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# DEVELOPING A NEW SURROGATE SAFETY INDICATOR BASED ON MOTION EQUATIONS 


#### Abstract

Collision avoidance system (CAS), with the help of surrogate safety measures is a beneficial tool for reducing driver errors and preventing rear-end collisions. One of the most well-known surrogate safety measures to detect rear-end conflicts is Time-to-collision (TTC). TTC refers to the time remaining before the rear-end accident if the course and the speed of vehicles are maintained constant. Different surrogate measures have been derived from TTC; however, the most important are Time Exposed Time-to-collision (TET) and Time Integrated Time-to-collision (TIT). In this paper a new surrogate safety measure based on TTC notion has been developed. This new indicator merges TET and TIT into one measure and gives a score between 0 and 100\%, as the probability of collision. Applying this indicator in CAS as a safety measure will be more useful than TET\&TIT, to reduce driver errors and rear-end collisions.


## KEY WORDS

Time-to-collision; Collision avoidance systems; car-following; motion equation;

## 1. INTRODUCTION

Automated in-vehicle systems are now being developed by car manufacturers with the aim of reducing driver's workload and errors. Therefore, they can help in improving traffic safety. A group of such systems is called advanced driver assistance systems (ADAS), and consists of systems such as intelligent speed adaptation (ISA), adaptive cruise control (ACC), collision
avoidance system (CAS), etc. Among these systems, CAS is designed to alert the drivers on time if there is a possibility of a rear-end collision, to avoid such accidents [1-4].

The critical parameter in CAS system is the system reliability. It means that the system errors must be at the minimum level and the system should alert the driver just when there is high probability of collision. In addition, the alert must be soon enough to receive a proper reaction from the driver. Thus, in developing CAS, first it is essential to define a warning strategy that warns the drivers when there is a rear-end collision risk and there is enough time to receive the driver reactions [2-6].

Time-to-collision (TTC) is one of the most important and well-known safety indicators, which can be useful in detecting rear-end conflicts in car-following scenarios. Car-following is a situation in which one vehicle follows another and so there is an interaction between these two vehicles. If two vehicles in a car-following scenario maintain their speed and collision course, then TTC is the amount of time remaining to a rear-end collision.

TTC can be used as a surrogate safety measure in traffic conflict technique to assess the safety of a freeway. Traffic conflict technique is an indirect method for identifying high crash locations, with no crash data. In this method, TTC can be used both to assess the safety status of each vehicle on the road and by averaging the results to get a useful indicator to evaluate the overall safety of the road section [7-9].

In addition, TTC is an effective measure for CAS to detect rear-end conflicts [4]. There are some new surrogate safety measures derived from TTC, among which TET and TIT are the most important. TET and TIT are useful safety indicators for evaluating car-following scenarios during an interval. However, there are some challenges in applying these indicators to detect dangerous situations with rear-end collision potential. This paper attempts to develop a new safety indicator named rear-end collision probability (RECP), based on TTC notion to resolve these challenges. RECP merges TET\&TIT into one measure and gives a score in the range [0, 100\%].

The paper consists of 6 sections. Section 2 deals with the literature review, which deals with TTC, TET and TIT. Then in Section 3 RECP development is described. Section 4 contains data collection. In Section 5 , TET, TIT and RECP are calculated based on these data and compared. Finally, the paper ends with the conclusion in Section 6.

## 2. TTC NOTION AND IMPROVEMENTS

### 2.1 Definition

TTC is a time-based surrogate safety measure, which can be applied in different studies for evaluating collision risk [7-15]. TTC value at an instant $t$, for rearend conflicts, is defined as the time for the collision of two vehicles if they continue at their present speed and on the same path [13, 14].
$\operatorname{TTC}_{F}(t)=\frac{X_{L}(t)-X_{F}(t)-I_{L}}{\dot{X}_{F}(t)-\dot{X}_{L}(t)} \quad \forall \dot{X}_{F}(t)>\dot{X}_{L}(t)$
where
$X$ - vehicle position defined as the position of the point in the front of the vehicle ( m ),
$\dot{X}$ - derivative of $X$ with respect to time ( $\mathrm{m} / \mathrm{s}$ ),
$L_{L}$ - leading vehicle length (m),

L and F - indices referring to leading and following vehicles in a car-following situation.
The smaller the TTC, the higher is the risk of collision.
Note: TTC is only valid when the speed of the following vehicle is higher than the speed of the leading vehicle.

### 2.2 Time-exposed TTC (TET) and timeintegrated TTC (TIT)

To calculate these indicators the trajectories and TTC profiles of vehicles are used. If we assume that Figure 1 is the TTC profile of a driver and TTC* is the boundary between safe and unsafe car-following situations, then TET and TIT would be calculated as below.

TET is the duration of exposition to safety-critical TTC values over specified time duration. So all instants in which the driver is following the leading vehicle, with $0<T T C<T T C^{*}$ must be summed. For the subject vehicle $i$, it is assumed that the measured TTC values at an instant $t$ do not change during small time steps $\tau_{\mathrm{sc}}$ (here $\tau_{\mathrm{sc}}=0.1 \mathrm{sec}$ ). For the considered period $H$, there are $T=H / \tau_{s c}$ time instants. Therefore, TET is [14]:
$T E T_{i}=\sum_{t=0}^{T} \delta_{i}(t) \times \tau_{S C}$
$\delta_{i}(t)= \begin{cases}1, & 0 \leq T T C_{i}(t) \leq T T C^{*} \\ 0, & \text { otherwise }\end{cases}$
The average TET during $H$ seconds for vehicle $i$, is calculated as presented in Equation 3:
$\overline{T E T_{i}}=100 \times \frac{T E T_{i}}{H}$
However, TET does not consider the difference between the value of TTC* and TTC below the critical threshold. Therefore, TIT is introduced. TIT is the summation of TTC profile over $\tau_{\text {sc }}$ time intervals (with steps of 0.1 second), for TTCs below TTC*. Based on this notion TIT is [14]:


Figure 1 - An example of calculating TET\&TIT

$$
\begin{aligned}
T I T_{i} & =\sum_{t=0}^{T}\left[T T C^{*}-T T C_{i}(t)\right] \times \tau_{\mathrm{SC}} \\
& \forall 0 \leq T T C_{i}(t) \leq T T C^{*} \\
T I T_{i} & =T T C_{i}(t)-T T C^{*}=0 \\
& \forall T T C_{i}(t)>T T C^{*}
\end{aligned}
$$

The average TIT, during $H$ seconds for vehicle $i$ is calculated as shown in Equation 5:
$\overline{T I T_{i}}=100 \times \frac{T I T_{i}}{H . T T C^{*}}$
To realize the procedure of calculating TET and TIT, these two indicators are displayed schematically in Figure 1. The shaded areas $\left(S_{1}+S_{2}+S_{3}\right)$ below the critical threshold (TTC*) display TIT and the summation of all moments below the threshold $\left(T_{1}+T_{2}\right)$ would be TET.

In addition, in recent years other definitions have been developed based on TTC to consider conflicts in different angles and also motion characteristic [10, 11, 12, 15]. For simplicity, this paper uses the conventional definition of TTC for rear-end conflicts introduced in Equation 1.

### 2.3 Weak points of TET\&TIT

There are two main challenges when applying TET and TIT to assess the safety in a car-following situation.

Firstly, the critical threshold TTC* is not a certain and obvious value. Until now, different values have been suggested as TTC*. Hirst and Graham reported


Figure 2 - Comparison of TET and TIT for two typical car-following situations

3, 4 and 5 seconds as values of TTC threshold based on different laboratory experiments [6]. Hogema and Janssen suggested 2.6 and 3.5 seconds after they studied the driver behaviour [16]. Van Der Horst reported even lower critical TTC values at intersections [17]. Therefore, there is no definite value as TTC* of different drivers in different car-following scenarios.

Secondly, in Figure 2, TTC profiles for two schematic car-following situations are drawn. Comparing these situations indicates that, $T E T_{l}<T E T_{\| I}$ but $T I T_{l}>T I T_{I I}$. Now, there is a question: which situation is more critical? In fact, in literature there is not enough evidence about the priority of TET and TIT for safety evaluations [8].

## 3. REAR-END COLLISION PROBABILITY INDICATOR (RECP)

The idea to develop this new safety indicator has come from the concept of water pressure on a vertical gate, which is shown in Figure 3. At the water surface the pressure is zero and as the depth of the water increases so does the pressure, until the maximum pressure is observed at the bottom [18]. Here $\gamma$ is the specific gravity of water and h is the depth of water.

There might be a similar relationship between TTC values and the possibility of rear-end collisions. In fact, it is expected that the possibility of rear-end collision increases as the TTC value decreases. Consider the possibility of rear-end collision to be in the range of [0, 100\%], the relationship between this possibility and TTC values might be non-linear.

Assume that in a car-following scenario, there is a relationship that can determine the probability of collision at each instant based on the amount of TTC (Equation 6). If displaying this probability for vehicle $i$ at instant t by $R E C P_{t}^{i}$, then for an interval like H , there are

$$
n=\left[\frac{H}{\left(t=\tau_{S C}\right)}\right]+1
$$

time steps. Note that in different car-following scenarios, $n$ would be different. Now to determine the average rear-end collision risk during H seconds one can apply


Figure 3 - Water pressure on a vertical gate

Equation (7). Our intention is to obtain the formula for function (TTCt ${ }^{i}$ ) in (6).
RECP ${ }_{t}^{i}=$ function $\left(T T C_{t}^{i}\right)$
$\overline{R E C P}_{H}^{i}=\frac{\sum_{t=1}^{n} R E C P_{t}^{i}}{n}$
where
$R E C P_{t}^{i}$ - rear-end collision probability at instant $t$ for vehicle i,
$\overline{R E C P}_{H}^{i}$ - average rear-end collision probability during time interval H for vehicle i ,
$n$ - number of time-steps.
$\overline{R E C P}_{H}^{i}$ is our new safety indicator. It merges the characteristics of both TET and TIT. TET indicates the duration of time that the driver travels with TTC below the critical TTC*. Based on the fundament of probabilistic theory, for each following moment safety $R E C P_{t}^{i}$ is evaluated and has a score between 0 and $100 \%$. For a period of the following with $n$ time steps, $\overline{R E C P}_{H}^{i}$ is the average of all these scores (TET's specification).

TIT takes the impact of TTC values in the safety assessment according to the duration of the travelling time below the critical TTC*. As described above, we expect that RECP ${ }_{t}^{i}$ in each moment is to be calculated in a manner that it increases when TTC value decreases (TIT's specification). So, $\overline{R E C P}_{H}^{i}$ would have the characteristics of both, TET and TIT. We are trying to avoid the disadvantages discussed earlier, because there is no need to define a critical threshold for TTC. However, to determine the value of $\overline{\operatorname{RECP}}{ }_{H}^{i}$ for a carfollowing scenario, the relationship between TTC and $R E C P_{t}^{i}$ must be known (Equation 6).

### 3.1 Scenario 1

Assume that vehicle $F$ is following vehicle $L$ (like in Figure 4). When $t=t_{1}, \dot{X}_{F}\left(t_{1}\right)>\dot{X}_{L}\left(t_{1}\right)$, TTC can be computed ( TTC >0). In order to investigate if this moment in the car-following situation is safe or not, Equation (8) is used. This equation implies that, if vehicle $F$ wants to avoid the collision, TTC must be 0 or not defined.

It is assumed that during the deceleration period $\left(t_{1} \rightarrow t_{2}\right)$, the leading vehicle travels at constant speed


Figure 4 - Scenario 1 for RECP determination
and the following vehicle has a movement with constant deceleration so:

$$
\left\{\begin{array}{l}
D_{2}=D_{1}+s_{2}-s_{1}  \tag{8}\\
s_{1}=\frac{\dot{X}_{L}^{2}-\dot{X}_{F}^{2}}{} \\
s_{2}=\left(t_{2}-t_{1}\right) \times \dot{X}_{L}=\frac{\dot{X}_{L}-\dot{X}_{F}}{a_{\max }} \times \dot{X}_{L}
\end{array}\right.
$$

where
$D_{1}$ - clearance between vehicles at $t_{1}(m)$,
$D_{2}$ - clearance between vehicles at $t_{2}(\mathrm{~m})$,
$s_{1}$ - distance that the following vehicle travels to decelerate from $\dot{X}_{F}$ to $\dot{X}_{L}(m)$,
$\mathrm{s}_{2}$ - distance that the leading vehicle travels at constant speed ( $X_{L}$ ) during deceleration of the following vehicle (m),
$t_{2}-t_{1}$ - deceleration time interval of the following vehicle (s),
$a_{\max }$ - maximum deceleration rate (negative) for a typical vehicle, $\left(\mathrm{m} / \mathrm{s}^{2}\right)$.
In this scenario if $D_{2} \leq 0$, then the collision cannot be avoided and $R E C P_{t}^{i}$ is equal to $100 \%$; otherwise, we should go to the second scenario.

### 3.2 Scenario 2

If $D_{2}>0$, then collision is avoided at moment $t_{2}$ and $R E C P_{t}^{i}$ has a value less than $100 \%$ (and greater or equal to 0 ). Based on the previous scenario, the following vehicle (F) has decelerated to reduce its speed from $\dot{X}_{F}$ to $\dot{X}_{L}$. So, at moment $t_{2}$ both vehicles have the same speed, $T T C=\infty$ and car-following is safe (Figure 5). RECP $t_{t}^{i}$ now depends on further behaviour of the leader driver.

If in a few instants later vehicle $L$ decelerates and reduces its speed to $\dot{X}_{L}-\varepsilon$, TTC would be definable again and as the clearance between vehicles has been reduced, small TTCs are expectable and the car-following situation is unsafe (in time $t_{3}$, Figure 5).


Figure 5 - Scenario 2, for RECP determination

$$
\left\{\begin{array}{l}
D_{3}=D_{2}+s_{4}-s_{3} \\
s_{3}=\dot{X}_{L} \times\left(t_{3}-t_{2}\right)=\dot{X}_{L} \times \frac{\dot{X}_{L}-\varepsilon-\dot{X}_{L}}{\ddot{X}_{L}}  \tag{9}\\
s_{4}=\frac{\left(\dot{X}_{L}-\varepsilon\right)^{2}-\dot{X}_{L}^{2}}{2 \ddot{X}_{L}}
\end{array}\right.
$$

where
$D_{3}$ - clearance between vehicles after the leading vehicle deceleration (m),
$s_{3}$ - distance that the following vehicle travels at constant speed, when the leading vehicle is decelerating ( m ),
$\mathrm{S}_{4}$ - distance that the leading vehicle travels when decelerating (m),
$t_{3}-t_{2}-$ deceleration interval for the leading vehicle (s),
$\varepsilon$ - value of speed reduction of the leading vehicle ( $\mathrm{m} / \mathrm{s}$ ),
$\ddot{X}_{L}$ - deceleration rate of the leading vehicle during $t_{3}-t_{2},\left(\mathrm{~m} / \mathrm{s}^{2}\right)$.
At $t=t_{3}, \quad T T C=\frac{D_{3}}{\varepsilon}$, again the following vehicle would try to reduce its speed, from $\dot{X}_{L}$ to $\dot{X}_{L}-\varepsilon$, in order to make TTC meaningless (TTC $\leq 0$ or TTC $=\infty$ ). This process would happen in $t_{4}-t_{3}$ interval (Figure 5). At the end of this process $\left(t=t_{4}\right)$, the clearance between vehicles would be:

$$
\left\{\begin{array}{l}
D_{4}=D_{3}+S_{6}-S_{5} \\
S_{5}=\frac{\left(\dot{X}_{L}-\varepsilon\right)^{2}-\dot{X}_{L}^{2}}{2 a_{\max }}  \tag{10}\\
S_{6}=\left(\dot{X}_{L}-\varepsilon\right) \times\left(t_{4}-t_{3}\right)=\left(\dot{X}_{L}-\varepsilon\right) \times \frac{\dot{X}_{L}-\varepsilon-\dot{X}_{L}}{a_{\max }}
\end{array}\right.
$$

where:
$D_{4}$ - clearance between vehicles at $t_{4}(m)$,
$D_{3}$ - clearance between vehicles at $t_{3}(\mathrm{~m})$,
$\mathrm{S}_{5}$ - distance that the following vehicle travels when decelerating (m),
S6

- distance that the leading vehicle travels at constant speed, when the following vehicle is decelerating ( m ),
$t_{4}-t_{3}$ - deceleration interval of the following vehicle (s).
Now, if merging Equations 8, 9, and 10 and assuming $D_{4}$ is equal to 0 (collision occurs), then $\varepsilon$ can be computed. In fact, by solving this equation it would be known how much of reduction in the speed of the leading vehicle ( $\varepsilon$ ) might lead to a rear-end collision in the second scenario.
$\varepsilon=\sqrt{\left(D_{1}-\frac{\dot{X}_{F}^{2}+\dot{X}_{L}^{2}}{2\left|a_{\max }\right|}+\frac{\dot{X}_{L} \times \dot{X}_{F}}{\left|a_{\max }\right|}\right) \times \frac{2\left|a_{\max } \times \ddot{X}_{L}\right|}{\left|a_{\max }\right|+\left|\ddot{X}_{L}\right|}}$
The maximum deceleration rate, which is comfortable for 90 percent of drivers based on [19] is about $3.4 \mathrm{~m} / \mathrm{s}^{2}$, by replacing 3.4 for $a_{\max }$ in Equation 11, we have:
$\varepsilon=\sqrt{\frac{6.8\left|\ddot{X}_{L}\right| \times D_{1}-\left|\ddot{X}_{L}\right| \times\left(\dot{X}_{L}+\dot{X}_{F}\right)+2\left|\ddot{X}_{L}\right| \times \dot{X}_{L} \dot{X}_{F}}{3.4+\left|\ddot{X}_{L}\right|}}$
But $\ddot{X}_{L}$, which is the deceleration of the leading vehicle in the second scenario is unknown in Equation 12. To determine this parameter, the car-following models must be used; however, here for more simplicity $\ddot{X}_{L}$ is assumed to be $3.4 \mathrm{~m} / \mathrm{s}^{2}$. Now we have:
$\varepsilon=\sqrt{3.4 D_{1}-0.5\left(\dot{X}_{L}^{2}+\dot{X}_{F}^{2}\right)+\dot{X}_{L} \dot{X}_{F}}$
Note that if $\varepsilon>\dot{X}_{L}$ then $R E C P_{t}^{i}$ would be equal to $100 \%$. It is assumed that both the leading and the following vehicles just move forward (as prevalent in freeways) and we assume that the vehicle speed cannot be negative ( $\dot{X}_{L}-\varepsilon<0$ ), which means concisely that, $\forall \varepsilon \quad \dot{X}_{L}-\varepsilon>0$.

In order to calculate $R E C P_{t}^{i}$ in the second scenario, the probability of reduction $\varepsilon$ of the leading vehicle speed must be known. This can be achieved just by the statistics and assuming that there is a specified probability density function for the vehicle speed variations. This procedure will be demonstrated in further sections.

For the sake of simplicity and to review the procedure of developing a new surrogate safety measure, a flowchart is presented in Figure 6. In this step RECP ${ }_{t}^{i}=$ function $(\varepsilon)$.

## 4. DATA COLLECTION

The data, which are used to calculate TET, TIT and $R E C P_{t}^{i}$, are the parts of a comprehensive database obtained from the Next Generation Simulation (NGSIM) web site (NGSIM, June 5, 2009). NGSIM is a Federal Highway Administration (FHWA) supported project and obtains microscopic traffic data. The NGSIM Table 1 - Microscopic traffic data gathered in NGSIM project

| Row | Parameter | Unit |
| :---: | :--- | :--- |
| 1 | Vehicle ID | Number |
| 2 | Frame ID | $1 / 10$ of a second |
| 3 | Total Frames | $1 / 10$ of a second |
| 4 | Local X | Feet |
| 5 | Local Y | Feet |
| 6 | Vehicle Length | Feet |
| 7 | Vehicle Width | Feet |
| 8 | Vehicle Class | Text |
| 9 | Vehicle Velocity | Feet/Second |
| 10 | Vehicle Acceleration | Feet/Second Square |
| 11 | Lane Identification | Number |
| 12 | Preceding Vehicle ID | Number |
| 13 | Following Vehicle ID | Number |
| 14 | Spacing | Feet |
| 15 | Headway | Seconds |



Figure 6 - Research flowchart


Figure 7 - Interstate-80, six-lane section under study
project maintains some data sets from freeways and arterials gathered using high resolution cameras that are able to record the vehicle position every 0.1 seconds, which means that the time steps will be [0, 0.1]. Vehicle trajectory data collected in the afternoon peak hour on Wednesday, April 13, 2005 from 4:00 p.m. to 4:15 p.m. on a segment of Interstate 80 in Emeryville, San Francisco are used in this research [20], (Figure 7). The microscopic data contain the parameters pre-
sented in Table 1. The ID in Table 1 is vehicle Identification code; here this variable changes from 1 to 2,052 .

There are three classes of vehicles (motorcycles, passenger cars, and heavy vehicles) on this freeway which has six lanes. Here, Lane 1 is the leftmost lane and at the same moment it is an HOV (High Occupancy Vehicle) lane. In addition, as seen in Figure 6, Ianes 5 and 6 are the rightmost lanes influenced by an onramp and an off-ramp [20].

In order to use these trajectory data for the proposed analysis, the car-following scenarios must be selected. For this purpose three criteria are used:

- Both the leading and the following vehicles are passenger cars.
- The two cars had to be adjacent during the whole period in which they were both observed through the $1,600 \mathrm{ft}$ study segment, e.g. none of the vehicles changes its lane in this segment and no third vehicle comes between the two vehicles.
- The period during which both cars were observed should have a duration of at least 30 s (at least 300 observations).
Of the above mentioned condition, 491 car-following time series are achieved.


## 5. RESULTS

As discussed in Section 3, to determine the probability of the speed reduction like $\varepsilon$, first the PDF (probability density function) of speed variations of the leading vehicles must be known. Here, based on the car-following scenarios, speed changes for the leading vehicles are calculated ( $\varepsilon$ ). The distribution has been calculated with at least 147,300 points $(491 \times 300)$. Therefore, the distribution is constructed with:

$$
N=\sum_{i=1}^{491} n_{i},
$$

where: $n_{i}$ is the number of observations in the "carfollowing situation" i.

Then, the best distribution that can be fitted to these data is determined (Figure 8). Results indicate
that the distribution is normal with 0 and 12.7 as the mean and variance respectively, $X \square N(0,12.7)$. In fact, this distribution is obtained based on the driver behaviours on I-80 freeway.

Now, based on this normal distribution, one would be able to calculate the probability of the speed reduction more than or equal to $\varepsilon$.

Finally, the relationship between TTC and $R E C P_{t}^{i}$ values for each moment of following in which $0<T T C<10$ and $R E C P_{t}^{i} \neq 0$ is presented with a scatter plot in Figure 9. This plot is drawn based on the microscopic data of the car-following scenarios. TTC of more than 10 seconds is considered as a safe situation; in literature the maximum value as the critical threshold of TTC is 5 seconds; here, for more confidence 10 seconds have been selected.

In fact, in this figure each point has two coordinates; one relates to the value of TTC at a moment and the other one is $R E C P_{t}^{i}$, which is calculated based on the algorithm described in Figure 6. To find a general relationship between TTC and $R E C P_{t}^{i}$, the best curve fitted to these data is depicted in Figure 9 by a red line. Results show that the initial expectation in Section 3, to have a non-linear relationship between RECP and TTC is valid and the relationship is as Equation 14. According to the relationship between $R E C P_{t}^{i}$ and TTC which are presented in Figure 9, there is a formula (14) of the function desired in (6). The $\overline{R E C P}_{H}^{i}$ for different car-following scenarios on I-80 freeway can be calculated afterwards with the help of Equation (7).

$$
\begin{align*}
& R E C P_{i}^{t}=0.00581 .\left(T T C_{i}^{t}\right)^{4}-0.1575 .\left(T T C_{i}^{t}\right)^{3}+  \tag{14}\\
& \quad+1.658 .\left(T T C_{i}^{t}\right)^{2}-8.628 .\left(T T C_{i}^{t}\right)+25.27
\end{align*}
$$



Figure 8 - The best distribution function fitted to speed variations of leading vehicles


Figure 9 - Data for a relationship between $R E C P_{t}^{i}$ and TTC


Figure 10- $\overline{T E T}$ calculated for car-following situations

The relationship presented in Equation (14) is suitable for $2<T T C<10 \mathrm{~s}$, because data for $0<T T C<2 s$ are not sufficient (since in real world, drivers usually avoid to follow each other with a small TTC). In order to develop a comprehensive relationship between RECP ${ }_{t}^{i}$ and TTC, further research is needed based on experimental data.

Also, to have a comparison between these three indicators; $\overline{T E T}, \overline{T I T}$ and $\overline{R E C P}$ are determined for

150 car-following scenarios and results are presented in Figures 10 to 12. Note that $\overline{T E T}$ and $\overline{T I T}$ calculated by Equations 4 and 5, respectively, have no measure units. From this figure it can be deduced that $\overline{T E T}$, $\overline{T I T}$ and $\overline{R E C P}$ variations for different car-following scenarios are the same, e.g. for car-following IDs between 55 to 60,75 to 80,90 to 95 all these three surrogate measures display a peak in bar charts. This means that $\overline{R E C P}$ is also a valid surrogate measure


Figure 11- $\overline{\text { TIT }}$ calculated for car-following situations


Figure $12-\overline{\operatorname{RECP}}(\%)$ calculated for car-following situations
as $\overline{T E T}$ and $\overline{T I T}$ are. However, $\overline{R E C P}$ has some advantages like:

1. Giving a value as the probability of collision;
2. Having both characteristics of $\overline{T E T}$ and $\overline{T I T}$;
3. Being more considerate in safety assessment (considering the worst case scenarios that might happen in the near future);
4. Being founded based on the equations of motion;
5. Not needing to define a critical threshold.

Therefore, applying this measure in the car-following situations to detect rear-end collisions would be more desirable than $\overline{T E T}$ and $\overline{T I T}$.

From the above results it can be concluded that applying RECP ${ }_{t}^{i}$ in CAS as a warning strategy can help in reducing driver errors, because in a car-following, $R E C P_{t}^{i}$ calculates the risk of collision by considering
the worst-case scenarios that might happen in the near future. Also, $\overline{R E C P}$ is an appropriate indicator to detect risky vehicles in traffic stream based on rearend collision probability.

## 6. CONCLUSION

This paper aims to develop a new surrogate safety measure for rear-end collisions. The new safety index, named rear-end collision probability (or RECP) is derived from the concept of TTC which is the basic surrogate safety measure. Since TTC is unable to consider the full course of vehicles over space and time, two new safety indicators, namely TIT and TET are introduced in the literature. TET expresses the duration of time in which the driver is following another vehicle with unsafe TTCs and TIT takes into consideration the value of TTC below the critical threshold, during the following period. Although TET\&TIT are useful in traffic safety evaluations, there are some challenges to apply these indicators in the collision avoidance systems (CASs) to prevent rear-end collisions. First, the critical threshold of TTC is not a certain value (in literature different values are suggested). Second, the priority between TET and TIT is not obvious in safety evaluations; it means that it is not clear which index is more important.

RECP resolves the disadvantages of TET\&TIT. To compute RECP the traffic microscopic characteristics of vehicles in each time step (like 0.1 sec.) must be known. Then, based on a simple procedure (with the help of equations of motion), RECP would be calculated for each instant in which one vehicle is following another. Now, if applying RECP in CAS, in each moment of the following, one aggregated index (with specifications of TET and TIT) would be calculated in [0, 100\%] range. This can give a proper insight about the safety status. In all, RECP can be a fruitful warning strategy for CAS, because it will give the probability of collision based on the worst-case scenarios that might happen a few moments later in the following process. Therefore, the drivers would have enough time to make a decision and to undertake the best reaction to avoid a potential collision.
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كارشناس ارشـد راه و ترابرى

$$
\begin{aligned}
& \text { مينا صيادى } \\
& \text { دانشجوى كارشـناسـى ارشـد راه و ترابرى } \\
& \text { دانشگاه صنعتى نوشيروانى بابل } \\
& \text { حكيده } \\
& \text { براساس معادلات حركت ايمت }
\end{aligned}
$$


 جلوگيرى از برخوردهاى جلو به عقب اسـت. يكى از مصم ترين شاخص هاى ايمنى براى تشخيص تداخلات جلو به عقب، زمان نا تصادف (TTC) است. TTC به زمان باقيمانده تا برخورد در صورت ثابت فرض كردن مسير و سـرعت وسائل نقليه اطلاق
 TIT وTCT مشتق شـده كه از اين جمله مى توان به TET
اشاره كرد. در اين مقاله يكى شاخص ايمنى جديد بر پايه
 به يك شاخص تبديل و يك نمره بين 0 تا 100، به عنوان احتمال رخداد برخورد جلو به عقب ارائه مى دهد. بكارگيرى اين شاخص در CAS به عنوان يك شاخص ايمنى برای كاهش تع تعداد خطاهاى راننده بسيار مفيدتر از TET و TIT مى تواند باشـد.

## كليد وازه

زمان تا تصادف، سيستم جلوگيرى از تصادف، نعقيب خودرو و معادله حركت

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