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DYNAMIC USER OPTIMAL SIGNAL DESIGN AT ISOLATED INTERSECTIONS

ABSTRACT

This paper presents a proposal of novel signal design problem at isolated intersections, which assumes that the effective green times assigned to each signal phase follow dynamic user optimal (DUO) principle. At the DUO state, the average delays of vehicles using the signal phases with positive additional green times (the assigned effective green times minus the minimum effective green times) are equal and maximum. The proposed signal design problem is formulated as a variational inequality (VI) problem. The point queue (PQ) model is applied to represent traffic dynamics and to generate the cumulative traffic flows, which is further used to estimate the average delay of each signal phase. The existence of the solution of the proposed VI problem is proved and a solution algorithm based on the method of successive averages (MSA) is developed to solve the proposed signal design problem. Finally, a sample intersection is used to illustrate the application of the proposed model and the solution algorithm.

KEY WORDS

signal design, dynamic user optimal, point queue model, variational inequality problem

1. INTRODUCTION

Signal control is a traditional method to improve traffic efficiency at intersection areas, and the related signal design problems have been investigated for several decades. According to the traffic flow state, two categories of signal design problems are addressed so far: static-flow-based problems [1-3] and dynamic-flow-based problems [4-6]. In general, the static signal design methods only consider traffic flows at a steady state and generate a fixed signal timing plan which can be easily implemented in real application. However, the fixed timing plan becomes inefficient when traffic flows are dynamic and fluctuating from time to time, especially during the rush hours. On the contrary, the dynamic signal design methods take traffic dynamics into consideration, and generate dynamic timing plans which can govern the traffic flows more efficiently. On the other hand, since the dynamic traffic flows are the input of the dynamic signal design methods, traffic flow forecasting is the basic component of those methods, which increases the difficulty of its implementation in practice.

Traditionally, the signal design problems aim to improve the traffic efficiency from a system point of view, such as minimizing the total delay, queue length or the total system travel time. If only the system performance is considered in signal design problems, the travel delay of some direction can be very large, which may lead the drivers to perceive this as unfair and to act with the lack of patience. As a result, they are more likely to undertake irregular driving or disobey traffic rules, which can increase the probability of conflict or accidents and reduce traffic safety. Based on this consideration, this paper proposes a novel signal design solution, which aims to embody fairness among all the travellers and to avoid dissatisfaction in travellers' minds, which can potentially improve the traffic safety. The proposed signal design problem assumes that the effective green times assigned to each signal phase follow dynamic user optimal (DUO) principle. At the DUO state, the average delays of vehicles using the signal phases with positive additional green times (the assigned effective green times minus the minimum effective green times) are equal and maximum. The DUO-based signal design problem is formulated as a variational inequality (VI) problem. The underlying reason for formulating the signal design problem based on DUO principle as a VI problem is that the VI formulation is simple in form and easy to use with many existing solution algorithms developed for VI problems to solve the proposed signal design problem. The point queue (PQ) model is used to simulate the traffic flow propagation and estimate the travel delays. The solution existence of the proposed VI problem is also discussed, and a solution algorithm is further developed to solve the proposed model.

The rest of the paper is organized as follows: In the next section, a DUO-based signal design problem is formulated as a VI problem, and the PQ model is used to simulate the traffic flow and estimate the travel delays. An algorithm based on the method of successive averages (MSA) is developed to solve the proposed VI problem in Section 3. Numerical examples are developed in Section 4 to illustrate the performance of the proposed model and solution algorithm. Finally, concluding remarks and future research directions are drawn in Section 5.

2. MODEL FORMULATION

2.1 Dynamic user optimal signal design problem

2.1.1 Constraints of the dynamic signal design problem

An isolated signal-controlled intersection is considered. E is the set of signal phases. $\lambda_e(m)$ is the effective green time assigned to signal phase $e \in E$ during signal period m. The sum of effective green times assigned to all signal phases equals the length of a signal cycle minus total signal loss time. Equivalently, there is

$$\sum_e \lambda_e(m) = C - C_l, \forall m, \tag{1}$$

where C is the length of a signal cycle and C_l is the total signal loss time due to both red phases and phase changing, which is assumed to be fixed in this paper.

The duration of effective green time is subject to a minimum value for safety reasons so as to avoid sudden stop-and-go movements and to provide sufficient time for vehicle drivers to prepare for the changes of signal states [6, 7], and can be calculated by $\lambda_{e,\min} = [4 + 2 * Integer(^{d_e}/_{6.1})]$, where d_e is the maximum distance between detector and stop-line of all flow directions belonging to signal phase e [7]. Therefore, we have the following constraint:

$$\lambda_e(m) \geq \lambda_{e,\min}, \forall e, m, \tag{2}$$

The minimum effective green time is required for all signal phases and independent of the traffic flow state. A signal design problem is to determine the additional green time of each signal phase, i.e. the difference between the assigned effective green time and the minimum effective green time. Let $\tilde{\lambda}_e(m)$be the additional green time of phase e. By definition, we have $\tilde{\lambda}_e(m) = \lambda_e(m) - \lambda_{e,\min}$ and $\lambda_e(m) = \tilde{\lambda}_e(m) + \lambda_{e,\min}$. Substituting the latter into Eqs. (1) and (2), we have

$$\sum_e \tilde{\lambda}_e(m) = C - C_l - \sum_e \lambda_{e,\min}, \forall m, \text{ and} \tag{3}$$

$$\tilde{\lambda}_e(m) \geq 0, \forall e, m. \tag{4}$$

Constraints (3) and (4) form the set of feasible timing plans:

$$\Omega = \left\{ \tilde{\lambda} \geq 0 \left| \sum_e \tilde{\lambda}_e(m) = C - C_l - \sum_e \lambda_{e,\min}, \forall m \right. \right\}, \tag{5}$$

where $\tilde{\lambda} = [\tilde{\lambda}_e(m)]$ is the vector of the additional green times.

2.1.2 Dynamic user optimal condition

The DUO principle has been widely used in dynamic traffic assignment (DTA) problems [8-11]. The DUO condition in DTA states that, for each origin-destination pair, any routes used by travellers departing at the same time must have equal and minimal travel time [8]. This principle can be straightforwardly extended to the signal design problem, and a DUO-based signal control can be stated as follows:

Definition 1: If, for each phase, during any signal period, the average delays of vehicles using all phases with positive additional green time are the same and equal the maximal delay, the dynamic signal control is in the travel-delay-based DUO signal state.

Then the DUO-based signal control problem at an isolated intersection can be mathematically formulated as a nonlinear complementarity problem (NCP), given by:

$$[\sigma(m)-\sigma_e(m)]\tilde{\lambda}_e(m)=0, \forall e, m, \quad (6)$$

$$\sigma(m)-\sigma_e(m)\geq 0, \forall e, m, \quad (7)$$

$$\tilde{\lambda}_e(m)\geq 0, \forall e, m, \quad (8)$$

where $\sigma_e(m)$ is the average delay of vehicles that enter the directions governed by signal phase e during signal period m, and $\sigma(m)$ is the maximum average delay of all phases during signal period m, i.e., $\sigma(m)=\max_e\{\sigma_e(m)\}$.

The basic concept of the DUO-based signal design is to assign the additional green times to the congested signal phases. Under the DUO conditions, Eq. (6) implies that if the additional green time assigned to phase e is positive, i.e., $\tilde{\lambda}_e(m)>0$ and $\lambda_e(m)>\lambda_{e,\min}$, the average delay with respect to this phase equals the maximum average delay; otherwise, the average delay with respect to the phases without additional green time is no more than the maximum average delay. Therefore, travellers using the phases with positive additional green time endure the same average delay, and they will have fewer complaints on the unfairness of the signal control.

2.1.3 Variational inequality formulation

The DUO conditions (6)-(8) have an equivalent VI formulation, which can be stated as follows:

Theorem 1: The dynamic timing plan $\tilde{\lambda}^*\in\Omega$ is in a DUO state, if and only if it satisfies the following VI problem:

$$\sum_m\sum_e[\sigma^*(m)-\sigma_e^*(m)][\tilde{\lambda}_e(m)-\tilde{\lambda}_e^*(m)]\geq 0, \forall\tilde{\lambda}\in\Omega. \quad (9)$$

Proof: Firstly, it is proven that the DUO conditions (6)-(8) imply the VI problem (9). Let $\tilde{\lambda}^*\in\Omega$ be an optimal solution of the DUO conditions (6)-(8). According to Eqs. (6)-(8) we have $[\sigma^*(t)-\sigma_e^*(m)]\tilde{\lambda}_e(m)=0$ and $\sigma^*(m)-\sigma_e^*(m)\geq 0$. According to Eq. (4), the additional green times are non-negative, i.e., $\tilde{\lambda}_e(m)\geq 0$, hence we have $[\sigma^*(m)-\sigma_e^*(m)]\tilde{\lambda}_e(m)\geq 0$. Therefore, we have

$$\sum_m\sum_e[\sigma^*(m)-\sigma_e^*(m)]\tilde{\lambda}_e^*(m)=0, \text{ and} \quad (10)$$

$$\sum_m\sum_e[\sigma^*(m)-\sigma_e^*(m)]\tilde{\lambda}_e^*(m)\geq 0. \quad (11)$$

The left hand side of Eq. (11) minus the left hand side of Eq. (10), we can easily obtain the VI problem (9).

Moreover, we prove that any optimal solution of the VI problem (9) satisfies the DUO conditions (6)-(8). Let $\tilde{\lambda}^*\in\Omega$ be an optimal solution of the VI problem (9). Since $\tilde{\lambda}_e(m)\geq 0$ and $\sigma^*(m)-\sigma_e^*(m)\geq 0$ are satisfied by definition, we only need to prove that $[\sigma^*(m)-\sigma_e^*(m)]\tilde{\lambda}_e^*(m)=0, \forall e, m$. Equivalently, we only need to prove that Eq. (10) is satisfied.

Assume that Eq. (10) is not satisfied, since $\tilde{\lambda}_e(m)\geq 0$ and $\sigma^*(m)-\sigma_e^*(m)\geq 0$, we have

$$\sum_m\sum_e[\sigma^*(m)-\sigma_e^*(m)]\tilde{\lambda}_e^*(m)>0. \quad (12)$$

We need to find a timing plan $\tilde{\lambda}^*\in\Omega$ so that the following equation

$$[\sigma^*(t)-\sigma_e^*(m)]\tilde{\lambda}_e^*(m)=0, \forall m,$$

always holds. For each signal period m, we can always find a phase $e\in E$ with maximum average delay. We can assign all the additional green time to this phase with maximum average delay, and assign zero additional green time to the other phases. This will generate a feasible timing plan $\tilde{\lambda}=[\tilde{\lambda}_e(m)]$, which satisfies the following equation

$$\sum_m\sum_e[\sigma^*(m)-\sigma_e^*(m)]\tilde{\lambda}_e^*(m)=0. \quad (13)$$

The left hand side of Eq. (13) minus the left hand side of Eq. (12) yields

$$\sum_m\sum_e[\sigma^*(m)-\sigma_e^*(m)][\tilde{\lambda}_e(m)-\tilde{\lambda}_e^*(m)]<0.$$

The above equation contradicts the VI problem (9). Therefore, any optimal solution to the VI problem (9) must satisfy Eq. (10), which further implies that the equality condition of the DUO conditions (6)-(8) must hold since the additional green times and the differences between the maximum average delay and the average delay must be non-negative by definition. This completes the proof.

2.2 Traffic performance model

In dynamic signal control problems, the time-dependent timing plans are determined by the real time traffic flow (i.e. traffic demand of each direction), and travel delay is one of the most important factors which should be taken into account. However, traffic demand does not directly provide travel delays. To do this, the traffic performance model is usually adopted to describe the traffic flow propagation at each direction and output the cumulative flow curves, which can be further used to estimate travel delay of each direction. A great many of traffic performance models can be used to simulate the propagation of traffic flows travelling through an intersection along each flow direction and generate the cumulative flows [12]. The traffic performance model used in this paper is the PQ model [9, 13]. The underlying reasons for choosing the PQ model are [13]: (1) the PQ model is easy to calibrate since its parameters, including free-flow travel time and bottleneck capacity, are all well defined physical quantities that are relatively easy to measure; and (2) the PQ model takes advantage of the computational efficiency, and behaves identically as the cell transmission model (CTM) [14, 15] if queue spillback does not occur. In this paper, only the isolated intersections are considered, and thus queue spillback will not occur.

The PQ model treats vehicles as points without physical length, and assumes the storage capacity of a link is infinite and there are no link inflow constraints. In the PQ model, vehicles travel through a link with free-flow speed and form queues at the link exit if the outflow capacity of the link is not sufficient. Travel delay of a vehicle is defined as the queue time of the vehicle at link exit, which does not include the free-flow travel time of the link. Without loss of generality, this paper assumes that the free flow travel time of each link takes a value of zero, and only the travel delay is considered.

It is assumed that the traffic flows belonging to different directions are separated from each other, and are governed only by the corresponding signal phases. Under this assumption, each flow direction can be viewed as a link. We discretise the time period T of interest into a finite set of time intervals $K=\{k=1,2,\dots,\underline{K}\}$. Let δ be the interval length such that $\delta\underline{K}=T$ and C/δ is an integer. Let $U_a(k)$ be the cumulative number of vehicles using direction a by the end of time interval k, $V_a(k)$ be the cumulative number of vehicles leaving direction a by the end of time interval k, and $v_a(k)$ be the number of vehicles leaving direction a during interval k. The outflow is constrained by both the traffic demand and the outflow capacity of the direction, and can be formulated as follows:

$$v_a(k)=\min\{U_a(k+1)-V_a(k),Q_a(k)\}, \qquad (14)$$

where $Q_a(k)$ is the outflow capacity of direction a during interval k.

According to the definition of cumulative outflow, we have:

$$V_a(k+1)=V_a(k)+v_a(k). \qquad (15)$$

Substituting Eq. (14) into Eq. (15), we have

$$V_a(k+1)=\min\{U_a(k+1),Q_a(k)+V_a(k)\}. \qquad (16)$$

The outflow capacity of each direction during each interval is governed by the signal timing plan. Let $t_e^1(m)$ and $t_e^2(m)$ be the start time and the end time of green for signal phase e during the m^{th} signal cycle, respectively. The difference of the two time instants is the length of green time of the signal phase during that signal cycle, and thus we have $t_e^2(m)=t_e^1(m)+\lambda_e(m)$. The outflow capacity of a direction during a particular interval is equal to the product of the saturation flow rate and the length of time covered by the green time of the corresponding signal phase during that interval. Therefore, the outflow capacity of direction a can be calculated as follows:

$$Q_a(k)=\begin{cases}[t_e^2(m)-(k-1)\delta]q_a, & \text{if } t_e^1(m)\le(k-1)\delta\le t_e^2(m)\le k\delta,\\ \delta q_a, & \text{if } t_e^1(m)<(k-1)\delta<k\delta<t_e^2(m),\\ [t_e^2(m)-t_e^1(m)]q_a, & \text{if } (k-1)\delta\le t_e^1(m)<t_e^2(m)\le k\delta,\\ [k\delta-t_e^1(m)]q_a, & \text{if } (k-1)\delta\le t_e^1(m)<k\delta\le t_e^2(m),\\ 0, & \text{otherwise,}\end{cases} \qquad (17)$$

where q_a is the saturation flow rate.

Proposition 1: The cumulative outflows are continuous with respect to the additional green times.

Proof: Since a small change in the timing plan will incur a small change in the start time and the end time of each signal phase. Eq. (17) implies that the outflow capacity of direction a is continuous with respect to the start time and the end time of each signal phase. Hence, the outflow capacity of direction a is continuous with respect to the green times. Eq. (16) implies that the cumulative outflows of direction a are continuous with respect to its outflow capacity. Hence, the cumulative outflows of direction a are continuous with respect to the green times. Since $\tilde{\lambda}_e(m)=\lambda_e(m)-\lambda_{e,\min}$, the green times are continuous with respect to the additional green times. Therefore, the cumulative outflows are continuous with respect to the additional green times.

2.3 Approach to estimate average travel delay

Using the PQ model, the cumulative outflows of all directions can be obtained. The cumulative number of vehicles that enter (leave) direction a at time instant t is indicated as $U_a(t)(V_a(t))$. As shown in *Figure 1*, the average delay $d_a(t)$ of vehicles entering flow direction a during time period $(t,t+\Delta t]$ can be formulated as follows:

$$d_a(t)=\frac{\int_{U_a(t)}^{U_a(t+\Delta t)}[V_a^{-1}(v)-U_a^{-1}(v)]dv}{U_a(t+\Delta t)-U_a(t)}, \qquad (18)$$

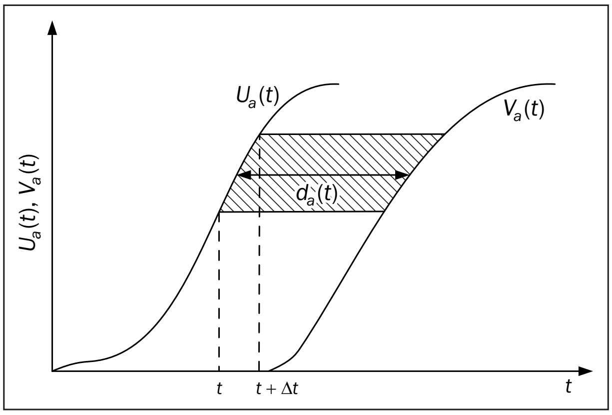

Figure 1 - Cumulative vehicle numbers as function of time

where $U_a^{-1}(\cdot)(V_a^{-1}(\cdot))$ is the inverse function of $U_a(\cdot)(V_a(\cdot))$. The numerator on the right hand side of Eq. (18) is equal to the shaded area in *Figure 1*, and the denominator is the number of vehicles that enter direction a during this time period. Eq. (18) directly formulates the actual average delay according to the cumulative entering and leaving flows, and hence all stated time losses are implicitly included in the calculation.

In general, the inverse functions $U_a^{-1}(\cdot)$ and $V_a^{-1}(\cdot)$ cannot be analytically obtained, and piecewise linear functions, such as the step function (SF) [12] and linear interpolation (LI) [16] are applied to approximate the profiles of cumulative flows, which are further used to estimate travel delays.

The following definitions will be used to estimate average travel delays of vehicles entering a particular direction during each time interval [17]:

Definition 2 (*Critical outflow interval*): A critical outflow interval with respect to interval k is defined as follows

$$n_k = \min\{l \mid U_a(k) \le V_a(l), l > k, l \in \{1,2,\dots\}\}. \quad (19)$$

As shown in *Figure 2*, let $y_a(k)$ be the number of vehicles entering direction a during interval k, $y_{ak}(l)$ be the number of vehicles that enter direction a during interval k and exit the direction during interval l, and $Y_{ak}(l)$ be the cumulative number of vehicles that enter direction a during interval k and exit the direction by the end of interval l. With those definitions, we have

$$Y_{ak}(l) = \begin{cases} 0, & \text{if } l < n_{k-1}, \\ V_a(l) - U_a(k-1), & \text{if } n_{k-1} \le l \le n_k, \\ U_a(k) - U_a(k-1), & \text{otherwise,} \end{cases} \quad (20)$$

$$y_{ak}(l) = Y_{ak}(l) - Y_{ak}(l-1), \text{ and} \quad (21)$$

$$y_a(k) = \sum_l y_{ak}(l) = U_a(k) - U_a(k-1) \quad (22)$$

If the cumulative flow curves are approximated by step functions, using Eqs. (20)-(22), we can mathematically formulate the average delay of vehicles entering direction a during interval k as follows [17]:

$$d_a(k) = \sum_{l=1}^{K} y_{ak}(l)(l-k)\delta / y_a(k) = = (n_k - k)\delta - \sum_{l=n_{k-1}}^{n_k - 1} \delta[V_a(l) - U_a(k-1)]/y_a(k) \quad (23)$$

Then the average delay of signal phase e during signal period m is obtained as follows:

$$\sigma_e(m) = \frac{\sum_{a \in A_e} \sum_{k \in K_m} d_a(k) y_a(k)}{\sum_{a \in A_e} \sum_{k \in K_m} y_a(k)}. \quad (24)$$

where K_m is the set of time intervals belonging to signal period m and

$A_e = \{a \mid \text{direction } a \text{ is controlled by signal phase } e\}$.

Proposition 2: The average delay calculated by Eq. (24) is continuous with respect to the additional green times.

Proof: Substituting Eq. (22) into Eq.(24), we have

$$\sigma_e(m) = \frac{\sum_{a \in A_e} \sum_{k \in K_m} \sum_l y_{ak}(l)(l-k)\delta}{\sum_{a \in A_e} \sum_{k \in K_m} \sum_l y_{ak}(l)}. \quad (25)$$

Eq. (25) implies that the average delay $\sigma_e(m)$ is continuous with respect to $y_{ak}(l)$. According to Proposition 1, the cumulative outflows are continuous with respect to the additional green times. Similar with the proof of Proposition 7 in Long et al. [17], we can prove that the traffic flow $y_{ak}(l)$ is continuous with respect to the additional green times. Therefore, the average delay $d_e(m)$ is also continuous with respect to the additional green times.

According to Eq. (5), the set of feasible timing plans Ω is a compact convex set, and Proposition 2 proves that the average delays are continuous on Ω. Therefore, the existence of the solutions of the VI problem (9) can be guaranteed.

2.4 Dynamic system optimal signal design problem

A great many of signal design problems consider the total system travel delay as the objective [5, 6, 18, 19], which refer to dynamic system optimal (DSO) signal design problems. For comparison, we also formulate the DSO signal design problem, given as follows:

$$\min_{\lambda \in \Omega} \sum_m \sum_e \sum_{a \in A_e} \sum_{k \in K_m} \sigma_e(m) y_a(k), \quad (26)$$

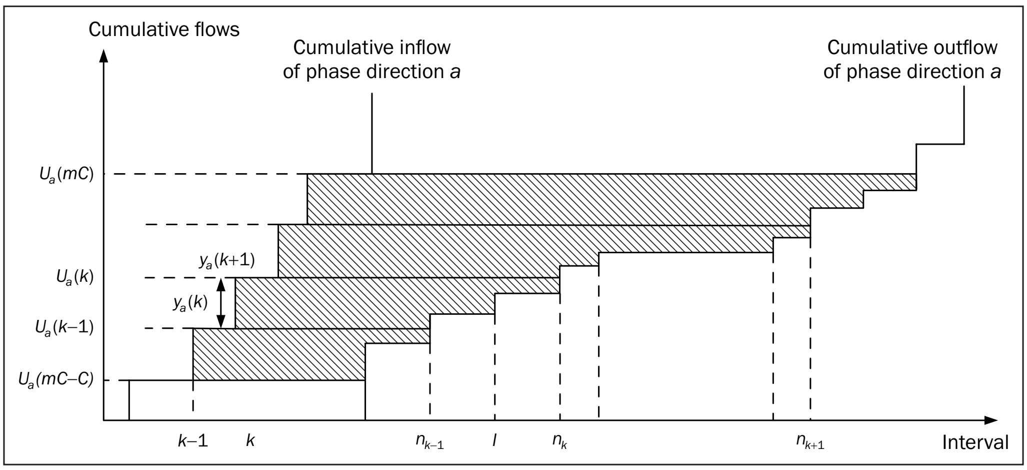

Figure 2 - Cumulative flow functions approximated by step functions

where $\tilde{\lambda} = [\tilde{\lambda}_e(m)]$ is the vector of decision variables of the optimization model (26). $y_a(k)$ is traffic demand of direction a during interval k, and is independent with respect to $\tilde{\lambda}$. $\sigma_e(m)$ is the average travel delay of vehicles that enter the directions governed by signal phase e during signal period m, and is an implicit function of $\tilde{\lambda}$. Therefore, the objective (26) is also an implicit function of $\tilde{\lambda}$.

Nie and Zhang [13] stated that if there is no spillback of flow from downstream links, the CTM with triangular flow-density diagram (*Figure 2* in Lo [19]) and the PQ model produce the same exit flow. Hence, if we restrict Lo's model [19] to an isolated intersection, and set the capacity of the cell before the intersection to be infinite, it is just the same as the DSO signal design problem of (26).

3. SOLUTION OF ALGORITHM

There are many iterative methods to solve the VI problem (9), such as the projection method [10], the diagonalization method [20], the alternating direction method [21], the MSA [22], the day-to-day swapping method [9, 23], and so on, provided that the convergent requirements are satisfied. In this paper, the MSA is adopted to solve the VI problem (9) due to its simplicity and convergence property. The solution algorithm is outlined as follows:

Step 1: *Initialization*. Select a feasible initial solution $\tilde{\lambda}^1 = [\tilde{\lambda}_e^1(m)]$, where $1 \le m \le \underline{K}$, and $\underline{K}$ is the number of signal cycles during the study period. Set index $n = 1$ and convergence tolerance $\varepsilon > 0$.

Step 2: *Updating average delay*. Implement the PQ model, obtain the cumulative flow curves, and then calculate the average delay of every phase $\sigma_e^n(m)$ by Eq. (25).

Step 3: *Implementation of an "all-or-nothing" assignment*. Assign all the additional green times to phase $e = \arg\max\{\sigma_b^n(m)\}$. Equivalently, set $\gamma_e^n(m) = C - C_l - 4\lambda_{e,\min}$ and $\gamma_b^n(m) = 0, \forall b \in E - \{e\}$.

Step 4: *Updating timing plan*. Update the additional green times as follows using the MSA:

$$\tilde{\lambda}_e^{n+1}(m) = \tilde{\lambda}_e^n(m) + (\gamma_e^{n+1}(m) - \tilde{\lambda}_e^n(m))/(n+1), \forall e, m. \tag{27}$$

Step 5: *Checking convergence*. The iteration terminates if

$$\max\{\delta_e^n(m)[\sigma^n(m) - \sigma_e^n(m)]\} < \varepsilon, \tag{28}$$

where $\delta_e^n(m) = 1$ if $\tilde{\lambda}_e^n(m) > 0$, otherwise, $\delta_e^n(m) = 0$. Otherwise, set $n = n + 1$, and return to Step 2.

Since the objective function of the DSO signal design problem (26) is an implicit function with respect to its decision variables, it is very difficult to obtain the gradient of the objective function. Hence, the descent methods are not convenient to solve the optimization problem (26). In this paper the genetic algorithm is applied to solve the DSO signal design problem (26).

4. NUMERICAL RESULTS

The isolated intersection in *Figure 3* is used to illustrate the performance of the proposed framework. The intersection is controlled by a general four-phase signal: east-and-west-through, east-and-west-left-turn, south-and-north-through, and south-and-north-left-turn. The right-turn flow and the through flow are governed by the same signal phase due to pedestrian flow. The signal phase sequence is given in *Figure 4*. We assume that the signal loss due to signal phase changing is omitted. The length of each signal cycle is 100s. There are 3 lanes for through directions and 2 lanes for left-turn directions in the intersection, and the saturation flow rates are 0.375 vel/s/lane and 0.5 vel/s/lane for turn directions and through directions, respectively. The setting implies that there is a reduction rate of 0.75 between left-turn and through direction.

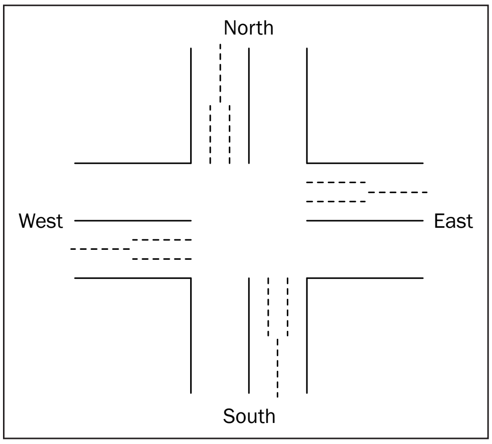

Figure 3 - Sketch map of an intersection

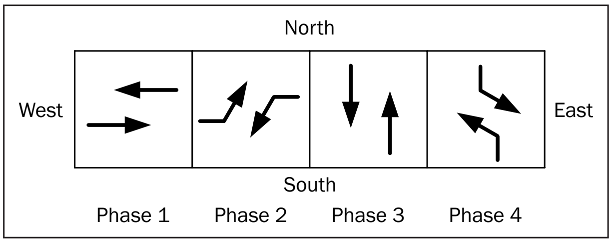

Figure 4 - Signal phase sequence of the used intersection

The basic demands of all directions during the whole peak hour (one hour) are shown in *Figure 5*. The OD demand matrix was uniformly varied by multiplying by a constant demand factor (denoted by ϑ), where the factor represents the demand level relative to the base demand matrix presented in *Figure 5*. One can observe from the figure that the traffic demands on

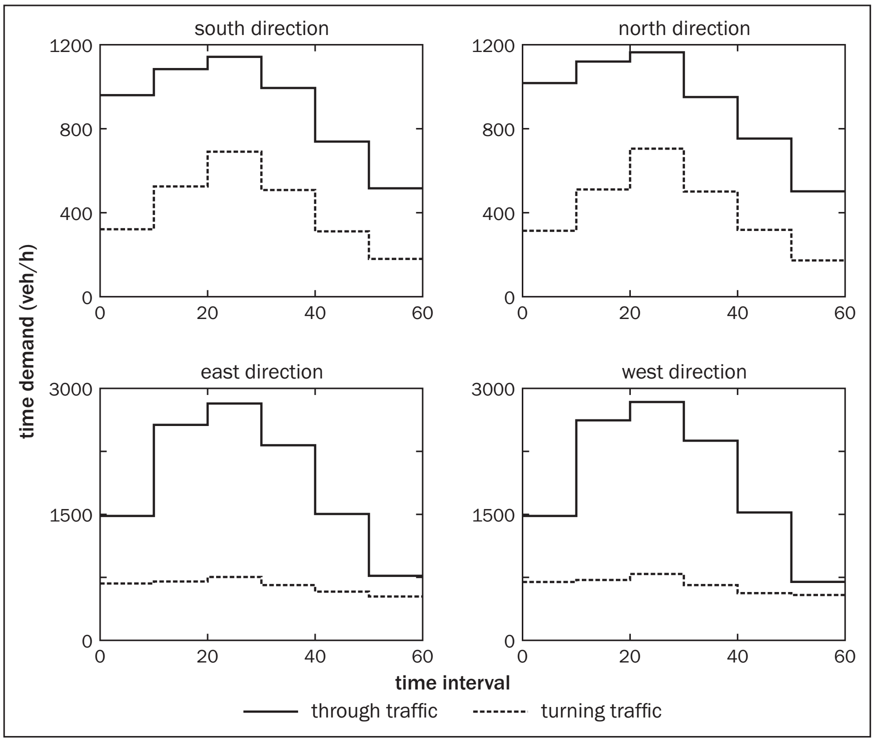

Figure 5 - Demands of all directions during the whole peak hour

east-and-west directions are much larger than those on south-and-north directions, especially during the mid-period. In this example, we will design two signal patterns: fixed timing plan and dynamic timing plan, and make a comprehensive comparison on their different performances. Furthermore, we will also compare the performance of the DUO model with that of the DSO model.

Firstly, the convergence of the iterative algorithm was examined. *Figure 6* gives the convergence of the solution algorithm when $\vartheta = 1$, where the convergence index is calculated by the left hand side of Eq. (28). The maximum iterative number is 10,000. We found that the value of the convergence index decreases rapidly from 2,665.8s at the first iteration. The values of convergence index after the 60th iteration are graphically displayed in *Figure 6*. One can observe that the proposed solution algorithm indeed guarantees convergence.

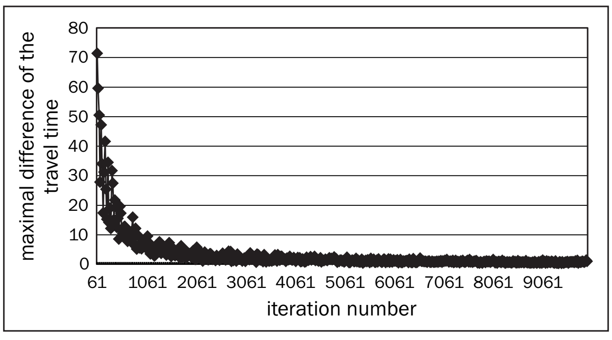

Figure 6 - Convergence of algorithm when $\vartheta = 1$

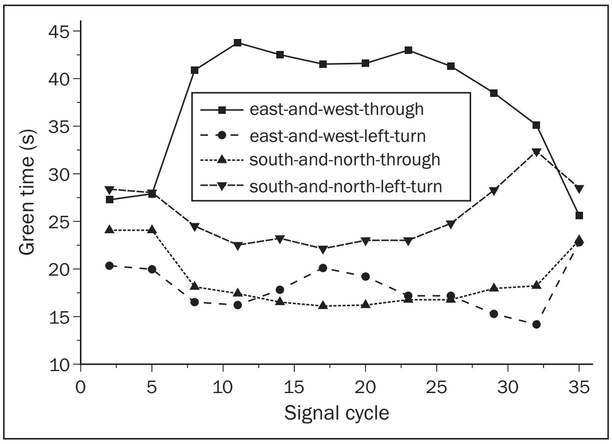

Figure 7 - Dynamic signal results when $\vartheta = 1$

The DUO timing plan is shown in *Figure 7*. We can observe that the effective green times assigned to all four phases are very close for the first two signal cycles. When the demand differences between these directions increase, the differences of the green times between the four signal phases increase accordingly. Since the demands of east and west directions are much larger than of other directions, the effective green time assigned to east-and-west-through is larger than the other directions for most of the signal cycles. The south-and-north-left-turn has the second largest demand, and the corresponding effective green time is also the second largest.

We also examined two fixed timing plans for different OD demand levels: one uses the Webster's signal

Table 1 - Comparisons on performances of different timing plans

ϑ	Travel delays (veh·h)			Improvements (veh·h)	
	Webster's Fixed timing plan	Optimal Fixed timing plan	DUO timing plan	Optimal Fixed timing plan	DUO timing plan
0.88	138.46	101.05	97.91	37.41	40.55
0.91	177.26	138.35	131.26	38.91	46.00
0.94	227.34	188.49	176.33	38.85	51.01
0.97	286.14	252.17	237.26	33.97	48.88
1.00	354.22	333.33	306.90	20.89	47.32
1.03	431.64	425.36	384.42	6.28	47.22

assignment model [20], and the other is the user optimal signal assignment which is similar to Definition 1 in this paper except that the timing plan is fixed for the whole study period. *Table 1* provides comparisons on performances of these two fixed timing plans and the DUO timing plan under various traffic demand levels. It can be seen from the table that the total travel delays of both optimal fixed timing plan and the DUO timing plan are lower than that of the Webster's fixed timing plan. The results presented in *Table 1* also show that the DUO timing plan outperforms the optimal fixed timing plan.

Table 2 provides the average travel delays of each signal phase during each signal cycle under the fixed timing plan and the DUO timing plan when $\vartheta = 1$. We can observe that the maximal gap between the average travel delays of each signal phase is very large when the fixed timing plan is adopted. On the contrary, the average travel delays of all signal phases during all the signal periods are very close when the DUO timing plan is adopted. Therefore, the DUO timing plan can embody more fairness for the travellers travelling through the intersection.

It is also interesting to compare the performances of the DUO timing plan with the DSO timing plan in terms of fairness. Each entrance of the studied intersection includes two traffic approaches: one for left turning, and the other for through and right turning. The smaller the difference of the average travel delays of the two approaches, the timing plan can be viewed as fairer. We solved the signal design problems (9) and (26), and presented the differences of the average travel delays of the two approaches belonging to the same east entrance in *Figure 8*(a). We can observe that the differences of the average travel delays of each direction with respect to the DUO timing plan are very small, while the differences with respect to the DSO timing plan are remarkable, especially when the intersection is congested. The differences of the average travel delays of two approaches belonging to the north entrance are presented in *Figure 8*(b). We can also observe that the average travel delay differences with respect to DSO timing plan are a lot greater than those with respect to the DUO timing plan. Therefore, the DUO timing plan is fairer than the DSO timing plan, especially when traffic demand level increases.

Table 2 - Average travel delays under the fixed and the DUO timing plans

Signal period	The fixed timing plan					The DUO timing plan				
	Average travel delays of the four phases (s)				Maximal gap (s)	Average travel delays of the four phases (s)				Maximal gap (s)
1	44.28	37.18	22.30	50.12	**27.82**	34.47	34.48	34.46	34.48	**0.02**
2	55.89	37.73	23.43	69.50	**46.07**	34.78	34.77	34.82	34.82	**0.05**
3	81.96	44.27	45.66	92.85	**48.58**	50.45	50.43	50.49	50.47	**0.06**
4	124.87	56.34	91.45	121.74	**68.53**	86.89	86.71	86.89	86.86	**0.18**
5	176.73	124.08	155.95	164.12	**52.65**	143.63	143.58	143.64	143.71	**0.13**
6	238.62	246.91	236.15	218.74	**28.17**	221.42	221.34	221.33	221.46	**0.13**
7	269.93	308.40	282.06	246.90	**61.50**	263.76	263.75	263.72	263.86	**0.14**
8	274.23	312.11	299.47	250.99	**61.12**	273.38	273.27	273.31	273.33	**0.11**
9	243.87	257.30	258.57	233.28	**25.29**	238.50	238.32	238.34	238.32	**0.18**
10	177.55	145.18	160.19	195.19	**50.01**	157.09	156.89	156.94	156.97	**0.20**
11	80.54	48.52	42.49	149.62	**107.13**	55.65	55.53	55.53	55.57	**0.12**
12	37.93	35.82	19.63	93.40	**73.77**	31.74	31.77	31.77	31.77	**0.03**

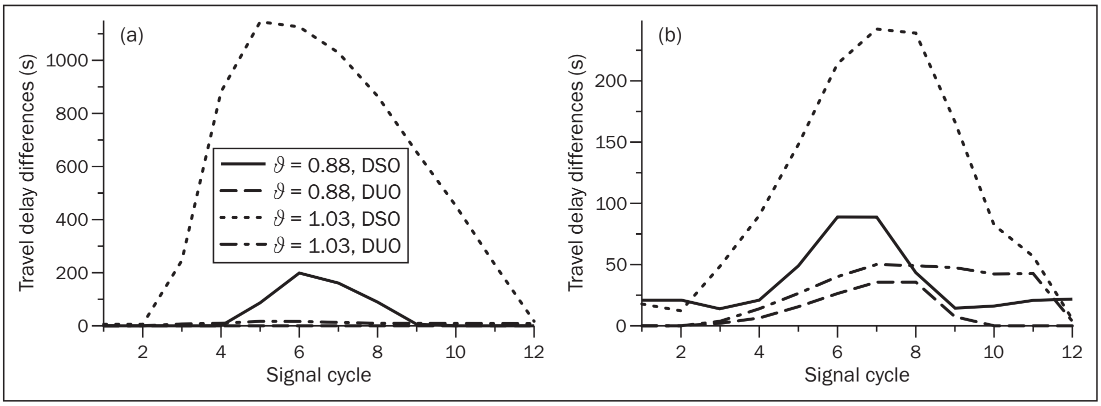

Figure 8 - Differences of average travel delays of directions: (a) east entrance; (b) north entrance

5. CONCLUSION

This paper has proposed a novel signal design problem at isolated intersections, which assumes that the effective green time assignment follows the DUO principle. The PQ model was used to simulate traffic flow propagation and obtain cumulative flows, which were further applied to estimate the travel delays. The proposed signal design problem was formulated as a VI problem, and the existence of the VI problem has been also proven. The proposed model formulation is general, and hence can be applied to all types of phase plans.

A sample intersection is applied to illustrate the application of the proposed model and the solution algorithm. The results show that (1) the proposed solution algorithm is convergent, and the obtained timing plan can well meet the traffic dynamics; (2) compared with the fixed timing plan, the dynamic signal timing plan can effectively improve the traffic performance at the intersection area, and the travellers endure the same travel delays on the average and feel fairer.

In this paper, we only considered the signal design problem at isolated intersections. Since a traffic network consists of a lot of intersections, we will extend the proposed signal design problem to general traffic networks, and evaluate its performance in the future.

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孤立交叉口的动态用户最优信号配时

本文提出一个全新的孤立交叉口的信号配时问题，其中各相位绿灯时间的分配遵循动态用户最优原则。在动态用户最优状态下，所有附加绿灯时间（分配绿灯时间减去最小绿灯时间）为正的相位，其平均车辆延误都相等并且最大。采用变分不等式问题来描述提出的信号配时问题。利用点排队模型来描述交通流的传播，得到累积流量，并进一步估计计每个相位的平均车辆延误。此外，还证明了提出的模型的解的存在性，并利用相继平均法来求解该模型。最后采用一个交叉口的例子来说明所给模型和算法的有效性。

关健词

信号设计，动态用户最优，点排队模型，变分不等式问题

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