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RISK ROUTE CHOICE ANALYSIS AND THE EQUILIBRIUM MODEL UNDER ANTICIPATED REGRET THEORY

ABSTRACT

The assumption about travellers' route choice behaviour has major influence on the traffic flow equilibrium analysis. Previous studies about the travellers' route choice were mainly based on the expected utility maximization theory. However, with the gradually increasing knowledge about the uncertainty of the transportation system, the researchers have realized that there is much constraint in expected utility maximization theory, because expected utility maximization requires travellers to be 'absolutely rational'; but in fact, travellers are not truly 'absolutely rational'. The anticipated regret theory proposes an alternative framework to the traditional risk-taking in route choice behaviour which might be more scientific and reasonable. We have applied the anticipated regret theory to the analysis of the risk route choosing process, and constructed an anticipated regret utility function. By a simple case which includes two parallel routes, the route choosing results influenced by the risk aversion degree, regret degree and the environment risk degree have been analyzed. Moreover, the user equilibrium model based on the anticipated regret theory has been established. The equivalence and the uniqueness of the model are proved; an efficacious algorithm is also proposed to solve the model. Both the model and the algorithm are demonstrated in a real network. By an experiment, the model results and the real data have been compared. It was found that the model results can be similar to the real data if a proper regret degree parameter is selected. This illustrates that the model can better explain the risk route choosing behaviour. Moreover, it was also found that the traveller' regret degree increases when the environment becomes more and more risky.

KEY WORDS

anticipated regret, expected utility, risk decision, route choice, traffic flow

1. INTRODUCTION

Travel behaviour is a core issue of the studies on transportation research. Travellers' decision behaviour affects directly the traffic flow distribution. The analysis of the travel behaviour is important for traffic planning, traffic policy formulation, road pricing, as well as the evaluation of the traffic management technology [1-3]. Travel behaviour can be described as a decision process. According to the certainty degree of decision objects, decision can be divided into assured decision and risk decision. Assured decision refers to the fact when the decision makers are sure about the event results, while risk decision denotes that the decision makers are uncertain about the event results, but they know the probability of the results [4-9]. So far, the researchers have proposed various risk decision theories from different aspects. Among these, the most classic one is the expected utility theory. This theory assumes that the decision makers follow the expected utility maximization principle. The utility maximization principle was first carried out by von Neumann and Morgenstern in 1944, and it was further improved by Savage and Anseombe et al. to 'subjective expected utility theory' [10-16]. At present, the expected utility theory is being widely applied to the decision of risk route choice [17-21]. Some studies have been further extended by applying the expected utility to the analysis process of the traffic equilibrium flow [22]. Due to relatively stronger interpretability, the expected utility theory has been the basis theory widely accepted in studying the route choice behaviour for a long time. However, with the constraint studies on risk theory, the researchers have begun to question the expected utility theory. The expected utility maximization requires that the travellers must be "absolutely rational", but in fact, the travellers are not truly 'absolutely rational'. Therefore, a 'bounded rationality'-based prospect theory is being gradually developed in the travellers' decision behaviour studies [23-28]. Based on the prospect theory, literature [29] analyzes the network equilibrium and further investigates the parameter sensitivity of a prospect theory-based route choice model. Besides, literature [1] constructs an elastic demand traffic distribution model based on the prospect theory and discusses the influences of parameters variation on the network equilibrium. Even though the prospect theory is more efficacious to explain the travellers' route choice behaviour under risk condition, there is much controversy in using this theory to analyse the traffic flow equilibrium problem. The main reason is that the prospect theory-based route choice behaviour cannot find sufficient evidences in empirical studies and is even inconsistent with empirical results. Due to the limitations of the prospect theory, the researchers have attempted to seek new decision theories that can explain more effectively the travellers' risk route choice behaviour. Therefore, the anticipated regret theory (ART) proposed in literature [30] attracts attention. ART is a risk decision theory. It suggests that people always try to avoid that the selected program is inferior to the unselected ones. In the extensive empirical studies in other fields, i.e. financial field and others, ART presents excellent interpretability [31-32]. Moreover, literature [33] finds the scientific basis for people's regret decision behaviour using nerve imaging techniques. All of the above indicates that perhaps it is feasible for us to study the travellers' route choice behaviour based on the anticipated regret theory.

In this study, at first, we use the regret degree function which has formulated by Chorus 2012 [23]. Then, it is further used in risk route choice process to analyze the travellers' route choice decision under different risk preferences and regret degrees. After discussing the transportation network equilibrium condition, an equilibrium traffic flow distribution model with minimum expectation regret is proposed. Besides, the basic characteristics of this model are analyzed and the solution algorithm of this model is proposed. By an actual experimental case, the effectiveness of this model is measured. The paper is arranged as follows: After the introduction, Section 2 introduces the basic theories of risk route choice. Based on ART, Section 3 analyzes the travellers' risk route choice decision process using a case. Section 4 gives the traffic flow equilibrium assignment model based on ART and designs the solution algorithm. Through an actual experimental case Section 5 verifies the effectiveness of the model. Some concluding remarks and discussion of future research are discussed in Section 6.

2. RISK ROUTE SELECTION THEORIES

2.1 EXPECTED UTILITY THEORY

The expected utility theory, as a famous theory of risk decision, was first proposed by Bernoulli in 1738. In 1944 von Neumann and Morgenstern presented the complete axiom system of this theory. In 1954 Savage improved this theory into subjective expected utility theory. At present, the expected utility theory is being developed into various forms and widely applied to risk decision analysis [6]. In the expected utility theory, the mean value and risk value of objectives need to be weighed to quantitatively describe the "preference" of the decision makers. One of the methods is by finding the utility function of the decision makers. Utility is an evaluation index which can reflect the preference degree of somebody to something. In decision issues, the utility value can express the preference degree of decision makers to a certain possible situation.

The von Neumann-Morgenstern expected utility theorem is one of the most fundamental results of individual decision-making theory. It shows that a preference relation defined on a decision space has an expectation utility representation, provided that it is a complete and transitive binary relation that satisfies the standard independence and continuity axioms.

Let R be the decision space, $R = \{R_1, R_2, \dots, R_i, \dots, R_n\}$ representing a set of all the possible decision results, $x = \{x_1, x_2, \dots, x_i, \dots, x_n\}$ is the utility value of every possible decision result in R. For each decision maker, the decision result is always uncertain, it is a probability distribution on R. The von Neumann-Morgenstern expected utility theorem require the following assumption axioms.

Completeness axiom: For comparison of any two decision results R_i and R_j, it should be one of the following three results: R_i is superior to R_j, R_j is superior to R_i or R_i and R_j - no difference.

Transitivity axiom: For three decision results R_i, R_j and R_k, if R_i is superior to R_j, R_j is superior to R_k, then there must be R_i superior to R_k.

Continuity axiom: For three decision results R_i, R_j and R_k, if R_i is superior to R_j, and R_j is superior to R_k, then there must be a probability p, such that the linear combination of R_i and R_k (based on probability p) yields no difference to R_j.

Independency axiom: For three decision results R_i, R_j and R_k, if R_i is superior to R_j and R_j is superior to R_k, then there must be a probability p, such that the linear combination of R_i and R_k (based on probability p) is superior to the linear combination of R_j and R_k.

Based on these assumption axioms, the decision-making under uncertainty environment, the expected utility theorem gives the following conclusions: under the risk environment, the final decision-making result utility can be obtained by adding the weighted utility of decision results. The purpose of the decision-maker is to maximize the expected utility, the weights show the probability of every possible decision results. The expected utility theorem describes how the 'rational human' makes a decision when they are facing an uncertain situation. It can be described by

$$U = u\left(\sum_{i=1}^{n} p_i x_i\right) = \sum_{i=1}^{n} p_i u(x_i),$$

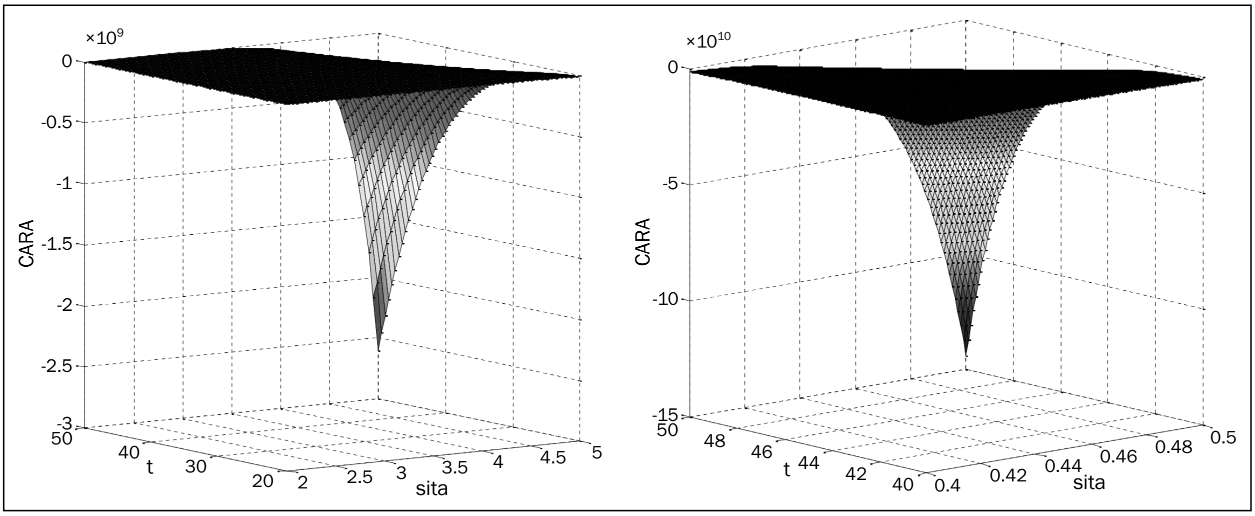


Figure 1 - CRRA and CARA

where U is the expected utility, u is the utility funcition, p_i is the probability of every decision result and it also represents the weights.

According to the expected utility theory axioms, Levy (1994) proposed two risk route utility functions based on empirical aspects, which are constant relative risk aversion (CRRA) utility function and constant absolute risk aversion (CARA) utility function. These two utility functions can be written as follows:

$$\left.\begin{array}{ll} U_{CRRA} = -t^{1+\theta}/(1+\theta) & (a) \\ U_{CARA} = [1-\exp(\theta t)]/\theta & (b) \end{array}\right\} \quad (1)$$

In (1), t is travel time and θ refers to risk aversion degree. Larger θ denotes that travellers tend more to risk aversion; $\theta \to 0$ suggests that travellers show more tendencies to risk-neutral attitude. *Figure 1* illustrates the variation law of the two utilities changing with travel time and risk aversion.

2.2 Anticipated Regret Theory (ART)

Anticipated regret is a negative emotion based on the cognition. It mainly occurs when individual realizes that better result will appear if other behaviours are taken previously [34]. Regret theory was first studied by the theoretical economists. Traditional rational decision theory (such as expected utility theory) considers that decision-making follows utility maximization principle. And people select the program with maximum utility value by calculating the expected utility

$$U = u\left(\sum_{i=1}^{n} p_i x_i\right).$$

Literature [30] considers that regret is a component of decision result. Especially, literature [23] give a explicit regret functional form. It is belileved that the regret function should be added to the expected utility theory to modify the original psychological utility curve, so that people's practical decision behaviour can be better explained. The definition of regret in literature [30] draws the focus of psychologist to regret theory. Regret, as a feeling that people are most eager to avoid in the decision-making process, has gradually become one of the most important fields of feeling in risk decision studies

According to ART, if the selected program shows fewer differences with the expected program, the regret utility will be lower; the regret utility value will be zero; if the selected program shows to be superior to the expected program, people will not regret, and they will feel lucky, the regret utility will be positive. Otherwise, if the expected program is superior to the selected program, people will feel regret and the regret utility value will be negative. Moreover, with the increase of utility difference, regret degree decreases. This rule can be expressed as:

$$\left.\begin{array}{ll} R(\Delta U) > 0 & \Delta U > 0 \\ R(\Delta U) > 0 & \Delta U < 0 \\ R(\Delta U) = 0 & \Delta U = 0 \\ dR(\Delta U)/d\Delta U > 0 & \end{array}\right\} \quad (2)$$

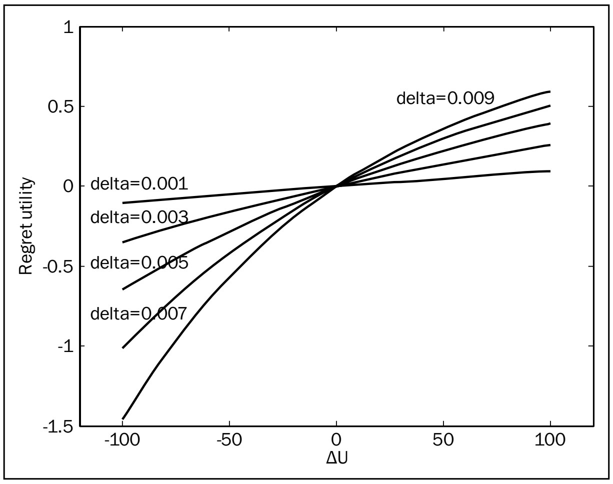


Figure 2 - Regret function under different regret degrees (cited from [23])

where $R(\Delta U)$ is regret degree function,

$$\Delta U = U_{choice} - U_{anticipate}.$$

In [23], the regret function is formulated in the following form:

$$R(\Delta U) = 1 - \exp(-\delta \Delta U) \tag{3}$$

where $\delta \in [0, +\infty]$, δ is regret degree parameter. The larger the δ, the more obvious is the regret degree tendency. $\delta = 0$ represents that there is no regret tendency. *Figure 2* shows the variation law of regret degree changing with regret degree and utility difference.

3. RISK ROUTE CHOICE ANALYSES

3.1 Anticipated regret utility

In the expected utility theory, the decision makers only consider themselves and make decisions by utility maximization. However, more recent studies suggest that the feeling of the decision makers in selection process fluctuates with the decision result; i.e., when travellers find that the selected route is the optimal one, the utility felt by them will increase (because the selected route is the best one); when they find that the selected route is not the optimal one, the utility felt by them will decrease (because the selected route is not the best one). Therefore, the real utility should include the regret utility on the basis of expected utility [21, 23]. This new utility function is:

$$U^*(t_{(i)s}) = U(t_{(i)s}) + R(U(t_{(i)s}) - U_0) \tag{4}$$

where U_0 represents psychological expectation; subscript s denotes the state, U is utility function and provided with two forms as indicated in section 2.1. R is regret degree function; its form is shown in section 2.2., $t_{(i)s}$ is a strictly increasing function about flow. Often, it can be expressed by BPR (Bureau of Public Road) function form.

$$t_{(i)s}(x) = t_{(i)s}^{free} \cdot \left(1 + \alpha\left(\frac{x}{c}\right)^{\beta}\right) \tag{5}$$

where $t_{(i)s}^{free}$ is road free flow travel time, α and β are the parameters that need to be calibrated, x is traffic flow volume, c is road traffic capacity. According to the expected utility theory, the expected utility in anticipated regret condition can be obtained as follows:

$$\begin{aligned} EU^*(t_{(i)s}) &= \sum_s [p_s \cdot U^*(t_{(i)s})] = \\ &= \sum_s \{p_s \cdot [U(t_{(i)s}) + R(U(t_{(i)s}) - U_0)]\} \end{aligned} \tag{6}$$

The expected utilities based on CRRA and CARA in regret condition are:

$$\begin{aligned} EU^*_{CRRA}(t_{(i)s}) = \sum_s p_s \Bigg\{ \left(\frac{-t_{(i)s}^{1+\theta}}{1+\theta}\right) + \\ + \left[1 - \exp\left(-\delta \cdot \left(\frac{-t_{(i)s}^{1+\theta}}{1+\theta} - U_0\right)\right)\right] \Bigg\} \end{aligned} \tag{7a}$$

$$\begin{aligned} EU^*_{CARA}(t_{(i)s}) = \sum_s p_s \Bigg\{ \left(\frac{1 - \exp(\theta t_{(i)s})}{\theta}\right) + \\ + \left[1 - \exp\left(-\delta \cdot \left(\frac{1 - \exp(\theta t_{(i)s})}{\theta} - U_0\right)\right)\right] \Bigg\} \end{aligned} \tag{7b}$$

where $\delta = 0$, formula (7) is degraded into common CRRA and CARA expected utility function.

3.2 Network equilibrium condition

From the perspective of behaviour science, utility is a direct decision basis for the individuals. In practice, travellers choose their travel route according to their cognitive utility. The network will gradually evolve to user equilibrium when all of the travellers have chosen their travel route based on the rule of maximizing their anticipated expected regret utility. Under the anticipated regret theory, the network user equilibrium condition can be written as:

$$\begin{cases} f_l^{w*} \geq 0 & EU_l^* = E\hat{U}_l \\ f_l^{w*} = 0 & EU_l^* < E\hat{U}_l \end{cases}, \ \forall l \in K, w \in W \tag{8}$$

where W is the set of all Origination to Destination (OD) pairs of the transportation network, and $w \in W$. $E\hat{U}_l$ is anticipated expected regret utility of the selected route in equilibrium; f_l^{w*} is the equilibrium flow of route l in OD pair w, K is the route set.

3.3 Basic assumptions

Assuming that there are two routes to be selected for N travellers from the Origin to the Destination (OD), one is safe route A and the other is risk route B. These two routes have assured travel time $t_{(A)}(v)$ (v is the traffic flow) and stochastic travel time $T_{(B)}$, respectively. The form of $T_{(B)}$ depends on the external condition of the road. It is assumed that external condition shows two states, which are 'good state' and 'hazardous state'. In 'good state' $T_{(B)}(v) = t_{(B)G}$ is a constant, while in 'hazardous state' $T_{(B)}(v) = t_{(B)H}(v)$. Moreover, it is assumed that probability when external condition in 'good state' is p, and the probability when external condition in 'bad state' is $1 - p$, that is:

$$\begin{cases} P(T_B = t_{(B)G}) = p \\ P(T_B = t_{(B)H}(v)) = 1 - p \end{cases} \tag{9}$$

Moreover, it is assumed that travellers do not know the state of road B, but they know the probabilities of road in 'good state' or 'hazardous state'. When road B is in 'good state', for the travel time of risk route B there is no congestion effect of route B, namely $t_{(B)G}$ is a constant. It is also assumed that the following inequalities (10a and 10b) exist.

$$t_{(B)H}(0) > t_{(B)G} \tag{10a}$$

$$t_{(A)}(0) > t_{(B)G} \tag{10b}$$

$$\left.\begin{aligned} t_{(A)}(0) < t_{(B)H}(N) \\ t_{(A)}(N) > t_{(B)H}(0) \end{aligned}\right\} \tag{10c}$$

Formula 10a suggests that, when road B is in 'good state', the free flow travel time of road B is larger than the travel time in 'good state'; that is, the travel time of risk route B is always the smallest when the state is good; 10b denotes that the free flow travel time of safe route A is larger than the risk route B when route B is in 'good state'; Both 10a and 10b indicate that the travel time of risk route B in 'good state' is always the smallest. Namely, B is always relatively optimal when the external state is good; 10c explains that there are no absolute advantages or disadvantages of the travel time on safe route A and risk route B when the external state is hazardous. In 'hazardous state', the travel time of the two routes is determined by the congestion effect functions and the flow distribution, respectively. *Figure 3* shows the assumptions above.

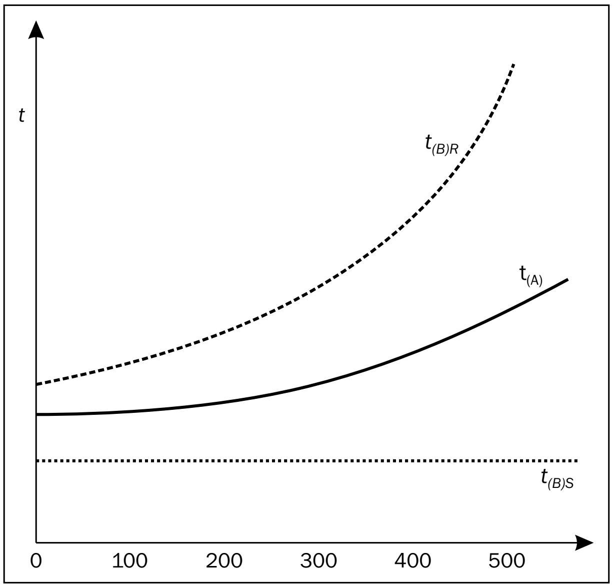


Figure 3 - Travel time in link A and B

3.4 Route selection analysis

Similar to the analysis of [23], we also analyze how the risk condition, travellers' risk aversion degree and travellers' regret degree impace the equilibrium of the network. Formula (7) gives the two modified utility function forms of CRRA and CARA. It can be seen from formula (7) that the route selection in risk condition is close to three parameters, which are probability p of road in risk condition, travellers' risk aversion degree θ and travellers' regret degree δ. This section mainly analyses how these three parameters influence travellers' route choice decision in the environment described in section 3.1. Let $N = 200$, $t_{anti} = 10$, $\alpha_{(A)} = \alpha_{(B)} = 2$, $c_{(A)} = c_{(B)} = 200$, $t_{(B)G} = 1$, $t_{(A)}^{free} = 10$, $t_{(B)H}^{free} = 25$. Let

$$p = 0, 0.25, 0.5, 0.75, 1;$$

$$\theta = 0.01, 0.02, 0.03, 0.04, 0.05;$$

$$\delta = 0, 0.001, 0.003, 0.005, 0.007.$$

Table 1 - Equilibrium flow of link B when p=0

p=0	θ=0	θ=0.01	θ=0.03	θ=0.05	θ=0.07	θ=0.09
δ=0	23	23	23	23	23	23
δ=0.01	25	25	25	26	26	26
δ=0.03	28	28	28	28	28	28
δ=0.05	30	30	30	31	31	31
δ=0.07	33	33	33	33	34	34
δ=0.09	37	37	37	37	37	37

Table 2 - Equilibrium flow of link B when p=0.25

p=0.25	θ=0	θ=0.01	θ=0.03	θ=0.05	θ=0.07	θ=0.09
δ=0	49	49	48	47	47	47
δ=0.01	52	52	51	51	50	49
δ=0.03	53	53	52	52	51	51
δ=0.05	55	55	55	54	54	53
δ=0.07	57	57	57	56	56	55
δ=0.09	59	59	59	58	57	57

Table 3 - Equilibrium flow of link B when p=0.5

p=0.5	θ=0	θ=0.01	θ=0.03	θ=0.05	θ=0.07	θ=0.09
δ=0	81	80	79	79	78	77
δ=0.01	83	82	81	80	79	79
δ=0.03	85	84	83	82	81	81
δ=0.05	86	85	85	84	83	82
δ=0.07	88	86	85	84	83	83
δ=0.09	89	88	87	86	85	84

Table 4 - Equilibrium flow of link B when p=0.75

p=0.75	θ=0	θ=0.01	θ=0.03	θ=0.05	θ=0.07	θ=0.09
δ=0	131	130	129	127	125	124
δ=0.01	132	131	129	128	126	124
δ=0.03	134	133	131	130	128	125
δ=0.05	136	134	133	131	129	127
δ=0.07	137	135	133	131	130	128
δ=0.09	139	137	135	133	132	130

Table 5 - Equilibrium flow of link B when p=0.895

p=0.895	θ=0	θ=0.01	θ=0.03	θ=0.05	θ=0.07	θ=0.09
δ=0	200	200	200	200	200	199
δ=0.01	200	200	200	200	200	200
δ=0.03	200	200	200	200	200	200
δ=0.05	200	200	200	200	200	200
δ=0.07	200	200	200	200	200	200
δ=0.09	200	200	200	200	200	200

The equilibrium flow of link A and B is shown in Tables 1-5.

The influences of the three parameters (risk environment probability p, traveller's regret degree δ, and risk aversion parameter θ) to the flow of risk route B can be observed in Tables 1-5. Among these parameters, risk environment parameter p shows the most significant influence on route B. With the increase of p, the flow of risk route B obviously grows. This is because with the increase of risk environment probability p, risk route B tends to be more in 'good state', thus the traveller shows preference to choose route B.

It can be observed from *Table 1*, *Table 2*, and *Table 5* that the changes of risk aversion parameter θ do not cause notable variations of the flow in risk route B. As shown in *Table 1* and *Table 5*, when $p=0$, risk route flow almost shows no changes with risk parameter increasing from 0 to 0.09 in different regret degree conditions. This shows that when risk environment tends to two extremes ($p=1$ is the extreme of "good state", while $p=0$ is the extreme of "bad state"), equilibrium flow will not generate significant variations with risk aversion parameter θ. That is to say, in deterministic state, the equilibrium flow in the transportation network will not experience obvious change with the traveller's risk aversion attitude. The reason lies in the fact that the travellers are fully aware of the results of their decision in non-risk environment, thus the expectation of obtaining better profits by risk aversion cannot be realized. But in risk environment (when p is away from the two extremes of 0 and 1), risk aversion parameter variation can cause significant changes of the flow on risk route B. In addition, the equilibrium results will gradually transfer to non-risk route A with the increase of risk aversion parameter, as shown in *Table 3* and *Table 4*. In *Table 3*, when $p=0.5$, the flow on risk route B changes by 4, 4, 4, 4, 5, 5 units in different regret degrees ($\delta=0,0.01,0.03,0.04,0.07,0.09$) separately with risk parameter θ increasing from 0 to 0.09. In *Table 4*, in the same conditions as above, the flow of risk route B changes by 7, 8, 9, 9, 9, 9 units, respectively. This is because, in risk condition, decision results are provided with certain risks, and it is possible for travellers to obtain better profits by "adventure".

Table 1-Table 4 also show that the equilibrium flow of risk route B grows with the increase of regret degree. This phenomenon suggests that regret degree can generate more notable influences to network equilibrium flow in arbitrary environment. The larger the regret degree, the more tendencies the travellers show to risk route choice. This behaviour is to reduce the psychological expectation brought by the feeling that the risk route is in 'good state' but is not selected. At the same time, it also indicates that there always exists anticipated regret behaviour in risk route selection.

4. EQUILIBRIUM MODEL BASED ON ART

4.1 Model development

According to the equilibrium condition of 3.2, we can get the equilibrium model based on ART, which can be written as:

$$\min Z(v)=\sum_{0}^{v_a}\int_{0}^{v_a}\left\{-\sum_{s}p_s\cdot\left[U(t_{(a)s}(v)+R(U(t_{(a)s}(v))-U_0))\right]\right\}dv \tag{11a}$$

$$St.\sum_{k}f_k^w=q_w \tag{11b}$$

$$f_k^w\geq 0\quad\forall k,w \tag{11c}$$

$$v_a=\sum_{w}\sum_{k}f_k^w\delta_{a,k}^w \tag{11d}$$

where w is OD pair, $t_{(i)s}(v)$ is the travel time function which increases by flow v, $\delta_{(a,k)}^w$ is '0' or '1' variable, $\delta_{(a,k)}^w=1$ means link a is in a part of link k of OD pair w, otherwise $\delta_{(a,k)}^w=0$, objective function $Z(v)$ is a non-linear function. Model (11) is minimum optimization problem with equality constraints and non-negative constraints. The Lagrangian function of (11) is

$$L(f,\mu)=Z(v)+\sum_{w}\mu_w\left(q_w-\sum_{k}f_k^w\right) \tag{12}$$

where μ_w is the Lagrangian parameter, the first-order conditions of (11) are equivalent to let $L(f,\mu)$ minimize in the constrains of $f_k^w\geq 0$. The variable of the Lagrangian function is the route flow f_k^w and Lagrangian parameter μ_w, is the first-order condition

$$f_k^w\frac{\partial L(f,\mu)}{\partial f_k^w}=0 \text{ and } \frac{\partial L(f,\mu)}{\partial f_k^w}\geq 0 \tag{13a}$$

$$\frac{\partial L(f,\mu)}{\partial \mu_w}=0 \tag{13b}$$

$$f_k^w\geq 0 \tag{13c}$$

According to (12), we can get

$$\frac{\partial L(f,\mu)}{\partial f_k^w}=\frac{\partial Z(v(f))}{\partial f_k^w}+\frac{\partial\sum_{w}\mu_w\left(q_w-\sum_{k}f_k^w\right)}{\partial f_k^w} \tag{14}$$

The first formula on the right of (14) is

$$\frac{\partial Z(v(f))}{\partial f_k^w}=\sum_{a\in A}\frac{\partial Z(v(f))}{\partial v_a}\cdot\frac{\partial v_a}{\partial f_k^w} \tag{15}$$

In (15)

$$\frac{\partial Z(v(f))}{\partial v_a}=\frac{\partial}{\partial v_a}\sum_{a}\int_{0}^{v_a}\left\{-\sum_{s}p_s\cdot\left[U(t_{(a)s}(v)+R(U(t_{(a)s}(v))-U_0))\right]\right\}dv \tag{16a}$$

$$\frac{\partial v_a}{\partial f_k^w}=\delta_a^{w,k} \tag{16b}$$

Substituting (16) to (15) yields

$$\frac{\partial Z(v(f))}{\partial f_k^w}=\sum_{a\in A}\left\{-\sum_{s}p_s\cdot\left[U(t_{(a)s}(v)+R(U(t_{(a)s}(v))-U_0))\right]\right\}\cdot\delta_a^{w,k}=-E(U_k^w)^* \tag{17}$$

The second formula on the right of (14) is

$$\frac{\partial \sum_w \mu_w \left(q_w - \sum_k f_k^w\right)}{\partial f_k^w} = -\mu_w \tag{18}$$

Since we know that q_w is a constant, μ_w is not a function of f_k^w. Substituting (17) and (18) to (14), yields

$$\frac{\partial L(f,\mu)}{\partial f_k^w} = -EU_k^w - \mu_k^w \tag{19}$$

So the first order condition of (14) can be written as

$$f_k^w\left(EU_k^w + \mu_k^w\right) = 0 \quad \forall k,w \quad \text{or} \tag{20a}$$

$$EU_k^w + \mu_k^w \leq 0 \quad \forall k,w \tag{20b}$$

$$\sum_w f_k^w = q_w \quad \forall k,w \tag{20c}$$

$$f_k^w \geq 0 \quad \forall k,w \tag{20d}$$

Let $-\mu_k^w = E\hat{U}_k$, we know that (20c) and (20d) are flow constraints and non-negative constraints of the route, they are satisfied in the minimum point of (11). We can always know from (20a) and (20b) that $EU_k^w < E\hat{U}_k$ when $f_k^w = 0$, and $EU_k^w < E\hat{U}_k$ when $f_k^w > 0$, then we know that (21) is an increasing function of v, so we can infer that

$$\partial\left(\frac{\partial Z}{\partial v_a}\right) \Big/ \partial v_b > 0 \text{ (when } a = b\text{)}.$$

This is equivalent to the equilibrium condition, and it also implies the equivalence of the model.

The optimization problem (11) is constructed by linear equality constraints (11b) and (11d), non-negative constraints (11c) and the objective function (11a). Obviously, the constraint sets are convex. If we know that the objective function (11a) is also convex, then we can say that the solution of optimization problem (11) exists and is unique. We can judge the convex of (11) by checking the positive of its Hessian matrix $\nabla^2 Z(v)$.

$$\frac{\partial Z}{\partial v_a} = -\sum_s p_s \cdot \left[U(t_{(a)s}(v_a)) + R(U(t_{(a)s}(v_a)) - U_0)\right) \tag{21}$$

Because the links are independent of each other, which means

$$\frac{\partial^2 Z}{\partial v_a \partial v_b} = \begin{cases} \dfrac{d\left(\dfrac{\partial Z}{\partial v_a}\right)}{dv_b} & a = b \\ 0 & a \neq b \end{cases} \tag{22}$$

the Hessian matrix of $Z(v)$ is

$$\nabla^2 Z(v) = \begin{bmatrix} \dfrac{\partial\left(\dfrac{\partial Z}{\partial v_a}\right)}{\partial v_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \dfrac{\partial\left(\dfrac{\partial Z}{\partial v_a}\right)}{\partial v_n} \end{bmatrix} \tag{23}$$

From (5) we know that $t_{(a)s}(v)$ is a strictly increasing function of *v*; from (1) we know that $U(t)$ is a strictly decreasing function of *t*, so $U(t_{(a)s}(v))$ is a strictly decreasing function of *v*. From (3) we know that $R(\Delta U)$ is a strictly increasing function of $\Delta U = U(t_{(a)s}(v)) - U_0$, and ΔU is a strictly decreasing function of *v*, so $R(U(t_{(a)s}(v)) - U_0)$ is a strictly decreasing function of *v*. Then we know (21) is an increasing function of *v*. So we can infer that

$$\frac{d\left(\dfrac{\partial Z}{\partial v_a}\right)}{dv_1} \geq 0.$$

From the above we can infer that matrix (23) is positive, the objective function (11a) is convex, and the solution of optimization problem (11) exists and is unique.

4.2 Solution algorithm

We can use the 'Frank-Wolfe algorithm' to solve the model of (11). 'Frank-Wolfe algorithm' was developed by M. Frank and P. Wolfe in 1956 [35]. For a long time this algorithm was proven to be an efficacious algorithm to solve some non-linear optimization problems. In particular, it has been successfully used to solve the traffic equilibrium assignment model [36]. The algorithm process can be written as follows:

Step 0: Initialization.

For every link a, compute the expected regret utility with zero flow.

$$EU_{(a)s}^0 = \sum_{p_s} \{p_s \cdot [U(t_{(a)s}(0) + R(U(t_{(a)s}(0)) - U_0))]\}.$$

Load the OD flow quantity to the network with 'all and nothing' method. Set the iterations $n = 1$, then we can get the links flow vector $V^n = \{v_1^n, v_2^n, \ldots, v_i^n, \ldots, v_m^n\}$ (m is the total number of the links of the network);

Step 1: Compute new utility.

For every link a, compute the expected regret utility with flow v_a^n $(a = 1, 2, \ldots, m)$

$$EU_{(a)s}^n = \sum_{p_s} \{p_s \cdot [U(t_{(a)s}(v_a^n) + R(U(t_{(a)s}(v_a^n)) - U_0))]\};$$

Step 2: Find a feasible direction.

According to $EU_{(a)s}^n$, load the OD quantity to the network with 'all and nothing' method and get the new flow of every link $V^{n+1} = \{v_1^{n+1}, v_2^{n+1}, \ldots, v_i^{n+1}, \ldots, v_m^{n+1}\}$. The feasible direction is $d = V^{n+1} - V^n$;

Step 3: Compute the iterative length.

The iterative length can be computed by solving the one-dimensional search optimization problem

$$\min \sum_a \int_0^{v_a^{n-1} + \alpha_n (v_a^{n+1} - v_a^n)} \left\{ -\sum_s p_s \cdot [U(t_{(a)s}(v) + R(U(t_{(a)s}(v)) - U_0))] \right\} dv$$

solution result is α_{n^*}.

Step 4: Convergence conditions checking.

Let $n = n + 1$, then $V^{n+1} = V^n + \alpha_n (V^{n+1} - V^n)$; Check the convergence condition, if the convergence condition

$$\sqrt{\sum_a (v_a^{n+1} - v_a^{n+1})^2} \Big/ \sum_a v_a^n \leq \varepsilon$$

is satisfied, then stop, otherwise transfer to Step 1.

5. EMPIRICAL STUDIES

The central area of Liaoyang city in northeast of China is selected as the study objective. Based on CRRA and anticipated regret utility function, the traffic flow in this central area is distributed using the model proposed in this study. Besides, the distribution results of this model are compared with real link flow survey data to verify the effectiveness of this model.

5.1 Experiment design and data collection

The data in this study include the OD quantity, link travel time and the traffic flow data of Liaoyang city. OD data are mainly used to make preparations for the traffic flow distribution, and link travel time data are used for developing link travel time function. The experiment data in this study are divided into two sets, one set is the survey data in non-risk environment in July of 2011 (summer), the other is the field survey data in risk environment in December of 2011 (winter). The reason that these two sets of data in different time periods are chosen are indicated as follows: in summer, the external environment of the road in northeast China shows little variations, the traveller's behaviour characteristics in non-risk condition can be more easily reflected; while in winter, road is often in the frozen or melting state influenced by climate. So the road state is random, and the road state faced by travellers is a risk environment. Under this situation, the route choice behaviour in risk condition can be more effectively reflected. According to the geography characteristic of Liaoyang, the central area of Liaoyang is divided into 75 traffic zones.

OD data survey was carried out on the inhabitants in these 75 traffic zones using travel survey questionnaire on July 20th, 2011 and December 17th, 2011, respectively. The survey questionnaire content included the origin point of the trip, the destination of the trip and the travel times. For every experiment we distributed 100,000 questionnaires, and received 58,761 and 61,083 valid copies separately. Then, according to population percentage of the 75 zones, the OD travel data of these zones were obtained.

The road network of Liaoyang city is divided into four types, which are expressway, main road, arterial road, and branch road. For each type, 20 samples were selected. Then in 'good state' (sunny weather) and 'bad state' (weather after snow), respectively, the travel time of the road samples in time period from 8:00-20:00 was collected at 1-hour intervals using GPS vehicle equipment. In this way 4 sets of data samples in total were obtained, 20 links for each set, and 12 recorded travel times for each link. Then, according to the flow collection equipment in the corresponding time period, the link flows corresponding to each travel time were acquired. The main data condition in this study is listed in *Table 6*.

5.2 Model development

The unknown parameters in the model mainly contain BPR road impedance parameters α and β, environment parameter p, risk degree parameter θ, regret degree parameter δ, and expected utility U_0. According to the travel time data and the corresponding traffic flow data, a logarithm regression model is established. By this model, the travel time function parameters α and β of different types of road can be calibrated. Travel time function is calibrated into two sets; one set is the link travel time function in 'good state', and the other set is that in 'bad state'.

In 'non-risk condition', it is considered that road is in a specific environment basically, thus the environment probability of road is $p = 1$. In 'risk condition' the road is likely in frozen or unfrozen environments, thus there are two environment states for road which are noted as $p_1 = p_2 = 0.5$, respectively. $\theta = 0.01$ (smaller value) is employed to represent the traveller preference and risk-neutral. And δ is a parameter that has to be adjusted specifically.

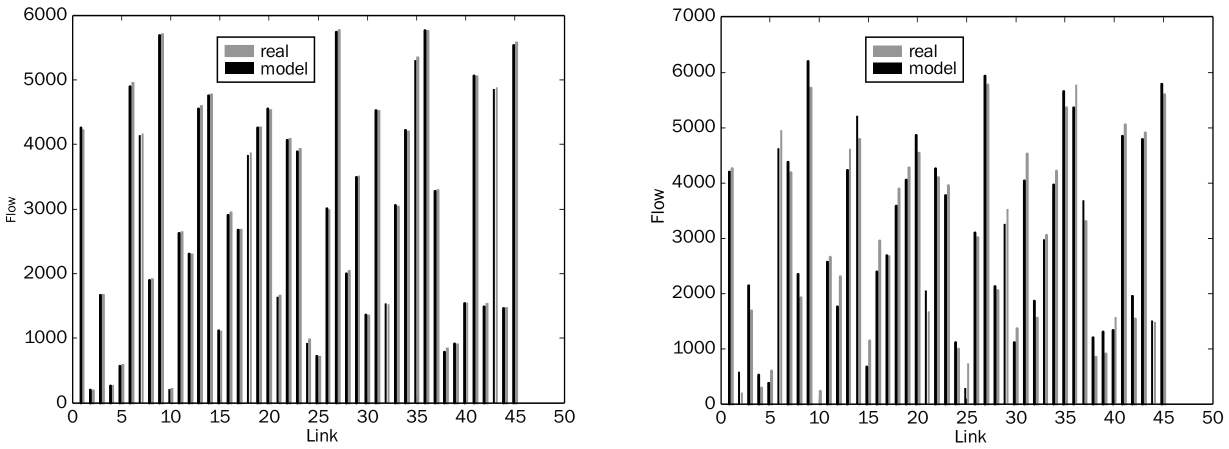
5.3 Model results

Using model (11) and the parameters calibrated in section 5.2, two sets of OD survey data (July 20, 2011 and December 17, 2011) are introduced to distribute the traffic flow in the central area of Liaoyang city, respectively. Then the flow distributed is compared with 45 actual road flows, and the results are shown in *Figure 4* and *Figure 5*.

*Figure 4*a illustrates that, in non-risk environment, when traveller's regret parameter is smaller ($\delta = 0.1$), the traffic flow distributed by the model is approximately consistent with the actual traffic flow investigated, and there are fewer differences; but when the traveller's regret degree parameter is larger ($\delta = 0.9$), as shown in *Figure 4*b, the traffic flow distributed by the model presents obvious differences with the actual traffic flow investigated. This indicated that in non-risk environment, the traveller's regret characteristic is not very obvious.

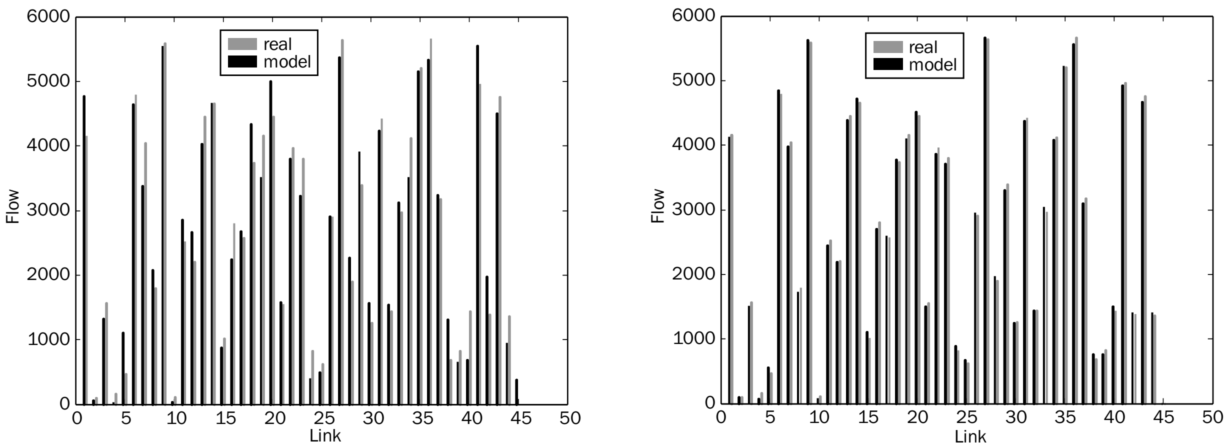
Table 6 - Summary of data

OD	Travel time	Link flow
OD data of 7/20 2011, and 12/17, 2011, include 75 areas	Fast road, main road, roads, branch, 20 each, get 12 group travel time data of every link. Use these data to develop travel time function	The main 45 link flow data of 7/20, 2011, and 12/17, 2011. in order to compare with the model results



a) δ = 0.1 b) δ = 0.9

Figure 4 - Comparing the results of the model and the field data (non-risk environments)



a) δ = 0.1 b) δ = 0.75

Figure 5 - Comparing the results of the model and the field data (risk environments)

*Figure 5*a demonstrates that in risk environment, when the traveller's regret parameter is smaller (δ = 0.1), the traffic flow distributed by the model shows bigger differences with the actual traffic flow investigated; but when the traveller's regret degree parameter is larger (δ = 0.75), as shown in *Figure 5*b, the traffic flow distributed by the model presents little differences with the actual traffic flow investigated. This indicated that in risk environment, the traveller's regret characteristic is very obvious.

6. CONCLUSION

Recently, the regret psychology in social sciences has been confirmed by more and more psychologists and has begun to be used in risk decision process gradually. In this study, the anticipated regret behaviour is introduced into risk route selection, and the route selection behaviour in anticipated regret condition is applied to the network equilibrium analysis. Based on the expected utility theory, we use the regret degree function which has formulated by Chorus 2012 [23]. CRRA and CARA expected utility functions are modified to obtain the anticipated regret expected utility function. Meanwhile, through a simple sample with two routes (one is a safe route, the other is a risk route), the effect mechanisms of environment risk degree p, regret degree δ, and risk aversion degree θ to risk route selection are analyzed. The results show that, (1) environment risk degree p presents the most significant influence on traveller's route selection. When environment tends to be in 'good state', the travellers show more preference to risk route. When environment tends to be in 'bad state', the travellers show more tendencies to safe route. (2) When environment tends to be in 'non-risk condition', i.e. when $p \to 1$ or $p \to 0$, risk aversion degree θ has no significant effect on route selection. But when the environment tends to be in 'risk condition' i.e. when p is away from two extremes of 0 and 1, risk aversion degree θ exhibits more obvious influence on route selection. As risk aversion degree increases, the travellers tend more to select the safe route. (3) Regardless of the environment, the traveller's regret degree δ shows more obvious influences on route selection. Finally, the anticipated regret utility constructed in this study is utilized to analyze the risk route selection. In this way, the network equilibrium condition with anticipated regret condition is obtained. Besides, a network equilibrium model is proposed based on the anticipated regret theory. The equivalence of the model, the existence and uniqueness of solution are also proven. Furthermore, a test is designed to compare model results with real data. The comparison results show that model distribution results can be better fitted with real data by adjusting regret degree value δ. This phenomenon proves the effectiveness and practicability of the model proposed in this study. In addition, it can be seen from the test process that travellers show different regret intentions in different risk environments, i.e. regret is weaker in the environment with weaker risk degree, but it is stronger in the risk environment.

The analysis and the model in this study have to be further studied and improved. Firstly, real travel demand is practically elastic, then what influences will be generated by environment risk degree *p*, regret degree δ, and risk aversion degree θ on travel demand? Secondly, in the empirical process of this study, parameters *p*, θ are calibrated by experience values, while δ is calibrated by adjustment. In practice, how these parameters are calibrated by actual investigated data is an issue that needs to be further investigated in the future studies. Thirdly, the equilibrium distribution model in this study is based on specific demands and complete information, while actual demand is elastic. And the assumption that drivers have complete information is invalid. Thus, how this model is extended to elastic demand needs to be further investigated. Lastly, we think that it is more reasonable and meaningful to assume that the driver's route selection is characterized by certain randomness in the future investigation.

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摘要

预期后悔条件下的风险路径选择分析及网络均衡模型

出行者行为假设对交通网络的均衡分析有着重要的影响。以往对出行者行为的假设主要依据于期望效用最大化理论，然而随着对交通系统不确定性的认识的深入（决策环境变得具有风险性），以‘绝对理性’为基础的期望效用理论受到制约。以‘预期后悔’理论为基础的风险决策理论为风险路径的选择提供了新的分析框架。将预期后悔理论应用到风险路径选择的分析中，构造了预期后悔效用函数。通过一个简单的、具有两条平行路径的风险网络环境，分析了不同风险环境条件下出行者的风险规避度、后悔度对风险路径选择的影响。并在此基础上建立了基于预期后悔理论的交通网络用户均衡分配模型；对模型的等价性及解的特征进行了证明，并给出了求解算法。通过设计实验，将模型结果于实际采集到的数据进行了对比，结果显示通过调整模型中的后悔度参数，可以是模型结果与实际结果更接近于实际结果，表明了模型可以较好地用于分析风险环境下的出行行为。此外模型的结果还表明，随着风险环境度的增加，出行者的后悔心理表现的越发明显。

关键词

预期后悔、期望效用、风险决策、路径选择、交通流

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