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MONTE CARLO SIMULATION-BASED APPROACH TO OPTIMAL BUS STOPS ALLOCATION IN THE MUNICIPALITY OF LAŠKO

ABSTRACT

The paper addresses the problem of optimal bus stop allocation. The aim is to achieve reduction of costs on account of appropriate re-design of the process of obligatory transportation of children from their homes to the corresponding schools in the Laško municipality. The proposed algorithm relies on optimization based on the Monte Carlo simulation procedure. The number of calculated bus stops is required to be minimal possible, which can still assure maximal service area within the prescribed radius, while keeping the minimal walking distances pupils have to go across from their homes to the nearest bus stop and vice versa. The main issues of the proposed algorithm are emphasised and the working mechanism is explained. The presentation of calculated results is given and comparison with some other existing algorithms is provided. The positions of the calculated bus stops are going to be used for the purpose of physical bus stops implementation in order to decrease the current transportation costs.

KEY WORDS

facility location, location problems, maximal covering problems, optimization, Monte Carlo simulation, Geographic Information System (GIS), reduction of transportation costs

1. INTRODUCTION

Since the public sector has been also captured by the consequences of the current economic crisis, the reduction of all types of costs has to be taken into consideration. The paper addresses the question of reducing specific type of costs, which are generated by obligatory transportation of specific categories of children in the Laško municipality. This transportation, which is enforced by government law, includes the transport of children from their homes to the corresponding primary schools and back to their homes.

Figure 1 shows the positions of pupils addresses (PA) in Laško municipality (shaded area). In the current situation, the total number is 562 pupils, which are marked with little circles in *Figure 1*. *Figure 1* also shows the positions of pupils' schools (little house symbols) and their total number is 11.

In the present time, the treated category of transportation reaches very high level of costs, which can rise even up to one million euro per year. Since this is unbearable for such a small municipality as Laško, there is very strong desire to reduce these costs as much as possible.

The main reason for such high level of costs is temporary organization of transport. In the current situation, namely, buses and other transportation vehicles, hired by the municipality are used for picking up and delivering every single individual pupil at their homes. Moreover, these transport routes are not well organized in the sense of unnecessary driven additional kilometres, vehicles are usually not fully loaded, routes are sometimes unnecessarily doubled or even tripled, etc.

As a way of reducing the costs mentioned above, a two-stage optimization approach is proposed (c.f. *Figure 2*). In the first stage, optimal bus stop locations are determined, at which pupils should be picked up. The allocation of bus stops implies that, on the one hand, pupils are required to walk from their homes to the nearest bus stops, which means that some comfort will be lost. On the other hand, the transportation vehicles will have to drive along significantly shorter quantity of routes (distances).

The second stage (*Figure 2*) concerns optimization of driving routes, driving schedules and driving fleet, related to the transportation of pupils from optimal bus stops to their schools and vice versa. This paper deals only with the results achieved during the first stage.

Bus stops cannot be located at any place in the area under study, so that their number cannot be chosen completely arbitrarily. On the contrary, the positions of

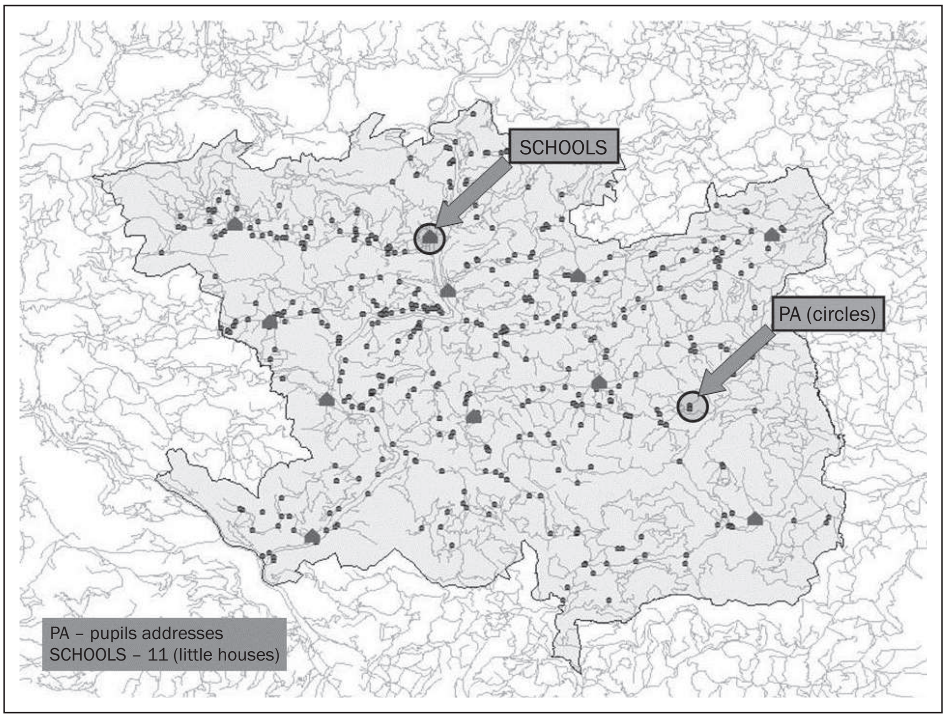

Figure 1 - Current positions of pupils' addresses (little circles) and the positions of their schools (little house symbols)

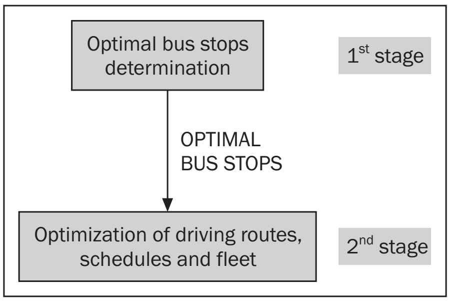

Figure 2 - The two-stage optimization approach to the reduction of transportation costs

bus stops must be determined in such a way that as many pupils as possible should be serviced from an individual bus stop, while the total number of bus stops must be reduced to the lowest possible level.

Obviously, this problem is similar to the so-called “maximum location covering problem” where the minimum fixed number of facilities must be located, which should maximize the service area.

A number of algorithms have been proposed so far for solving the maximum covering problems [1], [2], [3], [4], [5], [6]. For example, the so-called Greedy heuristic algorithm is one of the most typical algorithms for solving such problems [1]. The latter is called Greedy, since it does what is best at each step of the algorithm without looking ahead to see how the current decisions will affect the latter decisions and alternatives [1].

The algorithm, which is introduced in this paper, is slightly different from the existing algorithms. It is based on the Monte Carlo simulation method, which helps us to find the optimal bus stops locations during the combined simulation - optimization procedure.

The paper is further organized as follows. Section 2 presents a brief literature review of the existing algorithms for solving maximum covering location problems. The description of the treated data and initial road data reduction are given in Section 3. The proposed optimization procedure for the purpose of optimal bus stops allocation is described in Section 4. In Section 5, an excerpt of the numerical results is given. Finally, a comparison of proposed algorithm with well-known Greedy Add and Greedy Drop heuristic procedures is provided in Section 6.

2. LITERATURE REVIEW

The maximum covering location problem was first introduced by Church and ReVelle in 1974 [2]. It seeks the maximum population that can be served by a limited number of facilities within a stated service distance or time [2], [7].

The maximal location covering problems arise in a variety of public and private sector problems [1], [5], [6], [7], [8], [9], [10]. For example, state governments need to determine locations for bases for emergency highway patrol vehicles. Similarly, local governments must locate fire stations, police stations and ambulances. In all of these cases, poorly chosen locations can increase the possibility of damage or loss of life [1].

In the private sector, industry must locate offices, production and assembly plants, distribution centres and retail outlets. Poor location decisions in this environment lead to increased costs and decreased competitive position [1].

The maximum location covering problem is one of the typical problems in the facility location theory. Facility location is a resource allocation problem that deals with placement of different types of facilities, which should be provided to the customers on demand [11].

Besides maximum location covering problem, some other significant location problems are: the set covering problem, p-center problem, and p-median problem [1], [2], [3], [4], [12], [13].

The set covering problems are problems where the locations of the minimum number of facilities are found necessary to cover all demands. P-center problems are problems where the maximum distance (or travel times) from any demand node to its nearest facility has to be minimized. P-median problems are problems where the locations of a given number of facilities are found which are necessary to minimize the average distance between customers and the nearest facility.

The classification of the location problems mentioned above also determines the objectives which are needed to classify the facility location models. This is the first important criterion for the classification of the location models [7]. The second important criterion which is also needed for the classification of the location models, are the topological characteristics of the facility and demand sites. The latter leads to different location models including continuous location models, discrete network models, hub connection models, etc [7]. In each of these models, facilities can only be placed at the sites where it is allowed by topographic conditions.

The third criterion, which also determines the classification of facility location models, is the spectrum of solution methods [7]. This paper deals with the maximum covering location problems only which can be treated by use of discrete network models [1]. Therefore, the sequel is restricted only to the short overview of the methods, which are related to this type of problems.

At the beginning of maximum covering problems solving, Church and ReVelle used relaxed linear programming, supplemented by occasional use of branch and bound procedure to provide solution to this type of problems [2], [6]. Since then, a number of new algorithms and heuristics have been derived for this purpose.

For example, Greedy heuristic algorithms [1], [2], [7], [14] are often used, when facility location problems must be solved. Some of the most typical greedy algorithms are: Basic Greedy Adding (Add) algorithm, Improved Greedy Adding algorithm with Substitution, Greedy Dropping (Drop) algorithm, etc.

Basic Greedy Add starts with an empty solution set and then adds to this set one at a time best facility site, which covers the most of uncovered demand points. At the beginning, it picks the first facility, which covers the most of the total demand population. After that, it picks the second facility, which covers the most population not covered by the first facility. In further iterations, the sites for the next facilities are selected, which cover the largest number of demand points uncovered by the facilities in previous iterations. This process is continued until either chosen number of facilities has been selected or all the demand population is covered [1], [2].

Greedy Drop is based on the reverse strategy of the Greedy Add heuristic approach and uses the naive drop heuristics [14]. Initially, all facility candidates are part of the solution configuration. At each iteration one candidate is then removed until only the given number of candidates is left in the configuration. The mechanism of removing works in the sense that at each iteration the candidate that produces the smallest decrease in the objective function is dropped.

Of course, besides Greedy algorithms there are many other more advanced heuristics like Genetic Algorithms [15], [16], Simulated Annealing [17], [18] and Tabu Search [19], [20]. They may be also applied to obtain a better solution if necessary but should be compensated by a significantly longer computation time [7].

Some other significant methods which have been proposed for solving of maximum covering problems are: Lagrangean relaxation [1], [6], [13], Lagrangean/Surrogate heuristics [21], Heuristic concentration [6], etc. Detailed reviews of the solution procedures for the maximum covering problems can be found in [5].

3. PROBLEM DEFINITION AND INITIAL ROAD DATA REDUCTION

Figure 3 shows the positions of entire set of road data points in Laško municipality, which are also possible bus stops candidates (BSC). One can see that these points are marked with little squares. They have been generated by 300 metres segmentation of every single road within the treated surface. The total number of segmented road data points is 14,295, which were collected by the use of Geographic Information System (GIS) [22]. The positions of 562 pupils addresses (PA) are marked with little circles in *Figure 3* and these points were also collected by means of GIS system.

The distribution of pupils data points can be treated over the control area of 247.9km^2 (marked area A in *Figure 3*), while the road data points are distributed within a much larger area (more than 300km^2). Since the number of bus stop candidates is enormous in the treated case (14,295 possible points) and the treated region is quite large (more than 300km^2), while the walking requirements are quite rigorous (pupils should

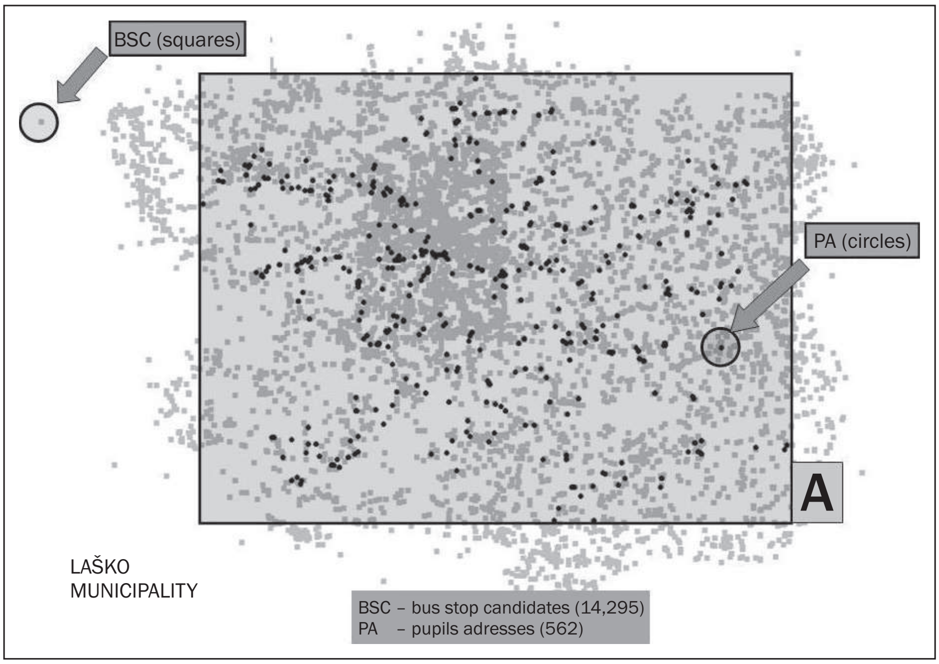

Figure 3 – 14,295 road data points - bus stops candidates (squares), and 562 pupils' data points (circles)

not be walking for more than 1.3km), this is likely to be an NP-hard problem. It is, namely that 1.3 km is quite a short distance in comparison with the enormity of the treated area.

Since the total number of bus stops candidates (14,295) is enormous, it has to be somehow reduced. For this purpose, the original set of road data is firstly reduced only to the control area A in *Figure 3*.

If only road data points inside area A as well as all pupils' addresses in A are considered, the resulting data points are shown in *Figure 4*. *Figure 4* shows the reduced situation of 11,820 road points, besides 562 pupils' data points.

Further reduction of the road data set in *Figure 4* relies on two heuristic rules:

1) Only those road data points will be considered which are not too close to the nearest neighbouring road data points. Hence, the calculations are simplified and the speed of computations is increased.
2) Only those road data points that are close enough to the pupils' data points will be taken into consideration.

If these heuristic rules are applied, the resulting data points are shown in *Figure 5*. *Figure 5* shows further reduced situation of 1,768 road points, besides 562 pupils data points.

One can see that the reduced number of road points is quite smaller in comparison with the original road data or data within the control area A (Figures 3 and 4). Further processing procedure, namely, is going to deal only with the 1,768 possible road points instead of the original number of 14,295 road points or by area A reduced number of 11,820 road data points, respectively.

Since the reduced number of 1,768 road data points is still too big in the sense of optimal bus stops allocation, further reduction of these points is somehow needed in order to lower the total number of optimal bus stops to the acceptable level.

4. OPTIMIZATION AND SIMULATION PROCEDURE

For the purpose of optimal bus stops allocation, the optimization procedure (c.f. *Figure 6*) must be applied in order to furthermore reduce 1,768 bus stops candidates, which were extracted during the initial road data reduction.

If the optimization, which is based on Monte Carlo simulation procedure, is directly applied for the whole control area A shown in *Figure 5*, it turns out that some serious problems would occur during the process of computations. Therefore, the observed surface with 1,768 road data points in *Figure 5* must be additionally divided into a certain number of subsectors (c.f. *Figure 6*) in order to efficiently use the Monte Carlo procedure.

Among several combinations of “ad hoc” partitions of area A in *Figure 5* into subsectors, it turns out that the partition into 8 subsectors can assure the most reasonable optimization results. Thus, if partition into 8 subsectors is applied, then *Figure 5* can be represented, as shown in *Figure 7*.

As it can be seen from *Figure 7*, each subsector covers a certain number of bus stops candidates and a certain number of pupils data points. Now, for each subsector it must be decided which bus stops candidates points provide service to the maximal possible number of pupils in the sense that their minimal walk-

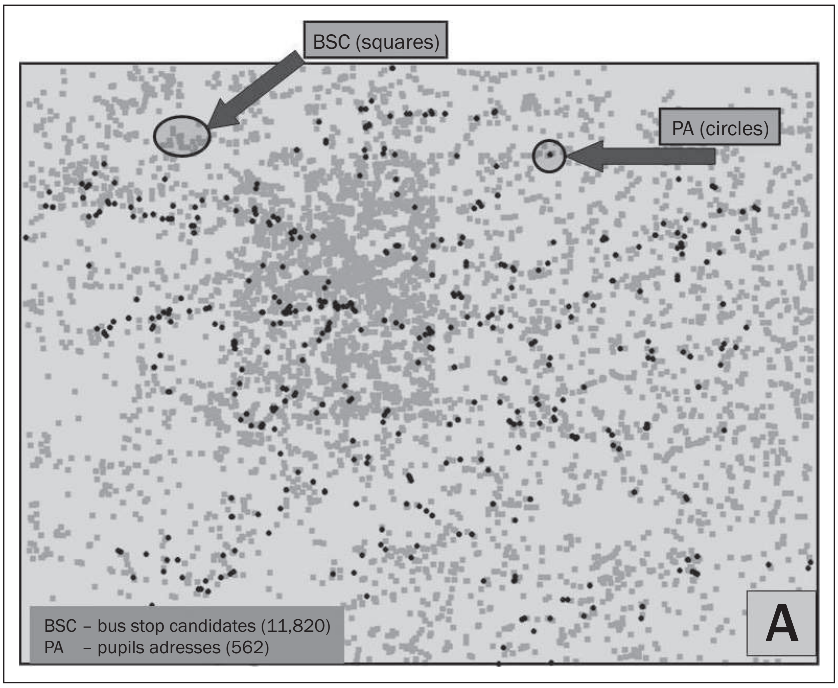

Figure 4 – 11,820 road data points (squares), reduced to the control area A; 562 pupils data points (circles)

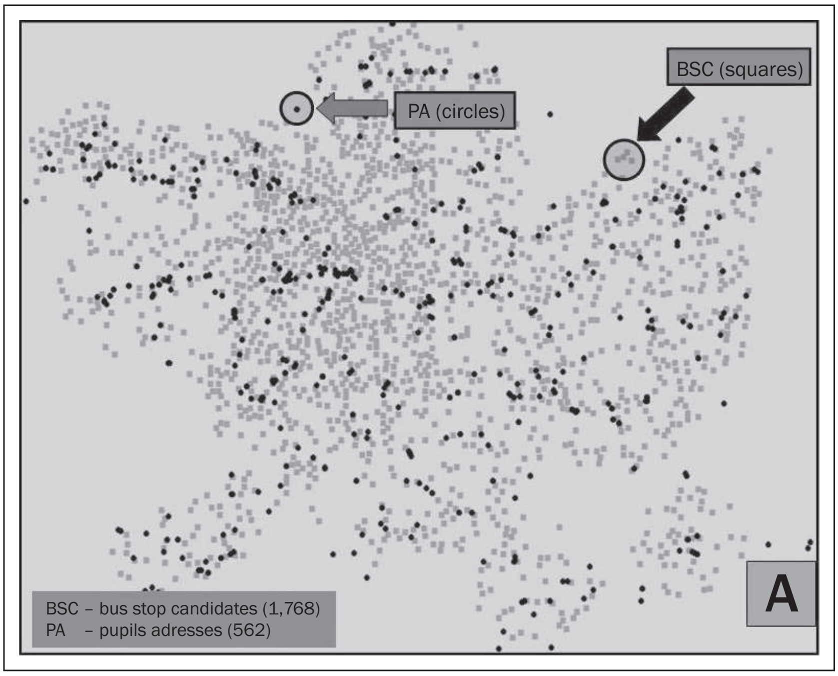

Figure 5 – 1,768 road data points (squares), further reduced according to the two heuristic rules; 562 pupils data points (circles)

ing distances lie within the prescribed radius r. Additional criterion is that the number of uncovered pupils (pupils who should walk more than the prescribed radius) is as low as possible.

It turns out that this problem can be efficiently solved by the use of Monte Carlo procedure. Let us take the *j*-th subsector ($j = 1, 2, \ldots, 8$) with randomly chosen road data points (for example 7 points $S_1, \ldots, S_7 = S_k, k = 1, \ldots, 7$ in *Figure 8*). Let each point be surrounded with circles $P_1, \ldots, P_7 = P_k, k = 1, \ldots, 7$ of prescribed radius r. If the first iteration of Monte Carlo simulation is executed, this situation can be presented as illustrated in *Figure 8*. In this simplified illustration, one can notice that there are only 30 bus stops candidates (squares) and 20 pupils addresses points (17 inside the circles P_k (little circles) and 3 outside the circles P_k (stars)). Of course, the true number of pupils data addresses to be served and road data points is certainly much bigger in individual subsector (c.f. *Figure 7*), than presented in *Figure 8*.

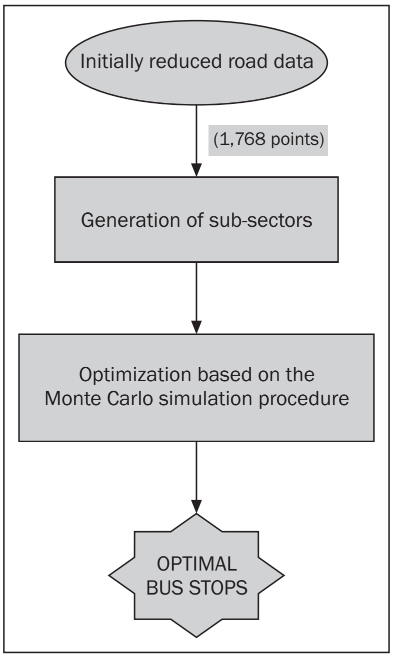

Figure 6 - Optimization procedure for the purpose of optimal bus stops allocation

The algorithm calculates the shortest distances from pupils addresses to the nearest chosen road points $S_k, k=1,\ldots,7$, at first (consider Euclidian distances $d_1, d_2, \ldots, d_{17}$ in *Figure 8*). Due to these calculations, the attempt is made to assign all pupils to the nearest bus stop candidate S_k, if the calculated mutual distance is shorter than the prescribed radius r. With respect to these conditions, 17 pupils data points are obviously assigned to (covered by) 7 randomly chosen road data points $S_k, k=1,\ldots,7$, with accompanying circles $P_1, \ldots, P_7$.

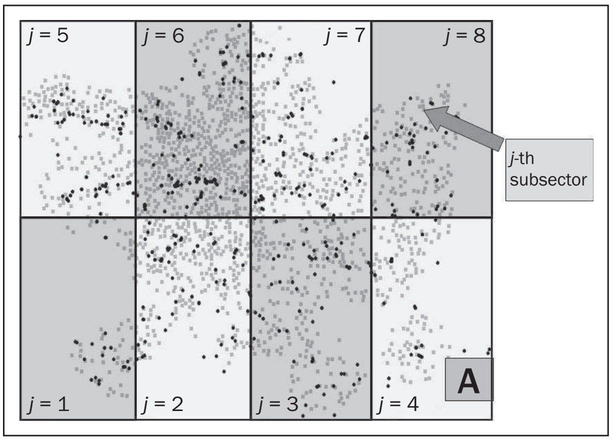

Figure 7 - Partition into 8 subsectors; 1,768 road data points – bus stops candidates (squares), 562 pupils data points (circles)

Figure 8 - Illustration of Monte Carlo simulation approach, applied for the *j*-th subsector, $j=1,2,\ldots,8$, during the first simulation

Since the distance from 3 pupils addresses to any of the chosen road points S_k is longer than the prescribed radius r, these points are treated as unassigned (uncovered) and are marked by stars in *Figure 8*.

Now, the following criterion function for *j*-th subsector can be applied as a sum of all distances of assigned pupils data points in *Figure 8*:

$$J_{crit1}(j,1) = \sum_{i=1}^{17} d_i, \quad j = 1,2,\ldots,8 \tag{1}$$

where 1 in the function argument denotes the number of first iteration during the simulation procedure. The meaning of the criterion function (1) is the following:

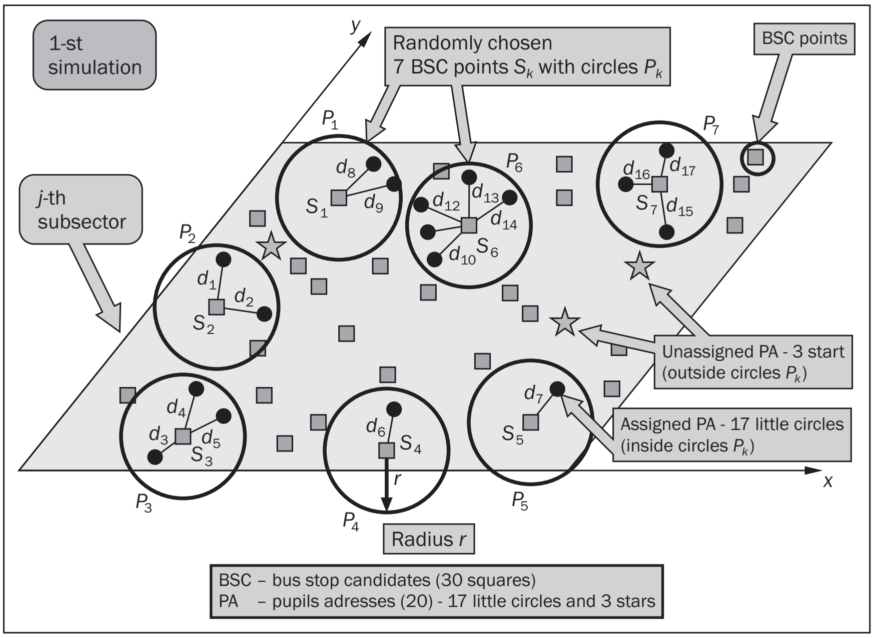

Figure 8 - Illustration of Monte Carlo simulation approach, applied for the j-th subsector, j= 1, 2,...,8, during the first simulation

the lower value of the criterion function (1) also means the shorter average walking distance of the assigned pupils.

Also, the number of unassigned pupils is applied as the second criterion function and can be presented in the following way for the j-th subsector and the first iteration:

$$J_{crit2}(j,1) = \text{number of unassigned pupils}, \quad j = 1,2,\ldots,8 \tag{2}$$

For the situation in *Figure 8*, the criterion function (2) evidently takes the value of 3 unassigned pupils. It is obvious, that there is a strong tendency to reach the lowest possible value of both criterion functions (1) and (2).

If the second iteration of Monte Carlo simulation is executed afterwards, the situation in *Figure 8* naturally changes, since the other set of 7 road data points is randomly chosen. Consequently, the other pupils addresses are assigned to the nearest chosen bus stops candidates and other pupils addresses are unassigned. The criterion functions (1) and (2) are changed to the different values, too.

Similar situation occurs for all other iterations. If, namely, the next $i = 3,4,\ldots,N$ iterations of Monte Carlo procedure are repeated afterwards, obviously the situation in *Figure 8* persistently changes, since the set of 7 randomly chosen road data points is different at every repetition. Consequently, the assigned and unassigned pupils data points are different at every repetition, and the criterion functions (1) and (2) are persistently changed.

Thus, when all N iterations of Monte Carlo procedure are finished, the following set of values of criterion functions (1) and (2) will be formed for the j-th subsector:

$$\begin{aligned} &J_{crit1}(j,1), J_{crit1}(j,2), \ldots, J_{crit1}(j,N),\ j = 1,2,\ldots,8 \\ &J_{crit2}(j,1), J_{crit2}(j,2), \ldots, J_{crit2}(j,N),\ j = 1,2,\ldots,8 \end{aligned} \tag{3}$$

For example, if the 1st subsector is taken into consideration and the prescribed radius is 1.3km, expressions (3) could take the form as shown in *Table 1*.

Table 1 - An example of both criterion functions values, generated by N iterations of Monte Carlo procedure, if the 1st subsector is observed and the prescribed radius is 1.3 km

iteration	sum of shortest distances	number of unassigned pupils
1	20.5 km	3
2	19.1 km	5
3	17.3 km	8
...		
N	22.7 km	0

Obviously, after running certain, sufficiently big number of N iterations during the simulation procedure, the whole set of values of both criterion functions is calculated for each subsector. Now, the two-criteria optimization procedure must be somehow applied to expressions (3), where the combined optimum is trying to be found for each subsector. The latter means searching for that "best" set of randomly chosen road data points for each subsector where the belonging criterion function (1) reaches the minimal value, while simultaneously keeping the criterion function (2) at the lowest possible level.

In case of *Table 1*, the sum of the shortest distances and the number of unassigned pupils in principle reach the minimum at different iterations. The optimization algorithm is then designed in a way that higher weight is assigned to the achievement of keeping the number of unassigned pupils at the lowest possible level. Consequently, the algorithm treats the solution with the sum of shortest distances 22.7km and 0 unassigned pupils as better than the solution with the sum of shortest distances 19.1km and 5 unassigned pupils.

The combination of calculated results for all subsectors, which are the "best" sets of randomly chosen road data points for each subsector, represents the positions of optimal bus stops. In this concept the following assumption can be given. If the optimal bus stops are calculated by means of the procedure described above, the bus stops cover as much pupils as possible within the prescribed radius, while they have to walk as little as possible. Also, the remaining number of unassigned children is supposed to be as minimal as possible.

At the end of this section two more facts should be stressed. Firstly, the algorithm is designed in order to randomly choose equal number of bus stops candidates in all subsectors while proceeding with the iterations of Monte Carlo procedure. It means that for the case represented in Figures 7 and 8 (8 subsectors, 7 randomly chosen road data points), obviously the total number of 56 optimal bus stops is going to be calculated during the optimization procedure.

Secondly, the algorithm performs Monte Carlo simulations for all subsectors simultaneously, which means savings of computational time during the simulation procedure.

5. PRACTICAL NUMERICAL RESULTS

The development of algorithm for optimal bus stops determination and all the computations of the optimization procedure were carried out in MATLAB. *Table 2* shows all relevant parameters which were used during the computation of results. It turns out that by use of these parameters most efficient results can be achieved.

The prescribed radius, which was used as an additional constraint during the computations, is 1.3km.

Table 2 - Relevant parameters which were used during the computation of results

prescribed radius r	1.3 km
number of subsectors	8
number of randomly chosen road points in each subsector	7
number of simulation iterations	1,000

The latter is not chosen randomly, but it is demanded as a maximum possible radius which is still acceptable for the Laško municipality. Thus, children should not walk more than 1.3km from their homes to the bus stops and vice versa.

Since the chosen number of subsectors is 8 and the number of randomly chosen bus stop candidates in each subsector is 7, the algorithm obviously deals with the determination of 56 optimal bus stops.

It turns out that the total number of 1,000 iterations of Monte Carlo procedure is big enough to satisfy the assumption that the results for calculated bus stops can be treated as close enough to the actually optimal results. It should be also stressed that all the needed computations, which have been executed, are calculated in a relatively fast way during the performance of the optimization procedure.

When the optimization procedure and Monte Carlo simulations are finished, the (X, Y) coordinates of 56 optimal bus stops are calculated. As it turns out, the latter are capable of covering 550 pupils within the prescribed radius of 1.3km. Therefore, the total number of 12 pupils remain unassigned to any bus stop. Since the total number of pupils is 562, the algorithm obviously managed to achieve service for 97.8% of all pupils by the determination of calculated optimal bus stop points. Hopefully, the unassigned 12 pupils should walk only several hundred metres more than the prescribed radius.

More useful information about the positions of calculated bus stop points can be illustrated on the observed surface of 1,768 bus stop candidates, determined by the initial road data reduction procedure (Figures 5 and 7). If the calculated 56 optimal bus stops (OBS) are extra marked in comparison with the other 1,768 – 56 = 1,712 bus stop candidates (BSC), the results can be represented as shown in *Figure 9*.

The situation shown in *Figure 9* can be also illustrated from a different point of view, if the non-optimal road points are removed and pupils data points are added to the Figure. Then the *Figure 10* can be used for the demonstration of results, where the distribution of optimal bus stops (OBS) can be observed with respect to pupils addresses (PA) distribution.

If the optimal bus stops in *Figure 10* are surrounded by circles $P_i, i = 1, \ldots, 56$ of prescribed radius r, the results can also be represented as shown in *Figure 11*. Twelve unassigned pupils data points with corresponding pupils addresses (PA) can be located by careful observation of *Figure 11*. Since the latter are not assigned to any optimal bus stop, they lie outside any circle P_i of the prescribed radius. The other 550 pupils data points are covered with the circles

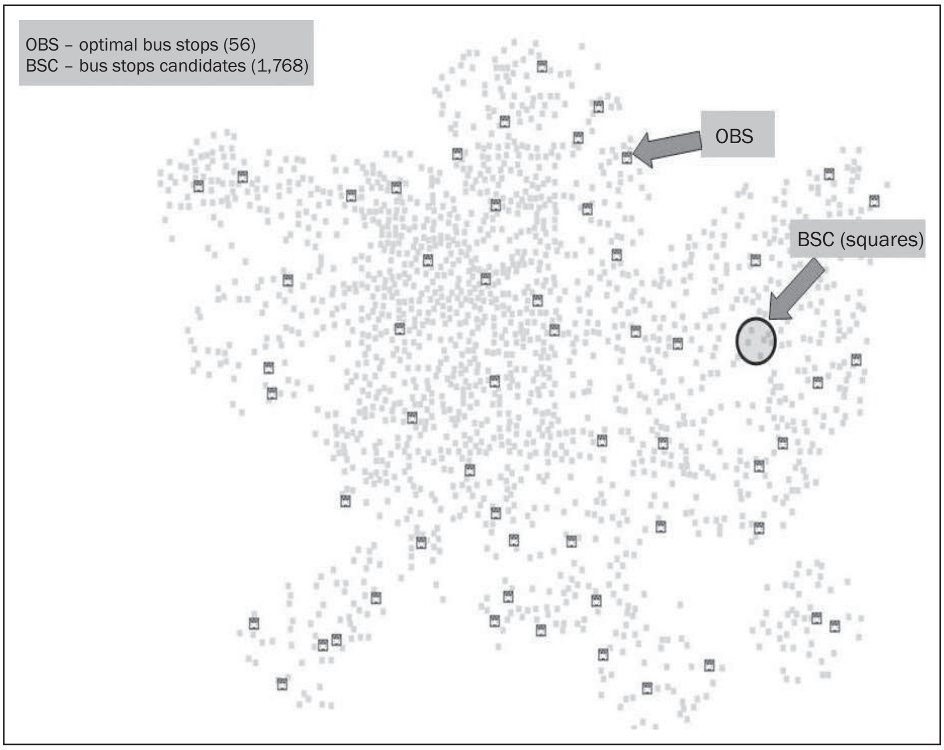

Figure 9 – 1,768 road data points (squares) extracted in the initial road data reduction; Extra marked positions of 56 optimal bus stops

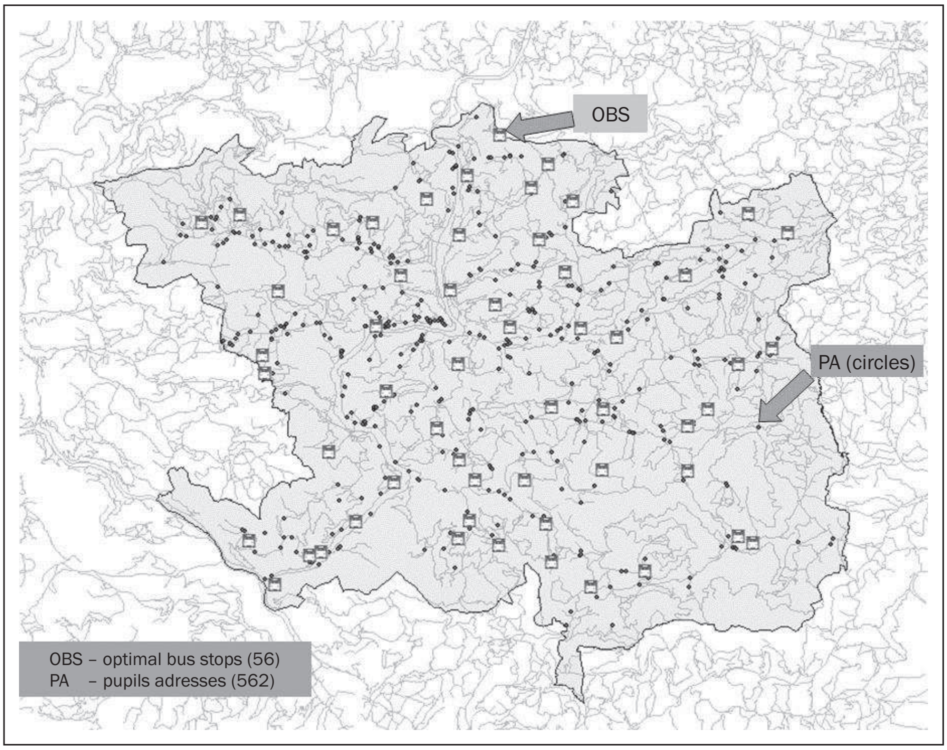

Figure 10 - Distribution of 56 optimal bus stops and distribution of 562 pupils addresses

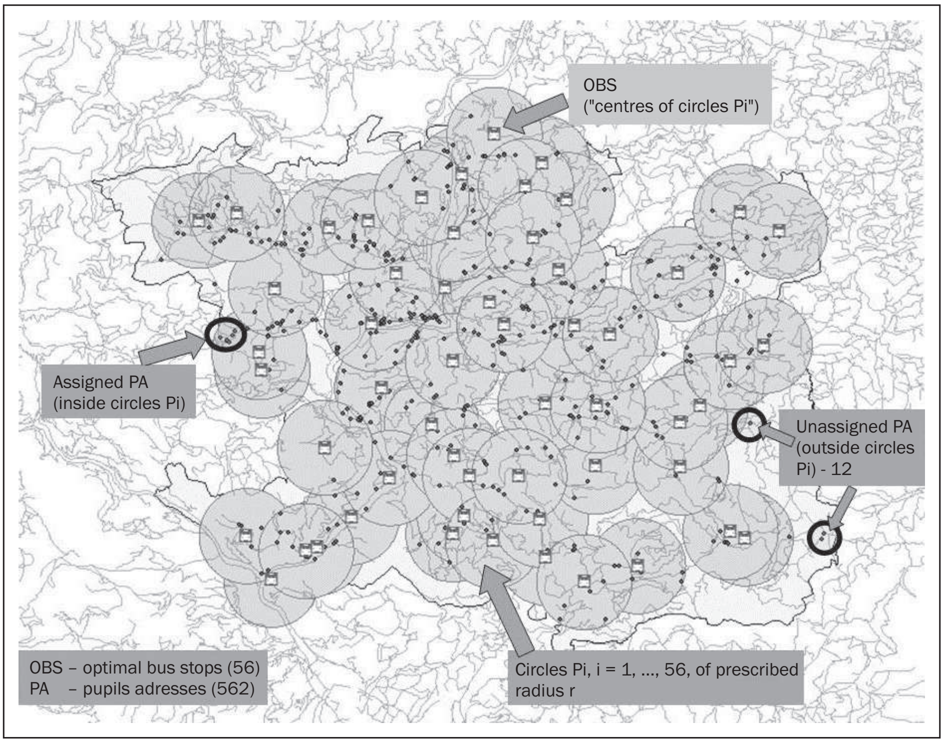

Figure 11 - Positions of 56 optimal bus stop points surrounded by circles Pi of the prescribed radius; 550 assigned and 12 unassigned pupils data points (little circles)

P_i of calculated 56 bus stops and therefore lie inside these circles.

Finally, the calculated results for optimal bus stops positions can be represented as shown in *Figure 12*. As it can be seen from *Figure 12*, only space distribution of 56 optimal bus stop positions is illustrated.

The coordinates of these locations have been submitted to the Laško municipality responsible personnel in order to trigger all necessary procedures for the purpose of bus stops physical implementation.

When this implementation is finished, the drivers of transportation vehicles will have to pick up the

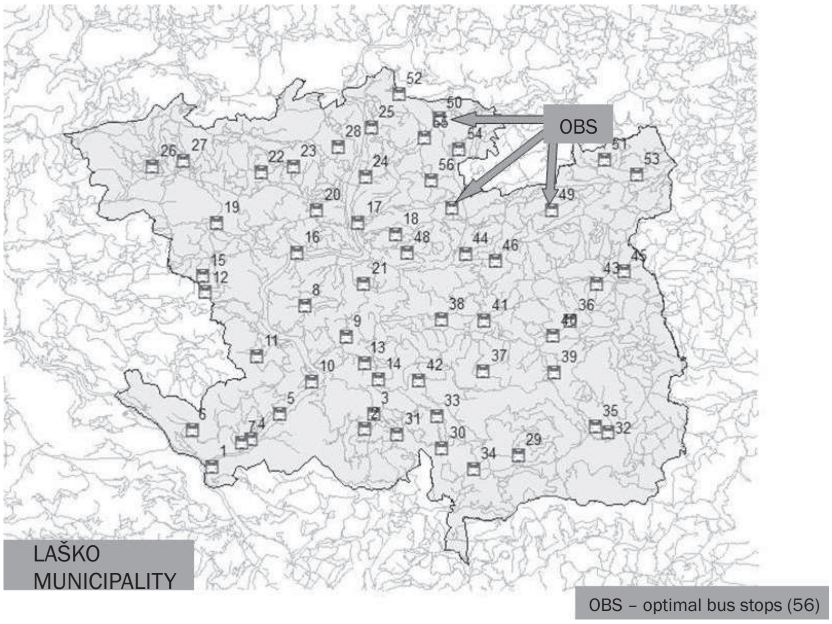

Figure 12 - Illustration of space distribution of 56 calculated optimal bus stops positions

treated pupils only at the bus stops, instead of picking them up at their homes individually. Since the number of all pupils is 562, while the number of planned bus stops is only 56, obviously the savings of transportation costs from this point of view are going to be significantly appreciable.

Of course, the current space distribution of the proposed optimal bus stops is going to change in the future as consequence of progressive changing of pupils addresses, since some of the children are progressively going to finish the school, while the others are progressively going to become pupils of the school. Thus, the design of optimal bus stop locations will have to follow the new trends of children's addresses distribution in the future.

6. COMPARISON WITH GREEDY ADD AND GREEDY DROP PROCEDURES

In order to verify the quality of the results achieved by the proposed optimization procedure, it is strongly recommended to make a comparison with some other already existing algorithms. For this purpose, basic Greedy Add and Greedy Drop procedures, briefly introduced in Section 2, were chosen.

As it turns out, the initial road data reduction described in Section 3 for these two procedures has to be applied as well; otherwise; great difficulties could occur in the optimal bus stops calculations.

As it can be seen in the previous section, the results for 56 optimal bus stops and 550 covered pupils have been presented in the case of Monte Carlo procedure. Naturally, in different combinations of the chosen number of subsectors and the number of randomly chosen bus stop candidates inside each subsector, the different number of optimal bus stops can be achieved. Thus, for example, the results ranging between 12 and 70 optimal bus stops can be calculated, where at each different number of optimal bus stops, the different number of pupils can be covered. The different numbers of optimal bus stops can be calculated not only when the optimization based on Monte Carlo method is used, but also when the other two procedures, Greedy Add and Greedy Drop, are used. In our opinion, namely, the comparison between the treated procedures is more authentic when it is done for different numbers of optimal bus stops and not only for one number of optimal bus stops.

Figure 13 shows the number of covered pupils, depending on the number of optimal bus stops, if the optimization based on Monte Carlo simulation is used (course a), or the Greedy Add procedure is used (course b), or the Greedy Drop procedure is used (course c), for bus stops allocation.

Naturally, the number of covered pupils is growing, when the number of optimal bus stops is increased. From *Figure 13* it is evident that the best results are achieved by the use of Greedy Add procedure, since it uses significantly lower number of optimal bus stops to cover as many pupils as possible, than the other two procedures. Slightly worse results are achieved by the use of procedure based on Monte Carlo simulation, which needs for the same coverage of pupils significantly more bus stops, while the Greedy Drop procedure gives the worst results. Thus, for example, the latter needs even 82 bus stops for covering 550 pupils (see point C in *Figure 13*), while the procedure based on Monte Carlo needs for the

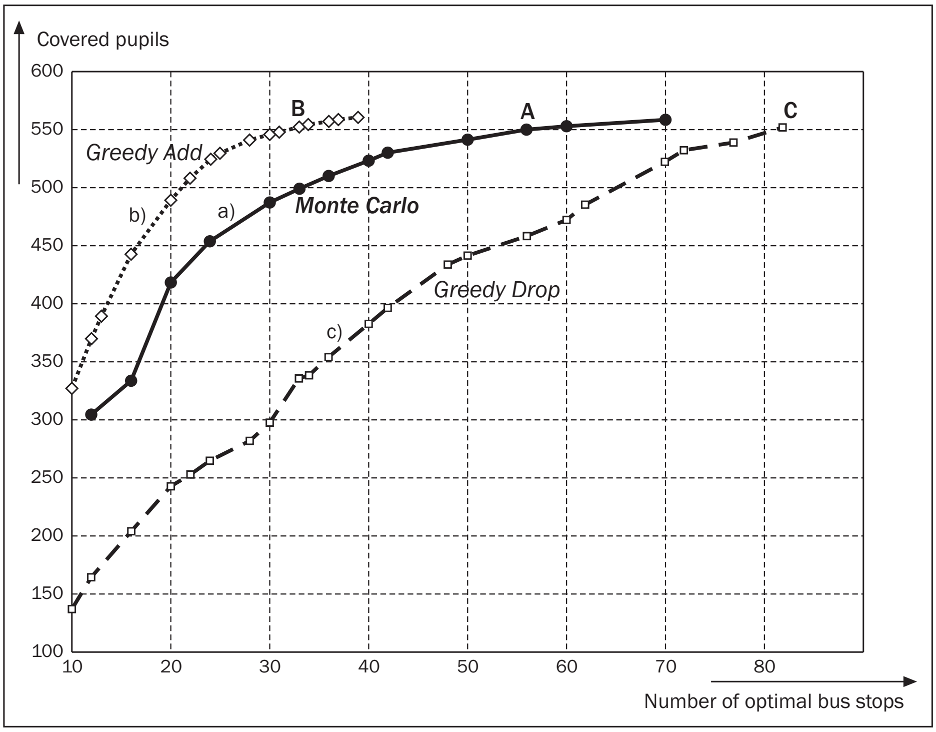

Figure 13 - The number of covered pupils, depending on the number of optimal bus stops. The latter are calculated by: a) Optimization based on Monte Carlo simulation; b) Greedy Add procedure; c) Greedy Drop procedure

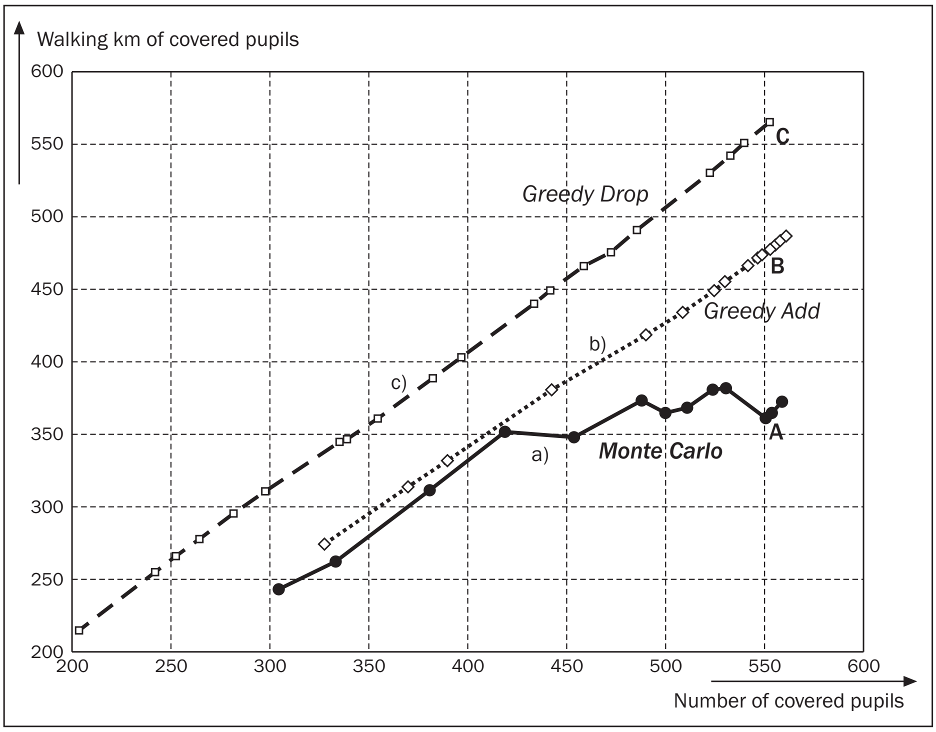

Figure 14 - The total number of walking kilometers of covered pupils, depending on the number of covered pupils. a) For the case of Monte Carlo simulation; b) For the case of Greedy Add procedure; c) For the case of Greedy Drop procedure

same coverage only 56 bus stops (point A in *Figure 13*), and the Greedy Add procedure needs only 33 bus stops for the same coverage (point B in *Figure 13*).

Figure 13 convinces us that the results, achieved by the use of procedure based on Monte Carlo simulation, are comparable with the results of other, existing procedures, such as for example Greedy procedures, and have similar quality. Even more, the results in *Figure 14* show that the total number of walking kilometres of all covered pupils (which are covered due to used optimal bus stops, see *Figure 13*) is in the case of Monte Carlo procedure (course a) significantly lower than in the case of Greedy Add procedure (course b), or in the case of Greedy Drop procedure (course c). Consequently, from this point of view, the procedure based on Monte Carlo even gives the best results in comparison with the other two procedures.

For example, the 550 covered pupils have to walk a total distance of 361.75km, when the procedure with Monte Carlo simulation is used (point A in *Figure 14*), while in the case of Greedy Add procedure they have to walk a total distance of 475.12km (point B in *Figure 14*), and in the case of the Greedy Drop procedure they have to walk a total distance of 564.72km (point C in *Figure 14*).

The reason for such a lower total walking distance of the pupils, when the Monte Carlo procedure is used, lies in the following fact: when a certain pupil is covered with the circles of prescribed radius of the several bus stops simultaneously, then he is assigned to that bus stop, which is closest to him. *Figure 15* shows an example with two optimal bus stops A and B and the location of one pupil U. Obviously, pupil U is covered by a prescribed radius r of both bus stops, but the distance $d(A,U)=d_1$ is lower than the distance $d(B,U)=d_2$, so pupil U is assigned to the closer bus stop, which is bus stop A.

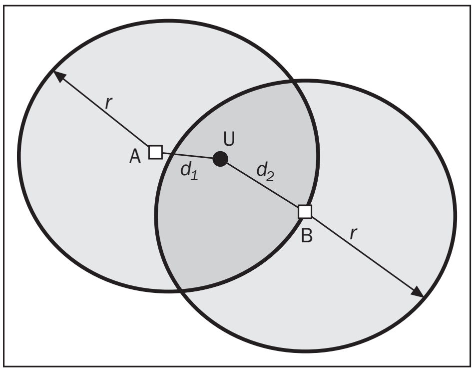

Figure 15 - An example of coverage of location of one pupil U with two optimal bus stops A and B

Since the applied Greedy Add and Greedy Drop procedures do not have this mechanism of additional distance calculations checking built in the frame of assignment of pupils to optimal bus stops, it is obvious why the total walking distance is much longer than in the case of procedure based on Monte Carlo simulation.

As the Laško municipality responsible personnel said, the most convenient for them are results for the coverage of 550 pupils, where the total walking distance of all the covered pupils is as low as possible, even at the expense of increasing the number of optimal bus stops. We therefore conclude that the solution achieved by means of procedure based on Monte Carlo simulation, with 56 optimal bus stops and the total walking distance of 361.75km is the most appropriate for the customer.

7. CONCLUSIONS

A simulation-based approach to the optimal bus stops allocation is presented and applied in order to achieve the reduction of costs generated by obligatory transportation of children from their homes to the corresponding schools in Laško municipality.

The original set of road data points (bus stops candidates) is first initially reduced by means of some heuristic rules. Then the optimization procedure based on Monte Carlo simulation is applied for the purpose of further reduction of road points in order to determine optimal bus stops.

If the optimal bus stops are calculated by means of proposed procedure, it is supposed that the minimal number of calculated bus stops cover as much pupils as possible within the prescribed radius, while they have to walk as little as possible from their homes to the nearest bus stop and vice versa.

The paper is believed to contribute in two ways. Firstly, the working mechanism of a relatively simple and efficient algorithm, which is slightly different from the existing “maximal covering location problem” algorithms, is introduced. Secondly, it is shown how this algorithm can be efficiently used to solve a real location problem.

In order to verify the quality of the results achieved by the proposed algorithm, a comparison with two Greedy algorithms, basic Greedy Add and Greedy Drop has been done. The comparison shows that the proposed algorithm, based on Monte Carlo simulation, gives comparable results with similar quality as calculated in case of both applied Greedy algorithms.

As evident from the results achieved by the proposed algorithm the total number of calculated optimal bus stops, which are able to cover the majority of 550 pupils within the prescribed radius, is 56. Only 12 pupils remain unassigned, but they should be walking only several hundred metres more than the prescribed radius.

When the calculated optimal bus stops are physically implemented, the transportation vehicles will have to drive along significantly shorter quantity of distances, since they will have to pass only the implemented 56 bus stops instead of all 562 pupils homes. Obviously, the savings of transportation costs from this point of view are going to be significantly noticeable.

In further research, the optimization of driving routes, driving schedules and driving fleet, related to the transportation of pupils from optimal bus stops to their schools and vice versa, is going to be developed and applied in order to achieve further reduction of transportation costs.

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POVZETEK

MONTE CARLO SIMULACIJSKI PRISTOP PRI DOLOČANJU LOKACIJ OPTIMALNIH AVTOBUSNIH POSTAJ V OBČINI LAŠKO

V prispevku je predstavljen problem določanja lokacij optimalnih avtobusnih postaj za potrebe zmanjševanja stroškov, nastalih zaradi prevoza otrok v občini Laško. Te je potrebno na osnovi zakonskih predpisov vsak dan razvoziti na pripadajoče šole ter jim zagotoviti vrnitev domov. Algoritem temelji na optimizaciji s pomočjo Monte Carlo simulacijske metode, pri čemer se izračunajo lokacije optimalnih avtobusnih postaj. Algoritem je sposoben izračunati najmanjše možno število avtobusnih postaj, ki pa bodo vseeno zagotavljale največje možno pokritje učencev v okviru predpisanega največjega radija pešačenja. Pri tem bo učencem potrebno prehoditi kar najkrajšo možno pot od doma do najbližje postaje in obratno. V prispevku so opisane glavne značilnosti delovanja mehanizma predlaganega algoritma. Prav tako je podan tudi prikaz vseh pomembnih izračunanih rezultatov in primerjava z nekaterimi drugimi obstoječimi algoritmi. Pozicije izračunanih avtobusnih postaj se bodo uporabile pri njihovi kasnejši fizični implementaciji z namenom kar največjega možnega zmanjšanja transportnih stroškov.

KLJUČNE BESEDE

lokacijski problemi, problemi maksimalnega pokritja, optimizacija, Monte Carlo simulacija, Geografski informacijski sistem (GIS), redukcija transportnih stroškov

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