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# STABILITY ANALYSIS OF A PREDECESSOR-FOLLOWING PLATOON OF VEHICLES WITH TWO TIME DELAYS

***ABSTRACT***

*The problem of controlling a platoon of vehicles moving in one dimension is considered so that they all follow a lead vehicle with constant spacing between successive vehicles. The stability and the string stability of a platoon of vehicles with two independent and uncertain delays, one in the inter-vehicle distance and the other in the relative velocity information channels, are considered. The main objectives of this paper are: (1) using a simplifying factorization procedure and deploying the cluster treatment of characteristic roots (CTCR) paradigm to obtain exact stability boundaries in the domain of the delays, and (2) for the purpose of disturbance attenuation, the string stability analysis is examined. Finally, a simulation example of multiple vehicle platoon control is used to demonstrate the effectiveness of the proposed method.*

***KEY WORDS***

*automated highway vehicle; stability; string stability; time delay;*

## 1. INTRODUCTION

Traffic congestion has been one of the most serious social, economic, and environmental problems in the world. The intelligent transportation system (ITS) paradigm is one possible solution for this problem. An ITS generally requires vehicles within each lane to drive in a platoon maintaining a small inter-vehicle spacing [1].

The control system for the vehicle platoon consists of a guidance model describing the interaction between vehicles, designing a spacing policy, and individual vehicle control. There are two major strategies for the control of a platoon of vehicles, constant spacing distance control [1-5] and constant time headway control [6-7]. Most of the studies on the platoon focused only on the unidirectional scheme [6-10]. Similar to a good human driver who usually takes advantages of information from the vehicles ahead and behind, it is expected that the vehicle controller behaves more safely using information from the front and rear vehicles. Bidirectional adaptive cruise control (ACC) can be found, for instance, in [1-3]. Because vehicles in a platoon are dynamically coupled by feedback control laws, the spacing and velocity errors of one vehicle may affect other vehicles or even amplify as they propagate upstream along the platoon [11-12]. Such a phenomenon is called string instability. String stability explains how errors are propagated through the group of vehicles as a result of disturbances or the reference trajectory of the formation lead. For this reason, one important aspect of platoon control, aside from stabilizing, is to guarantee string stability. Review of string stability can be found in [7, 13].

Up to date, a great deal of research work has been done in this area. Studies on the platoon stability of the unidirectional scheme are presented in [4, 5, 7-9]. Xiao et al. developed a sliding mode controller using fuel and brake delays and lags, which guarantees both homogeneous and heterogeneous string stability in unidirectional scheme [6]. Dunbar et al. [5] presented distributed receding horizon control algorithms to guarantee asymptotic stability and string stability. Due to practical design and implementation, the actuator lag must be considered [4, 6, 8]. Even though a considerable amount of research has been conducted on the robustness-to-disturbance and stability issues in the bidirectional scheme, most of them only investigated double integrator networks and the homogeneous case in which each vehicle has the same mass and employs the same controller (exceptions include [14-15]). Double integrator networks with decentralized bidirectional control are similar to mass-spring-damper systems [16]. Horn [17] proposed a method for suppressing traffic flow instabilities based on a bidirectional scheme.

The vehicles in an automated highway system (AHS) typically use radar to sense the relative spacing and velocity from their nearest neighbours. Since the signals are transmitted over a communication network of limited bandwidth, network induced delays are always inevitable, which makes the analysis and design of networked systems complicated. The effect of communication delay on string stability in vehicle platoons was investigated in [18-20]. A significant amount of research has been conducted on platoon stability issues without delays [1-3, 5, 7, 21, 22]. Since the presence of time delays in platoon control is often a source of instability, careful control design is critical in order to assure that the controlled system performs properly and remains stable despite the time delays. Recently, [23] considered the impact of communication delays on the platoon system and presented new algorithms to mitigate the delays to achieve the string stability of the platoon. The Lyapunov framework (e.g., linear matrix inequalities (LMIs) [4, 9] or Lyapunov-Razumikhin theorem [8]) was utilized in order to design controllers. This framework requires complex formulations, and can lead to conservative results and possibly redundant control.

Also, due to coupling delays, each vehicle cannot instantly get the information from others so that the effect of multiple delays, one in the inter-vehicle distance and the other in the relative velocity information channels, must be taken into account. To the best of our knowledge, despite the huge amount of relevant literature to date, a method and technique have not yet been reported to examine exact stability and string stability for a platoon of vehicles in the simultaneous presence of two communication delays. Therefore, this paper provides practical means to evaluate the ACC systems.

Hence, a computationally efficient approach based on the CTCR paradigm is presented in order to obtain stability boundaries in the domain of time delays. The process starts with holographic class coordinate transformation from the delay space to a new set of coordinates. This mapping reduces the dimension of the problem from infinite to a manageably small number. This methodology is then utilized to assess the stability of a platoon of vehicles. The method starts with the determination of all possible purely imaginary characteristic roots for any positive time delay. In practice, the delay is not fixed. The benefits of this paradigm could be exploited to examine multiple delay systems, especially in scenarios where delays are uncertain and/or unknown and complying with the necessary and sufficient conditions of the stability. Hence, the system is stable if the upper bound of delay is within stability zone. The main theme of the above methodology is not new and has been implemented for time delayed systems in the past [24-25].

The remainder of the paper is structured as follows. Section 2 briefly introduces the longitudinal vehicle model and states the problem. Section 3 deals with stability analysis and briefly describes the CTCR paradigm. Section 4 presents an analysis of string stability. In Section 5, the simulation results are given to show the efficiency of the proposed method. The paper ends with a conclusion.

## 2. VEHICLE MODEL AND PROBLEM STATEMENT

### 2.1 Vehicle model

The vehicle longitudinal control is generally composed of two loops: an inner force (acceleration) control loop which compensates the non-linear vehicle dynamics (acceleration and brake systems), and an outer inter-distance control loop which is responsible for guaranteeing good tracking of the desired inter-vehicle distance reference. In this paper, it is assumed that the inner control loop has already been designed to compensate for the internal vehicle dynamics, and the only interest here is in the outer control loop. Consider a group of vehicles in dense traffic with no overtaking. The formation control of a $N+1$ homogeneous string of vehicles is considered so that they all follow a lead vehicle, as shown in *Figure 1*.

The position, velocity and acceleration of the lead vehicle are denoted by $x_0(t)$, $v_0(t)$, $a_0(t)$, respectively. Also, $x_i(t)$, $y_i(t)$, $z_i(t)$ denote the position, velocity and acceleration of the $i$-th vehicle, respectively.

The longitudinal dynamics of the $i$-th vehicle in the platoon are modelled as follows (see e.g. [26-27] for details):

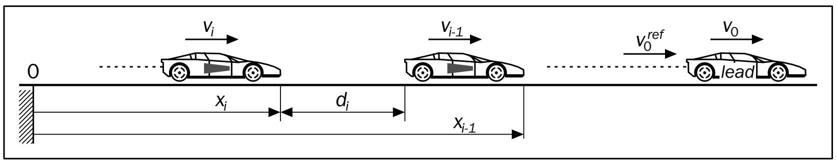


*Figure 1 - 1-D of Vehicle String*

$$\dot{F}_i = -\frac{F_i}{\chi_i} + \frac{c_i}{\chi_i}. \quad (1)$$

$$m_i a_i = F_i - \frac{\sigma A_i c_{di}}{2} v_i^2 - d_{mi}. \quad (2)$$

where $F_i$ denotes the driving force produced by the $i$-th vehicle engine, $c_i$ is the engine input, $\sigma$ is the specific mass of air, $A_i$, $c_{di}$, $d_{mi}$, $m_i$ and $\chi_i$ are the cross-sectional area, drag coefficient, mechanical drag, mass and engine time constant of the $i$-th vehicle, respectively. Equation (1) represents the $i$-th vehicle engine dynamics, and (2) represents Newton's second law applied to the $i$-th vehicle modelled as a particle of mass $m_i$.

Note that this simple model used to describe the engine dynamics (1) has been proved to be useful for preliminary system level studies in longitudinal control of a platoon of vehicles. In the sequel, exact linearization methods are used to linearize and normalize the input-output behaviour of each vehicle in the platoon. Differentiating both sides of (2) with respect to time and substituting the expression for $\dot{F}_i$, in terms of $v_i$ and $a_i$ from (I) and (2) the following is obtained:

$$\dot{a}_i = f_i(v_i, a_i) + g_i(v_i) c_i, \quad (3)$$

where $c_i$ is the engine input and $f_i(v_i, a_i)$ and $g_i(v_i)$ are given by:

$$f_i(v_i, a_i) = \frac{1}{\chi_i}\left(a_i + \frac{\sigma A_i c_{di}}{2m_i} v_i^2 + \frac{d_{mi}}{m_i}\right) - \frac{\sigma A_i c_{di} v_i a_i}{m_i}. \quad (4)$$

$$g_i(v_i) = \frac{1}{\chi_i m_i}. \quad (5)$$

The following control law has been adopted:

$$c_i = u_i m_i + 0.5\sigma A_i c_{di} v_i^2 + d_{mi} + \chi_i \sigma A_i c_{di} v_i a_i, \quad (6)$$

where $u_i$ is the additional input signal to be designed. Obviously, this control law can be achieved by feedback linearization, since after introducing (6), the third equation in (3) becomes:

$$\chi_i \dot{a}_i + a_i = u_i. \quad (7)$$

The feedback linearization controller in (6) plays the role of the first layer controller in our design. It helps to simplify the system model by excluding some characteristic parameters of the vehicle from its dynamics.

## 2.2 Problem statement

In this paper, the desired trajectory of the platoon is considered to be of a constant-velocity type, i.e. $x_0^*(t)$, so that spacing between successive vehicles does not change with time. The control objective is to make the string of vehicles track a pre-specified desired trajectory whilst maintaining desired formation geometry.

The desired geometry of the platoon is specified by the desired spaces $D_{i-1,i}$ for $i = 1, 2, \ldots, N$ where $D_{i-1,i}$ is the desired value of $x_{i-1}(t) - x_i(t) - L_{i-1}$, where $L_{i-1}$ is the length of the $i-1$ vehicle. The control objective is to preserve a rigid formation, i.e., to make neighbouring vehicles keep their pre-specified desired spaces and to make vehicle 1 follow its desired trajectory $x_0^*(t) - D_{0,1} - L_0$.

In this paper, a centralized control law is considered whereby this controller uses the distance and relative velocity to the preceding vehicle and the follower vehicle as well as relative velocity to the leading vehicle as an input. The communication between vehicles of the group is affected by two rationally independent time delays. The first delay is assumed to be in the inter-vehicle spacing information channels, whereas the second delay is in the relative velocity information exchange. These delays are considered as constant and uniform throughout the communication topology. A repeating structure is used so that the vehicle string can be easily extended:

$$\begin{aligned} u_i(t) = &-k_i^f(x_i(t-\tau_p) - x_{i-1}(t-\tau_p) + D_{i-1,i} + L_{i-1}) - \\ &-k_i^b(x_i(t-\tau_p) - x_{i+1}(t-\tau_p) - D_{i,i+1} - L_i) - \\ &-\beta_i(\dot{x}_i(t) - \dot{x}_0) - b_i^f(\dot{x}_i(t-\tau_v) - \dot{x}_{i-1}(t-\tau_v)) - \\ &-b_i^b(\dot{x}_i(t-\tau_v) - \dot{x}_{i+1}(t-\tau_v)) \end{aligned} \quad (8)$$

where $i = 1, 2, \ldots, N-1$, $k_i^f$, $k_i^b$ are the front and back proportional gains and $b_i^f$, $b_i^k$ are the front and back derivative gains, respectively, and $\beta_i$ is constant derivative gain related to the leader vehicle. The first two terms are used to compensate for any deviation away from nominal position with the predecessor (front) and the follower (back) vehicles, respectively. The superscripts $^f$ and $^b$ correspond to front and back, respectively. Notice that this protocol does not include self-delayed information, while all data coming from the informers are delayed, positions by $\tau_p$ and velocities by $\tau_v$. For the vehicle with index $N$ which does not have a vehicle behind it, the control law is slightly different:

$$\begin{aligned} u_N = &-b_N^f(\dot{x}_N(t-\tau_v) - \dot{x}_{N-1}(t-\tau_v)) - \beta_N(\dot{x}_N - \dot{x}_0) - \\ &-k_N^f(x_N(t-\tau_p) - x_{N-1}(t-\tau_p) + D_{N-1,N} + L_{N-1}). \end{aligned} \quad (9)$$

In the homogeneous and symmetric case, the following is obtained: $\chi_i = \chi$, $\beta_i = \beta$, $k_i^b = k_i^f = k$, $b_i^b = b_i^f = b$ for some positive constants $\beta$, $k$, $b$. Combining the open loop dynamics (7) with the control law (8), yields:

$$\begin{aligned} \chi \dot{a}_i(t) + a_i(t) = &-k(x_i(t-\tau_p) - x_{i-1}(t-\tau_p) + D_{i-1,i} + L_{i-1}) - \\ &-k(x_i(t-\tau_p) - x_{i+1}(t-\tau_p) - D_{i,i+1} - L_i) - \beta(\dot{x}_i - \dot{x}_0) - \\ &-b(\dot{x}_i(t-\tau_v) - \dot{x}_{i-1}(t-\tau_v)) - b(\dot{x}_i(t-\tau_v) - \dot{x}_{i+1}(t-\tau_v)) \end{aligned} \quad (10)$$

The dynamics of the $N$-th vehicle is obtained by combining (7) and (9). The desired trajectory of the $i$-th vehicle is:

$$\begin{aligned} x_i^*(t) &= x_0^* - D_{0,i} - \sum_{j=1}^{i-1} L_j = x_0^* - \sum_{j=1}^{i} D_{j-1,j} - \sum_{j=0}^{i-1} L_j = \\ &= x_0^* - \sum_{j=1}^{i} (D_{j-1,j} + L_{j-1}). \end{aligned} \quad (11)$$

To facilitate analysis, the following tracking error can be defined:

$$\tilde{x}_i = x_i - x_i^* \to \dot{\tilde{x}} = \dot{x}_i - \dot{x}_i^* \to \ddot{\tilde{x}}_i = \ddot{x}_i \quad (12)$$

Substituting (12) into (10), and using

$$x^*_{i-1} - x^*_i = D_{i-1,i} - L_{i-1},$$

yields:

$$\begin{aligned}\chi\dddot{\tilde{x}}_i(t) + \ddot{\tilde{x}}_i(t) = &- k(\tilde{x}_i(t-\tau_p) - \tilde{x}_{i-1}(t-\tau_p)) - \\ &- b(\dot{\tilde{x}}_i(t-\tau_v) - \dot{\tilde{x}}_{i+1}(t-\tau_v)) - b(\dot{\tilde{x}}_i(t-\tau_v) - \dot{\tilde{x}}_{i-1}(t-\tau_v)) - \\ &- k(\tilde{x}_i(t-\tau_p) - \tilde{x}_{i+1}(t-\tau_p)) - \beta\dot{\tilde{x}}_i(t) \end{aligned} \quad (13)$$

By defining state

$$X = [\tilde{x}_1, \dot{\tilde{x}}_1, \ddot{\tilde{x}}_1, \tilde{x}_2, \dot{\tilde{x}}_2, \ddot{\tilde{x}}_2, \dots, \tilde{x}_N, \dot{\tilde{x}}_N, \ddot{\tilde{x}}_N]^T,$$

the closed loop dynamics can be written as compact form from (13) as:

$$\dot{X}(t) = AX(t) + B_1 X(t-\tau_p) + B_2 X(t-\tau_v). \quad (14)$$

The explicit form of this equation is:

$$\begin{aligned}\dot{X}(t) = &\left(I_N \otimes \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\beta/\chi & -1/\chi \end{pmatrix}\right) X(t) + \\ &+ \left(I_N \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2k/\chi & 0 & 0 \end{pmatrix} + C \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k/\chi & 0 & 0 \end{pmatrix}\right) X(t-\tau_p) + \\ &+ \left(I_N \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -2b/\chi & 0 \end{pmatrix} + C \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b/\chi & 0 \end{pmatrix}\right) X(t-\tau_v), \end{aligned} \quad (15)$$

$$C = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 1 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix}, \quad (16)$$

where $\otimes$ denotes the Kronecker multiplication [28], $I_N$ is the identity matrix of dimension $N$ and $C$ is connectivity matrix of the communication topology [29]. Since the trajectory of the reference vehicle is equal to its desired trajectory, then $\tilde{x}_0(t) = \dot{\tilde{x}}_0(t) = 0$.

Due to the presence of transcendental terms, the characteristic equation of the closed loop dynamics (15) possesses infinitely many roots, some or all of which may determine stability. Because of this infinite dimensionality, the problem of determining the stabilizing controller without introducing conservatism into the stability analysis can be difficult. The objective is to obtain a range of control parameters to guarantee the stability and the string stability of a platoon of vehicles.

## 3. STABILITY ANALYSIS

Before analyzing any aspect of a platoon, it is important that the stability of a platoon is analyzed. Since the presence of time delays in platoon control is often a source of instability, a careful control design is vital in order to guarantee that a controlled system performs properly and remains stable despite the time delays. Finding the delay independent stable controller gains can be difficult. This is because of two reasons; one being infinite dimensionality and the other the complexity level of this dynamics which increases rapidly as the number of vehicles gets larger. The goal of this section is a structured methodology which can generate, very efficiently, an exact, exhaustive and explicit stability map of a platoon with respect to time delays. The methodology is based on the CTCR paradigm. The approaches are based on a non-conservative framework, and the stability regions are found to those compositions in the parametric space that produce stable operation for the entire system.

Stability analysis can be solved by investigating the characteristic equation of the closed loop dynamics (14). The corresponding characteristic equation of the system is:

$$CE(x, \chi, k, b, \beta, \tau_p, \tau_v) = \det\left(sI_{3N} - A - B_1 e^{-\tau_p s} - B_1 e^{-\tau_v s}\right). \quad (17)$$

To resolve the complexity level, the characteristic equation of this class of systems can be conveniently converted into a product of a set of factors whose order is the same as the individual vehicle dynamics. This procedure reduces the complexity of the problem. This is inspired by a recent thesis work [30].

**Lemma 1** (Factorization property): The transcendental function of the system (14) can always be expressed as the product of a set of n factors as:

$$\begin{aligned} CE(x, \chi, k, b, \beta, \tau_p, \tau_v) &= \det\left(sI_{3N} - A - B_1 e^{-\tau_p s} - B_1 e^{-\tau_v s}\right) = \\ &= \prod_{i=1}^{N} ce_i(s, \chi, k, b, \tau_p, \tau_v, \lambda_i) = \\ &= \prod_{i=1}^{N} (\chi s^3 + s^2 + \beta s + (2-\lambda_i) b e^{-\tau_v s} s + (2-\lambda_i) k e^{-\tau_p s}). \end{aligned} \quad (18)$$

The connectivity matrix is diagonalizable [31]. Then, there is a non-singular matrix $T$ such that $T^{-1}CT = \Lambda$ where $\Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_N)$ has non-zero entries equal to the eigenvalues of $C$, which are all real. Then a state transformation $X = (T \otimes I_3)\xi \in \Re^{3N}$ is introduced into (15) and using the chain multiplication features of the Kronecker product $(U \otimes V)(W \otimes Z) = UW \otimes VZ$ [28], (15) is transformed into:

$$\begin{aligned}\dot{\xi}(t) = &\left(I_N \otimes \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\beta/\chi & -1/\chi \end{pmatrix}\right) \xi(t) + \\ &+ \left(I_N \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2k/\chi & 0 & 0 \end{pmatrix} + \Lambda \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k/\chi & 0 & 0 \end{pmatrix}\right) \xi(t-\tau_p) + \\ &+ \left(I_N \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -2b/\chi & 0 \end{pmatrix} + \Lambda \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b/\chi & 0 \end{pmatrix}\right) \xi(t-\tau_v). \end{aligned} \quad (19)$$

Since $I_N$ and $\Lambda$ are diagonal matrices, Equation (19) is block-diagonalised, and it can be expressed as a set of $N$ decoupled subsystems of the form:

$$\dot{\xi}_i(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\beta/\chi & -1/\chi \end{pmatrix} \xi_i(t) +$$

$$+\left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2k/\chi & 0 & 0 \end{pmatrix} + \lambda_i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k/\chi & 0 & 0 \end{pmatrix}\right)\xi_i(t-\tau_p) +$$
$$+\left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -2b/\chi & 0 \end{pmatrix} + \lambda_i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b/\chi & 0 \end{pmatrix}\right)\xi_i(t-\tau_v) \quad (20)$$

with $i = 1,2,\dots,N$. It is obvious that the transcendental function of each block (19) is:

$$ce_i(s,\chi,k,b,\beta,\tau_p,\tau_v,\lambda_i) = \chi s^3 + s^2 + \beta s +$$
$$+(2-\lambda_i)be^{-\tau_v s}s + (2-\lambda_i)ke^{-\tau_p s}, \quad (21)$$

and the transcendental function of the complete system is the product of these *N* factors, as stated in (18). Furthermore, since the only differentiating element from one factor to the other is the eigenvalue $\lambda_i$, the stability analysis in the delays domain can be done just once. This property makes the stability analysis virtually independent of the number of vehicles. The complexity problem in determining the stability of a quasi-polynomial such as (17) is now shifted to find out the eigenvalues of a known matrix *C* and the repeated stability analysis over a very simple quasi polynomial as in (21).

The stability regions are intersected to determine those compositions in the parametric time delay space that produce stable operation for the whole system for a given setting of the control gains $k$, $b$ and $\beta$. For clarity of the development, this section is divided in two parts. The first part introduces the CTCR methodology, including the concept of the spectral delay space (SDS), whereas the second part applies this methodology to the analysis of the individual factors.

### 3.1 CTCR paradigm

In this section, an approach is followed to obtain this knowledge over a new domain which is called SDS. The following paragraphs present some preparatory definitions and key propositions of CTCR from [32-33].

**Definition 1** (Kernel hypercurves): The hypercurves that consist of all the points $(\tau_p,\tau_v) \in \Re^{2+}$ exhaustively, which cause an imaginary root $s = \omega i$, $\omega \in \Re^+$ and satisfy the constraint $0 < \tau_j\omega < 2\pi$, $j = p,v$ are called the kernel hypercurves. The points on this hypercurve contain the smallest possible delay values that create the given imaginary root at frequency $\omega$.

**Definition 2** (Offspring hypercurves): The hypercurves obtained from the kernel hypercurve by the following pointwise non-linear transformation can be calculated by:

$$\left\langle \tau_p \pm \frac{2\pi}{\omega}j_1, \tau_v \pm \frac{2\pi}{\omega}j_2 \right\rangle,\ j_1 = j_2 = 0,1,2,\dots \quad (22)$$

**Definition 3** (Root tendency): The root tendency (RT) indicates the direction of the imaginary root transition (to the right or the left half of the complex plane) as only one of the delays, $\tau_j$ increases by $\varepsilon$, $0 < \varepsilon << 1$ while all the others remain constant:

$$RT\big|_{s=wi}^{\tau_j} = \mathrm{sgn}\left[\mathrm{Re}\left(\frac{ds}{d\tau}\Big|_{s=\omega i}\right)\right]. \quad (23)$$

### 3.2 Spectral delay space

A procedure is described in this section for determining the kernel (and offspring) hypercurves. A mapping is introduced from time delays space to a $v_k = \tau_k\omega$ space for every point of $(\tau_p,\tau_v) \in \Re^{2+}$ on the kernel or the offspring hypercurve which is called SDS. The main advantage of SDS is that the representation of the kernel hypercurve in the SDS, called the building hypercurve, is confined to a square of edge length $2\pi$ (Definition 1). This finite domain is known as the building block (BB). There are several other intriguing properties of the SDS and building block concepts which can be found in [34].

### 3.3 Stability analysis of individual factors

The transcendental function (21) possesses infinitely many zeros due to the presence of transcendental terms. The closed loop dynamics (18) is asymptotically stable if and only if all these zeros have negative real parts. The continuity property of the roots of (21) on the complex plane holds, which indicates that stability analysis of (21) requires detecting the critical values of the time delays for which at least one root of (21) lies on the imaginary axis of the complex plane [35]. For delay-independent stability of the systems, it is necessary that the delay-free system is Hurwitz stable. This automatically guarantees that $s = 0$ cannot be a feasible solution of the corresponding transcendental function (21) for finite time delays, see [36] for details. Next, we analyze the stability of the delay-free controlled system (i.e. $\tau_p = \tau_v = 0$) by using the Routh-Hurwitz stability criterion.

**Lemma 2**: The system (14) without communication delays is stable if and only if

$$k < \frac{1}{\chi}\left(\frac{\beta}{2-\lambda_i} + b\right). \quad (24)$$

Then stability is guaranteed if the first column elements of the classical Routh's array do not change sign. Therefore, the system without communication delay is stable if and only if

$$2-\lambda_i > 0. \quad (25)$$

$$(2-\lambda_i)(b-\chi k) + \beta > 0. \quad (26)$$

Based on the Gershgorin circle theorem, every eigenvalue of *C* lies within a Gershgorin disc, $D(0,2)$, (the closed disc centred at zero with radius 2). Therefore, the condition $2 > \lambda_i$ has always been satisfied. Hence, if the control parameters are chosen so that $(2-\lambda_i)(b-\chi k)+\beta > 0$, the system without communication will be stable. By using elementary calculation, we can show this constraint will be satisfied if the following condition is held.

$$k < \frac{1}{\chi}\left(\frac{\beta}{2-\lambda_i}+b\right). \tag{27}$$

Here, the proof of the lemma is completed.

With the previous definitions and propositions, we now return to the particular problem. It is the exhaustive determination of all the imaginary roots $s=\omega i$ for the generic factor of the characteristic equation, as in (21) within the semi-infinite quadrant of $\tau_p, \tau_v \in \Re^+$. An enabling approach to study such solutions is by converting the infinite-dimensional characteristic equation (21) to a finite dimensional characteristic equation that has continuous coefficients as was done in [25, 26, 32-35]. This conversion does not lose the infinite-dimensional nature of the problem, and can be done via the exact Rekasius transformation [37]:

$$e^{-\tau_k s} = \frac{1-T_k s}{1+T_k s}, T_k \in \Re \tag{28}$$

where $s=\omega i$, $\omega \in \Re^+$. With the obvious mapping condition, we can obtain

$$\tau_k = \frac{2}{\omega}[\tan^{-1}(\omega T_k) \mp \ell\pi],\ \ell = 0,1,2,\ldots \tag{29}$$

With the substitution of $T_k$ as given in (28) into (21) and following this domain transformation from $\tau_k$ to $T_k$, we obtain a new characteristic equation $ce_j(s,T_k)=0$. It can be considered as a projection of an infinite dimensional equation in $\tau_k$ domain into a finite dimensional equation in the new $T_k$ domain. The equation $ce_j(s,T_k)=0$ is written slightly differently by using

$$u_k = T_k\omega = \tan(0.5v_k),\ v_k = \tau_k\omega \in [0,2\pi] \tag{30}$$

The set $(v_p, v_v)$ is used to represent the new coordination which is bound within $[0,2\pi]$. This new equation can be written as a polynomial in $\omega$ with complex coefficients that are parameterized in $u_p$ and $u_v$:

$$ce_j(\omega) = \left(\sum_{k=0}^{3} f_k(u_p,u_v)\omega^k\right) + i\left(\sum_{k=0}^{3} g_k(u_p,u_v)\omega^k\right). \tag{31}$$

Now, we search for an imaginary solution. If there is a solution $\omega \in \Re^+$ to (31), both its real and imaginary parts must be zero simultaneously. The condition that both real and imaginary parts of (31) have a common root is simply stated using a Sylvester's resultant matrix:

$$S = \begin{bmatrix} f_3 & f_2 & f_1 & f_0 & 0 & 0 \\ 0 & f_3 & f_2 & f_1 & f_0 & 0 \\ 0 & 0 & f_3 & f_2 & f_1 & f_0 \\ g_3 & g_2 & g_1 & g_0 & 0 & 0 \\ 0 & g_3 & g_2 & g_1 & g_0 & 0 \\ 0 & 0 & g_3 & g_2 & g_1 & g_0 \end{bmatrix}. \tag{32}$$

In order to satisfy both, the real and the imaginary parts of (31) simultaneously, $s$ should be singular. This results in the following expression in terms of $u_p$ and $u_v$:

$$\det(S) = F(u_p,u_v) = F(\tan(0.5v_p), \tan(0.5v_v)). \tag{33}$$

It is important to mention that purely imaginary roots of $CE(s,\tau_k)$ in (17) are invariant under Rekasius transformation. This point is proven in [39]. This point clearly represents that instead of the purely imaginary roots of the infinite dimensional system (17), one can study the purely imaginary roots of the finite dimensional system $CE(s,\tau_k)$. This is considerably easier for determining.

Equation (33) constitutes a closed-form description of the kernel hypercurves in the SDS $(v_p,v_v)$, i.e. the building block hypercurves. To obtain its graphical depiction, one of the parameters, say $v_v$, can be scanned in the range of $[0,2\pi]$ and the corresponding $v_p$ values are calculated again in $[0,2\pi]$. Notice that every point $(v_p,v_v)$ on these curves brings an imaginary characteristic root at $\pm\omega i$. That is, we have a continuous sequence of $(v_p,v_v,\omega)$ sets all along the kernel hypercurves. For more details on key properties of the building block concept as well as an earlier adopted longer procedure to find out the kernel and offspring hypercurves, the reader is referred to [34].

## 4. STRING STABILITY

For interconnected systems, in addition to the usual stability and robustness analysis, other properties may become important and must be considered such as string stability. The string stability deals with how errors are propagated through the vehicle string due to disturbances or the reference trajectory of the formation lead. String stability is usually answered by looking at the transfer functions that relate the spacing errors between two successive vehicle pairs. String-stability ensures that range errors decrease as they propagate along the vehicle stream.

This Section focuses on the analysis of string stability of a homogeneous platoon which is composed of identical ACC-equipped vehicles. This assumption is acceptable, because by using appropriate lower-level vehicle acceleration controllers, the dynamic behaviour of the vehicles can be approximated with one vehicle model.

The velocity and spacing error dynamic models can be derived based on the vehicle dynamics model (10). Differentiating both sides of (10) yields:

$$\begin{aligned}\chi\ddot{a}_i + \dot{a}_i(t) = &-\beta\ddot{x}_i - k(\dot{x}_i(t-\tau_p) - \dot{x}_{i-1}(t-\tau_p)) - \\ &-b(\ddot{x}_i(t-\tau_v) - \ddot{x}_{i-1}(t-\tau_v)) - k(\dot{x}_i(t-\tau_p) - \dot{x}_{x+1}(t-\tau_p)) - \\ &-b(\ddot{x}_i(t-\tau_v) - \ddot{x}_{i+1}(t-\tau_v)).\end{aligned} \tag{34}$$

The Laplace transforms can be used to analyze string stability with the conventional notation and the regular assumption of zero initial conditions for the derivation of transfer functions. Therefore, by taking the Laplace transform on both sides of (34), we obtain:

$$(\chi s^3 + s^2 + (2be^{-\tau_v s} + \beta)s + 2ke^{-\tau_p s})V_i =$$
$$= (ke^{-\tau_p s} + sbe^{-\tau_v s})V_{i-1} + (ke^{-\tau_p s} + sbe^{-\tau_v s})V_{i+1}. \quad (35)$$

Then Equation (34) can be rewritten in the following form:

$$V_i = GV_{i-1} + GV_{i+1}, \quad (36)$$

where

$$G = \frac{ke^{-\tau_p s} + sbe^{-\tau_v s}}{\chi s^3 + s^2 + (2be^{-\tau_v s} + \beta)s + 2ke^{-\tau_p s}}. \quad (37)$$

It is clear that the range error output must be smaller than or equal to the range error input to avoid range errors from propagating indefinitely along the string. For a vehicle string, a string-stability definition is widely used [39-40] and it is described as follows:

$$\left|\frac{V_i}{V_{i-1}}(j\omega)\right| < 1 \quad \forall\omega > 0. \quad (38)$$

By dividing both sides of the Equation (36) by $V_{i-1}$ we get:

$$\frac{V_i}{V_{i-1}} = G + G\frac{V_{i+1}}{V_{i-1}} = G + G\frac{V_{i+1}}{V_i}\frac{V_i}{V_{i-1}} \Rightarrow$$
$$\Rightarrow \frac{V_i}{V_{i-1}}\left(1 - G\frac{V_{i+1}}{V_i}\right) = G \Rightarrow \frac{V_i}{V_{i-1}} = \frac{G}{1 - G\frac{V_{i+1}}{V_i}}. \quad (39)$$

String stability is satisfied if the following condition is satisfied:

$$|G(j\omega)| < \left|1 - G(j\omega)\frac{V_{i+1}}{V_i}(j\omega)\right|. \quad (40)$$

By considering this fact that in this scenario, the last vehicle does not have a vehicle behind it. If we start from the end of the platoon by assuming that $|V_N/V_{N-1}(j\omega)| < 1$ if the following condition holds,

$$|G(j\omega)| < 0.5. \quad (41)$$

string stability is satisfied. Note the condition (41) is a sufficient condition for string stability.

**Lemma 3**: if the following conditions hold, string stability is satisfied.

$$\beta^2 - 4k - 4b\beta - 4k\beta\tau_p \geq 0 \quad (42(a))$$

$$1 - 2\beta\chi - 4b\tau_v - 4b\chi - 4k\chi\tau_p \geq 0 \quad (42(b))$$

$$\beta^2 - 2k - 2b\beta - 2k\beta\tau_p \geq 0 \quad (42(c))$$

Proof: $|G(j\omega)|$ can be expressed as $|G(j\omega)| = \sqrt{p/q}$ that

$$p = k^2 + b^2\omega^2 + 2kb\omega\sin(\tau_v - \tau_p) \quad (43)$$

$$q = 2k^2 + (4b^2 + \beta^2)\omega^2 + (1 - 2\beta\chi)\omega^4 + \chi^2\omega^6 +$$
$$+ 8kb\omega\sin\omega(\tau_p - \tau_v) \quad (44)$$

The magnitude of $|G(j\omega)|$ is less than 0.5 if the following condition is satisfied:

$$q - 4p = \beta^2\omega^2 + (1 - 2\beta\chi)\omega^4 + \chi^2\omega^6 -$$
$$- 4b\omega^2(\chi\omega^2 - \beta)\cos\tau_v\omega + 4k\omega(\chi\omega^2 - \beta)\sin\tau_p\omega -$$
$$- 4k\omega^2\cos\tau_p\omega - 4b\omega^3\sin\tau_v\omega > 0 \quad (45)$$

*Case I*: if $\chi\omega^2 > \beta$ then taking into account the fact $\sin x \geq -x$, $-\sin x \geq -x$, $-\cos x \geq -1$, $\cos x \geq 1$ for $(x \geq 0)$, (45) can be simplified as:

$$q - 4p \geq (\beta^2 - 4k + 4b\beta + 4k\beta\tau_p)\omega^2 +$$
$$+ (1 - 2\beta\chi - 4b\tau_v - 4b\chi - 4k\chi\tau_p)\omega^4 + \chi^2\omega^6 \geq 0. \quad (46)$$

*Case II*: if $\chi\omega^2 < \beta$ then taking into account the fact $\sin x \geq -x$, $-\sin x \geq -x$, $-\cos x \geq -1$, $\cos x \geq 1$ for $(x \geq 0)$, (45) can be simplified as:

$$q - 4p \geq (\beta^2 - 4k - 4b\beta - 4k\beta\tau_p)\omega^2 +$$
$$+ (1 - 2\beta\chi - 4b\tau_v + 4b\chi + 4k\chi\tau_p)\omega^4 +$$
$$+ \chi^2\omega^6 \geq 0, \quad (47)$$

While the coefficients of the inequality (45-46) are more than zero, the inequality (41-42) is satisfied. Hence, if the conditions $\beta^2 - 4k - 4b\beta - 4k\beta\tau_p \geq 0$ and $1 - 2\beta\chi - 4b\tau_v - 4b\chi - 4k\chi\tau_p \geq 0$ hold simultaneously, inequality (45) is satisfied.

Note that the spacing errors have most of their energy in the region of low frequencies [13]. Hence, the coefficient $\beta^2 - 4k - 4b\beta - 4k\beta\tau_p \geq 0$ is the most important value of the polynomial. To determine which delay has the most negative effect, the magnitude of $|G(j\omega)|_{\tau_p=0}$ was compared with the magnitude of $|G(j\omega)|_{\tau_v=0}$ at low frequencies, where the sign of $2b - \beta$ shows which delay has more negative effect if $\tau_p = \tau_v$. If $2b - \beta$ is not negative then $|G(j\omega)|_{\tau_v=0} < |G(j\omega)|_{\tau_p=0}$ i.e. the communication delay in the relative velocity information channel makes the larger negative effect on string stability. Otherwise $|G(j\omega)|_{\tau_v=0} > |G(j\omega)|_{\tau_p=0}$.

The next step of string stability analysis is to examine the previous assumption on $|V_N/V_{N-1}(j\omega)|$. By differentiating and by taking the Laplace transformation on both sides of the dynamics of the $N$-th vehicle, the following is obtained:

$$\frac{V_N}{V_{N-1}} = \frac{ke^{-\tau_p s} + sbe^{-\tau_v s}}{\chi s^3 + s^2 + (be^{-\tau_v s} + \beta)s + ke^{-\tau_p s}}. \quad (48)$$

$|V_N/V_{N-1}(j\omega)|$ can be expressed as

$$|V_N/V_{N-1}(j\omega)| = \sqrt{p_N/q_N}$$

that

$$p_N = k^2 + b^2\omega^2 + 2kb\omega\sin(\tau_v - \tau_p) \quad (49)$$

$$q_N = k^2 + (b^2 + \beta^2)\omega^2 + (1 - 2\beta\chi)\omega^4 + \chi^2\omega^6 +$$
$$+ 2kb\omega\sin\omega(\tau_v - \tau_p) \quad (50)$$

The magnitude of $|V_N/V_{N-1}(j\omega)|$ is less than one if the following condition is satisfied:

$$q_N - p_N = \beta^2\omega^2 + (1 - 2\beta\chi)\omega^4 + \chi^2\omega^6 - 2k\omega^2\cos\tau_p\omega +$$
$$+ 2\omega(\chi\omega^2 - \beta)(k\sin\tau_p\omega - 2b\omega\cos\tau_v\omega) -$$
$$- 2b\omega^3\sin\tau_v\omega \geq 0 \quad (51)$$

*Case I*: if $\chi\omega^2 > \beta$ then taking into account the fact $\sin x \geq -x$, $-\sin x \geq -x$, $-\cos x \geq -1$ for $(x \geq 0)$, (51) can be simplified as:

$$q_N - p_N \geq (\beta^2 - 2k + 2b\beta + 2k\beta\tau_p)\omega^2 +$$

$$+(1-2\beta\chi-2b\tau_v-2b\chi-2k\chi\tau_p)\omega^4+\chi^2\omega^6\geq 0\,, \quad (52)$$

*Case II*: if $\chi\omega^2<\beta$ then taking into account the fact $\sin x\geq -x$, $-\sin x\geq -x$, $-\cos x\geq -1$, $\cos x\geq 1$ for $(x\geq 0)$, (50) can be simplified as:

$$q_N-p_N\geq(\beta^2-2k-2b\beta-2k\beta\tau_p)\omega^2+$$
$$+(1-2\beta\chi-2b\tau_v+2b\chi+2k\chi\tau_p)\omega^4+$$
$$+\chi^2\omega^6\geq 0\,, \quad (53)$$

If the coefficients of the inequality (52-53) are greater than zero, inequalities (52-53) are satisfied. Hence, if the conditions $\beta^2-2k-2b\beta-2k\beta\tau_p\geq 0$ and $1-2\beta\chi-2b\tau_v-2b\chi-2k\chi\tau_p\geq 0$ hold simultaneously, inequality (51) is satisfied.

As mentioned earlier, the spacing errors have most of their energy in the region of low frequencies. Hence, the coefficient $\beta^2-2k-2b\beta-2k\beta\tau_p$ is the most important value of the polynomial. It can be concluded that the communication delay in the inter-vehicle distance information channel makes the negative effect on the string stability.

Here, the proof of the lemma is completed.

## 5. SIMULATION

In order to validate the performance of the proposed control algorithm, computer simulations have been carried out for the platoon system containing ten followers, i.e. $N=10$. As stated before, the ability to maintain the distance between vehicles is important for the safety of the vehicle platoon system. In the simulations, the desired vehicle spacing was set as $D_{i-1,i}=8\,\text{m}$ and the length of vehicle as $L_i=4\,\text{m}$, other parameters used in the simulations are the same as parameters mentioned in [1, 4, 10], that is, $\sigma_i=1\,\text{N/m}^3$, $H_i=2.2\,\text{m}^2$, $c_{di}=0.35$, $m_i=1{,}500\,\text{kg}$, $d_{mi}=150\,\text{N}$ and $\chi_i=0.2\,\text{s}$.

The most important disturbances in a platoon control system include lead vehicle acceleration/deceleration. The disturbance is defined as any source that causes the vehicle string to lose maintaining constant velocity. We assume that, without losing generality, the desired acceleration for the lead vehicle is given by:

$$a_{des}=\begin{cases}0 & t\leq 20\,\text{s}\\ 2\,\text{m/s}^2 & 20\,\text{s}\leq t\leq 30\,\text{s}\,.\\ 0 & t\geq 30\,\text{s}\end{cases} \quad (54)$$

The initial velocity is $v_{initial}=20\,\text{m/s}$ and the final desired velocity is $v_{final}=40\,\text{m/s}$.

For this communication structure, the complete eigenvalue set of the corresponding connectivity matrix is $\{\pm 1.919, \pm 1.6825, \pm 1.3097, \pm 0.8308, \pm 0.2846\}$. For all the factors which are generated by these eigenvalues, the stability boundaries in the SDS, i.e. the BB representations, are presented in Figures 2-4 using the control gains $k=2$, $b=1$ and $\beta=1$.

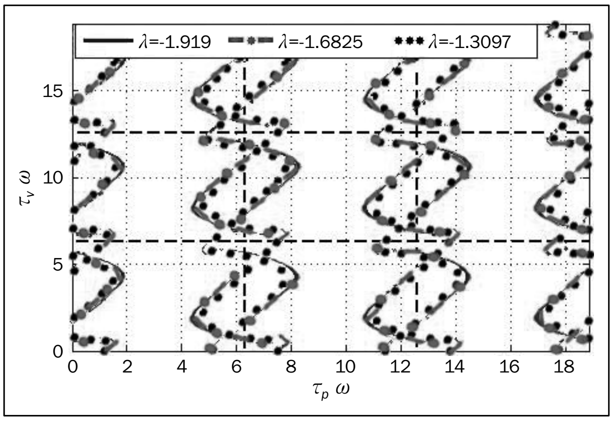


*Figure 2 - SDS representation of the individual factors $\lambda$=-1.919, $\lambda$=-1.6825 and $\lambda$=-1.3097*

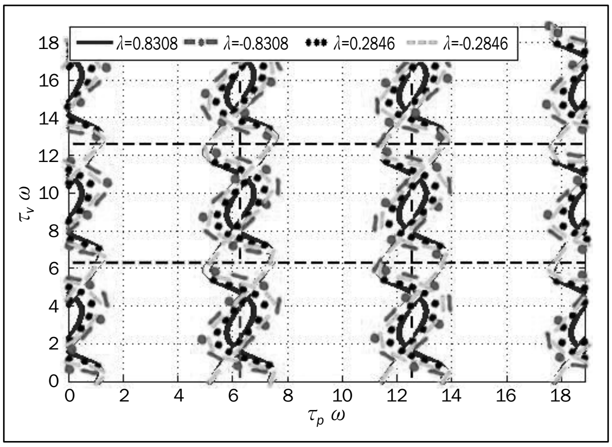


*Figure 3 - SDS representation of the individual factors $\lambda$=±0.8308, $\lambda$=±0.2846*

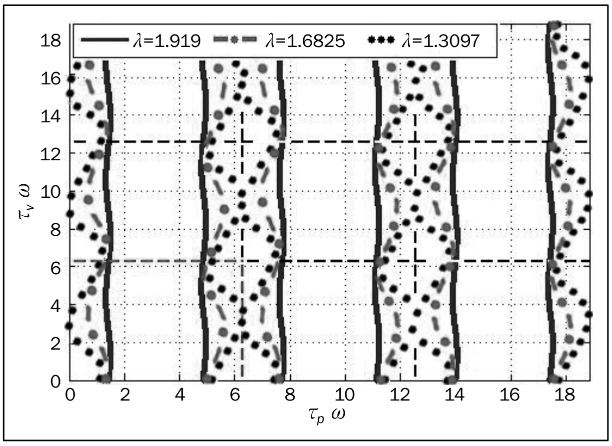


*Figure 4 - SDS representation of the individual factors $\lambda$=1.919, $\lambda$=1.6825 and $\lambda$=1.3097*

In Figures 2-4 the building curves (in the range of $[0,2\pi]$) are shown with their reflection curves in $(v_p,v_v)$ coordinates. It can be seen that the building block exactly repeats itself in its reflection curves with a period of $2\pi$. It is necessary and sufficient to detect the building block for the complete stability analysis in the $(\tau_p,\tau_v)$ domain. By using Definition 2, we create offspring curves from the kernel curves and depict

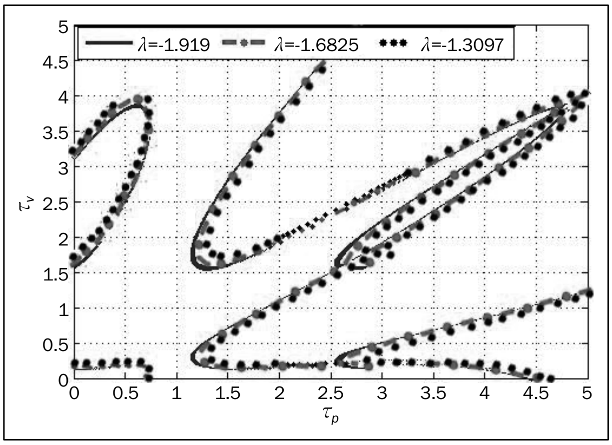


*Figure 5 - Stability boundaries of the individual factors $\lambda$=-1.919, $\lambda$=-1.6825 and $\lambda$=-1.3097*

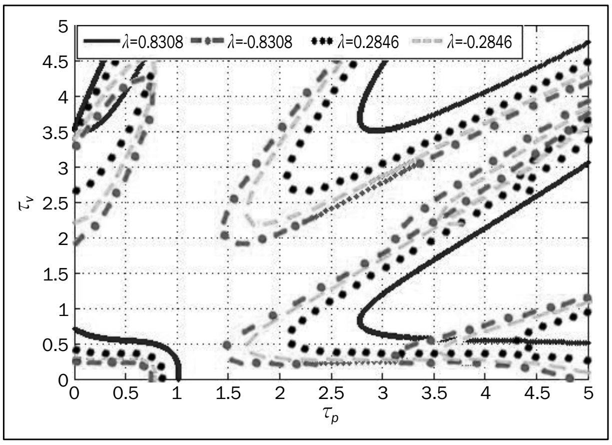


*Figure 6 - Stability boundaries of the individual factors $\lambda$=±0.8308, $\lambda$=±0.2846*

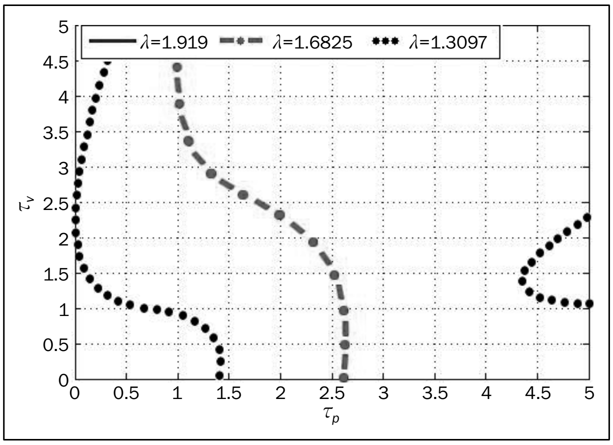


*Figure 7 - Stability boundaries of the individual factors $\lambda$=1.919, $\lambda$=1.6825 and $\lambda$=1.3097*

all of them in Figures 5-7 (kernel curves are shown in the range of $[0,2\pi]$). Notice that there is a one-to one mapping between the BB curves of Figures 2-4 and the kernel hypercurves of Figures 5-7.

Next, we look at the root tendencies on these curves using Definition 3. Deploying this feature, those regions where the number of unstable roots is zero, can be detected, and they represent stable regions, which are displayed as shaded in *Figure 8*. It is very simple to mark these regions using the D-Subdivision rule.

All $(\tau_p,\tau_v)$ compositions inside the shaded region of *Figure 8* result in the vehicles reaching stability at a constant velocity, whereas points outside that region render the system unstable. Furthermore, points on the boundaries of the shaded region create marginally stable behaviour.

*Figure 8*(b) shows that the system is more robust with respect to inter-vehicle distance channel ($\tau_p$). As a numerical validation of these results, *Figure 9* presents the spacing error of ten follower vehicles with the delay combination of $\tau_p = 0.6$ s and $\tau_v = 0.2$ s corresponding to point a in *Figure 8*. *Figure 10*, as a way of contrast, shows the results of an unstable case $\tau_p = 0.6$ s and $\tau_v = 0.25$ s (point b).

Next, the string stability condition is examined. The parameters are chosen as the string stability is maintained. By taking $k = 7$, $b = 1$, $\chi = 0.2$, $\beta = 9$, $\tau_p = 0.05$ and $\tau_v = 0.05$, *Figure 11* illustrates the performance of the string stability of the platoon under the above control parameters and *Figure 12* demonstrates excellent tracking in velocity.

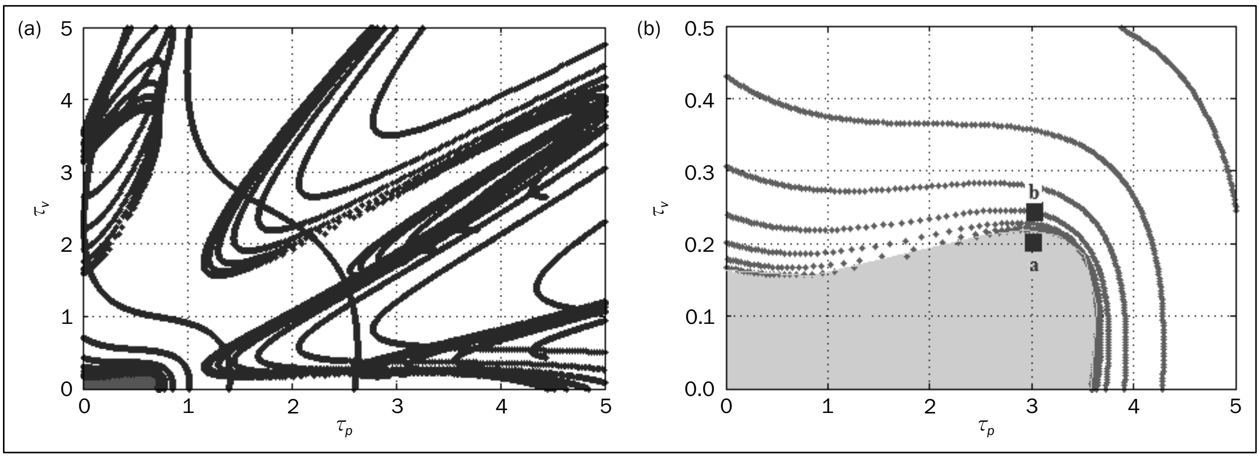


*Figure 8 - (a) Stability region (shaded) for the complete system (b) Detailed view of the same response*

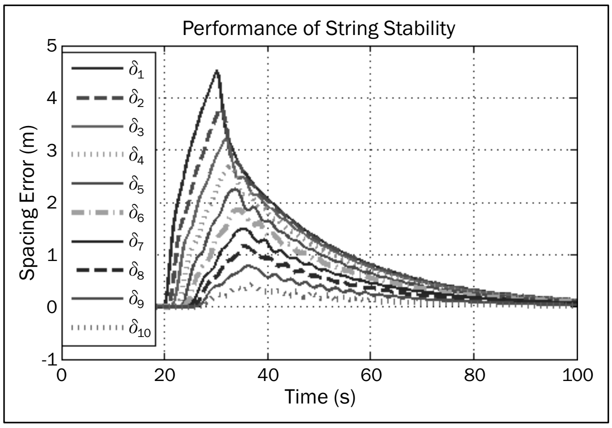


*Figure 9 - Stable behaviour for ten followers corresponding to point a in Figure 8*

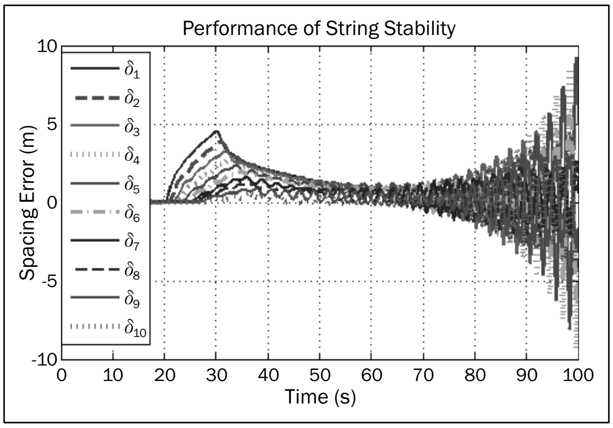


*Figure 10 - Unstable behaviour for ten followers corresponding to point b in Figure 8*

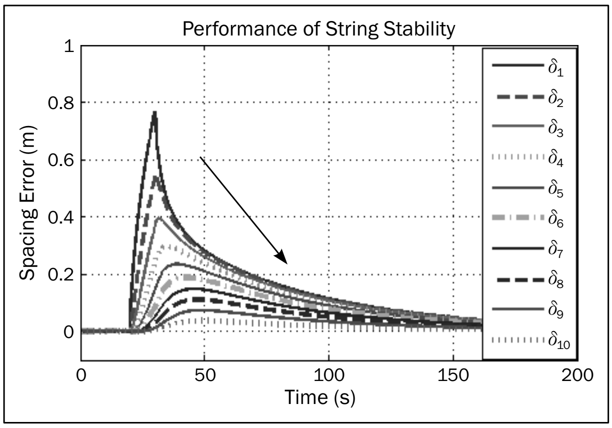


*Figure 11 - Performance of string stability of ten followers*

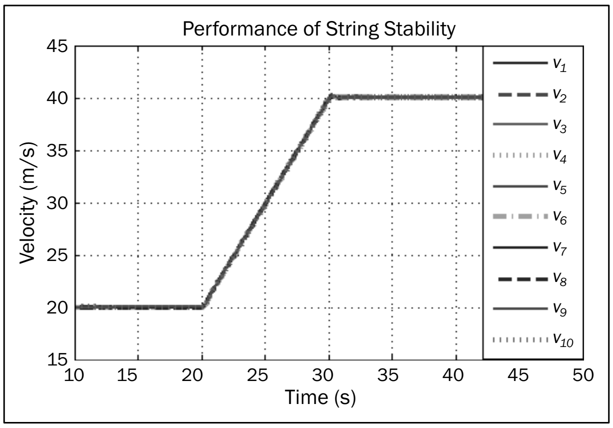


*Figure 12 - Performance of velocity tracking of ten followers*

## 6. CONCLUSION

The paper addresses the coordination problem of a multi-vehicle system with a leader. The determination of stability robustness against delay uncertainties of this system was the main objective in this work. The communication lines were affected by two rationally independent delays. The first delay was assumed to be in the inter vehicle distance information channels, whereas the second delay was in the velocity information exchange. To resolve this dilemma, the complexity of the problem is significantly reduced firstly by decomposing the transcendental function of the system. Then, the stability of the resulting subsystems was assessed exactly and exhaustively in the domain of the time delays by using the CTCR paradigm. The stability analysis demonstrated that the system was more robust with respect to the communication delay in the inter-vehicle distance information channel. The analytical analysis of error accumulation was also performed. The analysis of string stability demonstrated the negative effect of the communication delays on the string stability and the sign of $2b-\beta$ showed which delay had a more negative effect on string stability.

Because of the nature of traffic which gives more influence to the preceding vehicle, it is desirable that the forward term has more influence than the backward term. Hence, the analysis of stability taking into account the asymmetry in position and velocity feedback is still an open problem.

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**خلاصه**

## تجزیه و تحلیل پایداری یک دسته از خودروها با در نظر گرفتن دوتاخیر زمانی


کنترل طولی یک دسته از خودروها در نظر گرفته شده است به نحوی که خودروها یک خودرو راهنما را با فاصله بین خودروئی ثابت دنبال می کنند. پایداری و پایداری رشته ای یک دسته از خودروها با در نظر گرفتن دو تاخیر زمانی مستقل و نامعین، تأخیر اول در کانال ارتباطي موقعیت نسبی و تأخیر دوم در تبادل اطلاعات سرعت نسبی، بررسی شده است.

اهداف اصلی این مقاله عبارتند از:(1) استفاده از روش تجزیه ساده (دیکاپلینگ) و الگو رفتار خوشه ریشه هاي مشخصه (CTCR)، برای یافتن مرزهای دقیق پایداری در حوزه ی تاخیر زمانی و (2) تجزیه و تحلیل پایداری رشته ای به منظور تضعیف اغتشاش. در انتها، با انجام شبیه سازي برای یک دسته از خودروها اثربخشی روش ارائه شده، نشان داده شده است.



## کلمات کلیدی

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