificant. Moreover, we can see in *Figure 1* that the solution we have got is optimal.

6. CONCLUSION

The presented paper is focused on the problem of assigning the vehicles to the defined parking lots. The paper contains the mathematical model of the task; the functionality of the model was tested on an example from practice –the model was applied to the selected group of the Ostrava Transport's vehicles. The calculation experiment showed that the model is functional. Applying the model to our problem brought a 4% decrease of the total non-productively covered distance. It can be assumed that including other vehicles in the optimization process could bring an additional decrease of the non-productively covered kilometres.

The mathematical model formed by (1) – (7) does not consider additional costs that are associated with moving the specialized facilities for the types of the vehicles that have to be parked together in the same parking lot (in our case such moving did not occur). However, the additional costs can be easily implemented in the presented model; it can be incorporated in the additional costs in the objective function. It can be modified analogously to a model of a location problem [9].

Ing. **DUŠAN TEICHMANN**, Ph.D.
E-mail: dusan.teichmann@vsb.cz
VŠB – Technická univerzita Ostrava
Ing. **MICHAL DORDA**, Ph.D.
E-mail: michal.dorda@vsb.cz
VŠB – Technická univerzita Ostrava
Fakulta strojní, Institut dopravy
17. listopadu 15/2172, 708 33 Ostrava, Česká republika
Ing. **HELENA BÍNOVÁ**, Ph.D.
E-mail: michal.dorda@vsb.cz
České vysoké učení technické v Praze
Fakulta dopravní, Ústav logistiky a management dopravy
Konviktská 20, 110 00 Praha, Česká republika
Ing. **MARTIN LUDVÍK**
E-mail: ludvik@hsfsystem.cz
VŠB – Technická univerzita Ostrava
Fakulta strojní, Institut dopravy
17. listopadu 15/2172, 708 33 Ostrava, Česká republika

ABSTRAKT

OPTIMALIZACE PARKOVÁNÍ VOZIDEL MĚSTSKÉ HROMADNÉ DOPRAVY V OSTRAVĚ

Typickým problémem veřejné hromadné dopravy je prostorově roztroušená poptávka. Linková síť, která je provozována dopravcem (nebo skupinou dopravců), musí být pro tuto poptávku adaptována. Vozidla městské hromadné dopravy, která nejsou využívána v průběhu některé části dne, jsou obvykle parkována na definovaných parkovacích plochách s danou kapacitou. Pokud vozidlo přejíždí z místa, ve kterém vozidlo končí svou službu (zpravidla se jedná o konečnou zastávku posledního spoje, který vozidlo obsluhuje), do místa, kde je vozidlo zaparkováno, vzniká neproduktivní jízda. Tento problém nastává i před zahájením obsluhy prvního spoje. Hlavním cílem článku je představit matematický model, který umožňuje minimalizovat celkovou sumu všech neproduktivně ujetých kilometrů. Funkčnost navrženého matematického modelu byla testována v podmínkách reálné sítě veřejné autobusové dopravy.

KLÍČOVÁ SLOVA

optimalizace; matematický model; lineární programování; veřejná doprava;

REFERENCES

[1] Borndoerfer R, Groetschel M, Pfetsch ME. A Column-generation Approach to Line Planning in Public Transport. Transportation Science. 2007;41(2):123-132.

[2] Bussieck MR, Kreuzer P, Zimmermann UT. Optimal Lines for Railway System. European Journal of Operation Research. 1997;96(1):54-63.

[3] Černá A, Černý J. Erlander Principle in Managerial Decision Making on Czech and Slovak Urban Transport Routes. E + M Ekonomie a management. 2013;16(1):93-100.

[4] Černý J, Kluvánek P. Introduction to Mathematical Transport Theory [in Slovak]. Bratislava: VEDA; 1990.

[5] Dellólio L, Ibeas A, Ruisánches F. Optimizing Bus-size and Headway in Transit Networks. Transportation. 2012;39(2):449-464.

[6] Erlander S, Schéele S. A Mathematical Programming Model for Bus Traffic in a Network. Proceedings of the International Symposium Transportation and Traffic Theory.Vol VI; 1974 August 26-28; Sydney, Australia.

[7] Gass SI. Linear Programming-Methods and Applications. New York: Dover Publications; 2003.

[8] Hitchcock FL. The distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics. 1941;20:224-230.

[9] Janáček J, Buzna Ľ. Optimization in Networks. Žilina: Univerzity of Žilina; 2009.

[10] Janáček J, Širc J. The Column Generation Technique for Public Transport Line Planning by IP Solver. Komunikacie. 2010;12(3):60-64.

[11] Koháni M. Exact Approach to the Tariff Zones Design Problem in Public Transport. Proceedings of the International Conference Mathematical Methods in Economics 2012. Vol XXX; 2012 September 11-13; Karviná,Czech Republic.

[12] Kozel P. The Time Coordination of Bus Links in Both Transport Directions. Proceedings of the 29th International Conference on Mathematical Methods in Economics 2011; 2011 September 6-9; Janská dolina, Slovakia.

[13] Ludvík M. Allocation of Vehicle Optimization [in Czech]. Diploma thesis: VŠB – Technical University of Ostrava; 2010.

[14] Palúch S, et al. Direction for Software Communication Automated System for the Scheduling Vehicles - KASTOR [in Slovak]. Žilina: Transport Research Institut; 1984.

[15] Palúch S. Two Approaches to Vehicle and Crew Scheduling in Urban and Regional Bus Transport. Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XIV). Vol. XIV; 2008 June 5-7;High Tatras, Slovakia.

[16] Palúch S. A Vehicle Scheduling Algorithm Based on Vertex Covering by a Set of Disjoint Paths. Proceedings of International Conference Mathematical Methods in Economics 2010. Vol XXVIII; 2010 September 8-10; České Budějovice, Czech Republic.

[17] Peško Š. Flexible Bus Scheduling with Overlapping Trips. Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XIV). Vol XIV; 2008 June 5-7;High Tatras, Slovakia.

[18] Teichmann D, Dorda M, Frič J. An Extended Model for the Design of City Mass Transport Network. Proceedings of the 29thInternational Conference on Mathematical Methods in Economics 2011; 2011 September 6-9; Janská dolina, Slovakia.

[19] Teichmann D, Dorda M, Olivková I. Optimal assignment of vehicles to urban transport routes. Komunikacie. 2013;15(1):34-38.

[20] www.fico.com