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## LOCATION OF URBAN LOGISTIC TERMINALS AS HUB LOCATION PROBLEM

### ABSTRACT

*In this paper the problems of locating urban logistic terminals are studied as hub location problems that due to a large number of potential nodes in big cities belong to hard non-polynomial problems, the so-called NP-problems. The hub location problems have found wide application in physical planning of transport and telecommunication systems, especially systems of fast delivery, networks of logistic and distribution centres and cargo traffic terminals of the big cities, etc. The paper defines single and multiple allocations and studies the numerical examples. The capacitated single allocation hub location problems have been studied, with the provision of a mathematical model of selecting the location for the hubs on the network. The paper also presents the differences in the possibilities of implementing the exact and heuristic methods to solve the actual location problems of big dimensions i.e. hub problems of the big cities.*

### KEY WORDS

*city logistics, hub location problems*

### 1. INTRODUCTION

Because of competitiveness, the globalization process imposes, as one of the special objectives, the maintenance of adequate level of transport costs for the sake of their minimization. It is precisely in this requirement that the logistics plays an important role. The location of fixed facilities in the logistic network is an important problem of logistic decision-making since it provides the form, the structure, and the shape for the entire logistic system.

The location models are very often used in solving the problems that cover the setting of the desired facilities on the offered locations. This paper analyzes

the problem of locating urban logistic terminals as hub location problems.

Hub nodes, e.g. logistic and distribution centres are one of the possible solutions of optimizing the cargo flows in the big cities. As logistic concept, the hub nodes attract attention since they provide significant effects in the rationalization of cargo supply and dispatch flows from the urban environments. The decision-making process regarding the location of the logistic and distribution centres, warehouses, terminals and their capacities and organization in individual cities requires a series of quantitative and qualitative bases used by the decision makers to estimate the success and risk of investing into their construction project. The suitable methods for making decisions on the construction of hub nodes, their location, capacity, and organization of flows are the operations research methods, simulation methods, heuristic methods, analyses of solutions in other cities and other decision-making methods.

The location of hubs in big cities is a very serious and demanding problem since it provides rationalization of cargo transport in big cities. The location problems are interdisciplinary since they are subject of research in many scientific fields: the technology of traffic and transport, architecture and urban planning, electrical engineering, computing, mathematics and in other scientific fields and branches. Goods supply to the citizens of big cities and dispatch of cargo from the cities is the basic problem of urban traffic experts. The dimension of the goods transport problem in the cities is maybe the easiest to consider as the analysis of the needs for the supply of citizens. According to some research carried out in the developed European countries, the needs of one citizen of a big city for consumer goods exceed one tonne per year [1].

In this paper attention has been paid to the analysis of methods and suitable software to solve the hub location problems of big dimensions.

## 2. GENERALLY ABOUT METHODS OF DETERMINING THE LOCATIONS OF LOGISTIC AND DISTRIBUTION CENTRES

The problems of locating the distribution centres or warehouse facilities from which the distribution is performed started to be systematically solved for the first time at the beginning of the 20<sup>th</sup> century by the application of the Gravitation method for determining of the location of distribution warehouses. Since it takes into consideration the number of citizens in a certain area and their economic power, the method is suitable to determine the location of a warehouse or a distribution centre in a limited area. It may be used independently as a method for determining the location or as a method of the first approximation in other sophisticated models, especially if referring to the location of one facility. The methods of determining the location of the logistic and distribution centres can be generally divided into three categories: empirical methods, mathematical methods and simulation methods. Each method features a certain level of accuracy that depends on the selected model of determining the location.

The factors that influence the selection of the location of the logistic and distribution centre, mentioned in the literature, can be divided into two categories: factors that influence the selection of a wider location of a facility and factors that influence the selection of the micro-location of certain facilities. In the selection of the wider location of facilities, the factors are considered that are related to the role of the facilities in the logistic system – market characteristics, type and characteristics of goods in the distribution chain, transport possibilities and the availability of the expert personnel. In determining the micro-location of the facility, the following characteristics are considered: the area of the terrain, fitting of the location to urban plans, condition of the traffic infrastructure in the observed area, price of the land, attitudes and exemptions for the local community, etc. It is obvious here that this refers to the elements that serve to locate one facility on the network. Today's problems of locating facilities on the network are oriented to the optimization of the entire distribution network in order to harmonize and optimize the distribution process among individual subjects within the supply chain, meeting the market demand and reducing the total distribution cost.

When creating the distribution network model it is necessary to respect certain principles related to the type of data that are relevant for this purpose and the method of data collection that will be used in developing the model. Relevant data include for instance data

about the product demand within a certain time period, product characteristics, data about the location of production, fixed and operative costs of facilities and variables related to the cargo dispatch and storage requirements for individual type of distribution centres.

In order to create a successful model of the distribution system, three principles need to be applied: synthesis and evaluation of data, definition (forming) of the model and sensitivity analysis of individual models.

## 3. HUB LOCATION PROBLEMS ON THE NETWORK

A network is an ordered set of nodes, arcs, weights and function, i.e.  $M = (V, L, P, f)$ , where  $(V, L)$  is graph,  $P$  is a set of weights, e.g. of the distance between the nodes or transport costs, and  $f$  is the function which assigns weights to the arcs:  $f: L \rightarrow P$ . Nodes can be *cargo origins* (suppliers), *destinations* (customers), *transit nodes* and nodes that have a special role, which are in literature usually called *hub nodes* (main nodes). In this paper the hub nodes consist of logistic and distribution centres on the logistic network. Figure 1a shows a network of six nodes ( $n = 6$ ) with adequate distances on the arcs (weights).

If two hub nodes have to be located on the network ( $p = 2$ ), let us assume that they are located in nodes  $B$  and  $C$ , Figure 1b.

Let the transport price between the hubs be 0.25, and let the transport price customer-supplier, supplier-hub and hub-customer be 1 per unit of goods quantity at the unit distance. If nodes  $B$  and  $C$  are designated as hubs, then the transport price from supplier  $A$  to customer  $D$  via hubs  $B$  and  $C$  per unit of cargo will be:  $6 \cdot 1 + 16 \cdot 0.25 + 5 \cdot 1 = 15$ . If the goods were transported directly from node  $A$  to node  $D$ , the transport price would be  $20 \cdot 1 = 20$ .

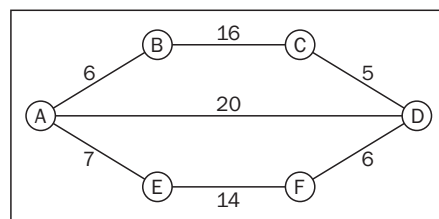


Figure 1a - Network with 6 nodes ( $n = 6$ )

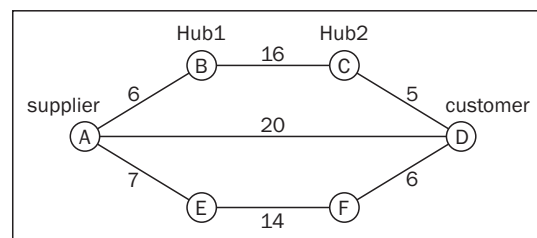


Figure 1b - Network with 6 nodes and 2 hubs ( $n = 6, p = 2$ ); Optimal path: A - B - C - D

Hub networks have a wide range of application in modern transport and telecommunication systems. Instead of directly supplying each customer by the assigned supplier, the concept in hub networks is the supplier-customer transport via one or several hubs. Hubs represent the centres of consolidation and bundling of cargo according to destination locations. Using hubs as flow redirecting points and by increasing the transport between hubs, the transport network capacity can be used in a much more efficient manner. Since the price of transport between hubs per unit of quantity is lower, because the capacities of the transport means are better used with the bundling of cargo, in this way the total transport costs in the transport network are reduced.

Unlike classical location problems, each hub location problem has two versions that use different allocation concepts. Two basic allocation concepts are distinguished [2]:

- single allocation scheme, in which every supplier/customer is assigned to precisely one hub, so that the entire transport from/to this node is performed exclusively via a certain hub.
- multiple allocation scheme, which allows every non-hub node to communicate with one or several hubs.

The multiple allocation scheme allows greater flexibility of the model, since, for the given set of hubs, the transport between the supplier and the customer can be realised by a route with the lowest transport price, independent of other nodes.

However, the majority of hub location problems understand the single allocation scheme, since every customer/supplier is usually assigned to their closest hub, i.e. hub with the lowest transport costs. For instance, the passengers in air traffic that are located in the same city (node of origin) can choose to fly via different cities (hubs) depending on the city of their destination (node of arrival). Similarly, in postal networks, post from the origin postal centre can be distributed via different postal terminals to the destination centre, so that the transport costs are minimal. However, if this refers to fast delivery postal networks (e.g. DHL for fast shipments) in which it is important to minimize the longest time of delivery for every customer-suppli-

er pair, in adequate hub models the single allocation scheme is used. Obviously, due to the flexibility of conditions in hub models with multiple allocation scheme, the transport costs will in that case be lower in relation to the models with single allocation scheme.

Figure 2a shows a network with five nodes ( $n = 5$ ) whose graph is strictly connected i.e. there is a path between every two nodes.

In the network presented in Figure 2a two hub nodes need to be established ( $p = 2$ ), so that the total transport costs are as low as possible, with no restrictions of node capacities. Here, transport between each pair of non-hub nodes has to pass via one or several hub nodes (direct supplier-customer transport is not permitted). Let the transport price between non-hub nodes and hubs be 1 money unit (m.u.), whereas the transport price between the hubs is 0.75 m.u. per quantity unit of goods. If the single allocation concept is adopted and all the possible combinations of choice of two hub nodes from the set of five nodes are analyzed the best solution is that the hubs are located in node B and D or C and D, Figure 2b. The total costs of all the transport combinations from any node of origin to any node of destination are the lowest:

$$\begin{aligned}
 &AB + AD + AC + AE + BD + BC + BE + CD + CE + DE = \\
 &= 13.5 + 6 + 16.5 + 10 + 7.5 + 3 + \\
 &+ 11.5 + 10.5 + 14.5 + 4 = 97
 \end{aligned}$$

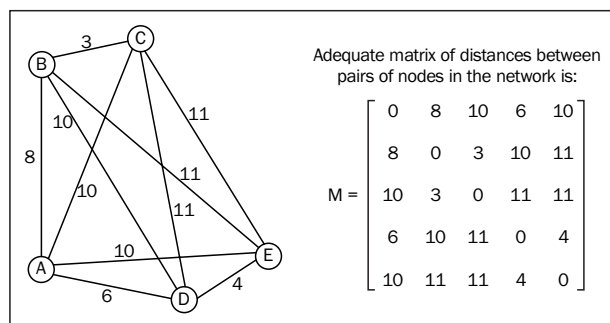


Figure 2a - Network with a strictly connected graph

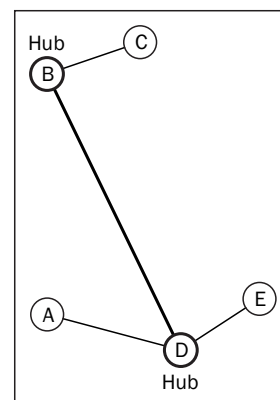


Figure 2b - Single allocation scheme with non-capacitated hubs

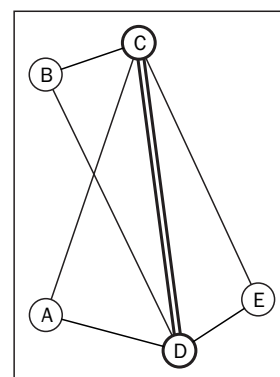


Figure 2c - Multiple allocation scheme with non-capacitated hubs

The same result is obtained if the hub nodes are *C* and *D*.

If the multiple allocation concept is adopted then the optimal choice of hubs is in nodes *C* and *D*, the total costs of all the transport combinations from any node of origin to any node of destination are the lowest, *Figure 2c*.

$$\begin{aligned} & \text{The total costs of all combinations of transport are:} \\ & AB + AD + AC + AE + BD + BC + BE + CD + CE + DE = \\ & = 13 + 6 + 10 + 10 + 10 + 3 + 14 + 8.25 + 11 + 4 = \\ & \qquad \qquad \qquad = 89.25 \end{aligned}$$

It can be seen that for multiple allocations the total unit costs are lower, since non-hub nodes *A*, *B* and *E* can communicate via hub *C* or via hub *D* depending on which price of transport is more favourable, the saving is about 8%.

#### 4. POSSIBLE FUNCTIONS OF OBJECTIVE OF HUB LOCATION PROBLEMS

The classical optimization problem is to find the minimum or the maximum of function  $f(X)$  with restrictions  $g_i(X) \leq 0, i = 1, 2, \dots, n$ ; where  $X$  is  $n$ -dimensional vector. Function  $F(X)$  is the function of objective or criteria. It is natural to expect that the majority of hub location problems will have the objective of minimizing the transport costs and possibly some other costs in the network (costs of establishing the hubs, new roads – arcs, costs of delay, etc.). The type of the function of objective or the type of optimization criteria of hub location model depend on the type of the real problem referred to by the given model.

Possible functions of objective of hub location problems are the following [3], [4], [5], [6]:

- a. Function of objective that minimizes the sum of transport costs for each customer-supplier pair via adequate hub/s, where the number of hubs that have to be established is predetermined and equals  $p$ .
- b. Problems that deal with the establishment of a certain number of hubs –  $p$ , in order to minimize the maximal distances/time/costs of transport between any two nodes in the network (*minmax* criterion). Defining the transport schedule so that the time of the last delivery is as short as possible is especially important in the fast delivery system.  
When the task is to set only one “undesired” location, it is necessary to find such a location that the shortest distance to the city is maximal (*maxmin* criterion).
- c. Problems in which the number of hubs is not set but rather the optimal number of hubs has to be determined, so that the total transport costs in the network be minimal. For each node that can be a hub node the costs of construction, possible

capacities, connection to other nodes, customers’ requirements, planned cargo flows, etc. need to be estimated. They have a wide implementation in the transport networks in which some of the nodes or lines are public or private property (airports, routes, railway lines, warehouses, etc.) which means that there is a fee for their usage. The function of objective does not have to refer to the minimization of costs / transport time, the function of objective can be maximizing the quantity of cargo in the network that is performed via hubs.

- d. The maximal covering location problem. The task is to select from a set of offered locations a certain number so as to cover as many customers as possible. It is often used in determining the location for emergency services such as police and fire stations, ambulance stations, etc.
- e. The locations of the military distribution centres have specific criteria that refer to the conditions of performing war operations. Among other criteria, the following conditions have to be met: secrecy of locations of military distribution centres, protection against enemy reconnaissance and attacks, insurance of alternative traffic communications, several smaller distribution centres, sufficient distance from active force, etc.

The majority of real problems have several functions of objective, several criteria. For the selection of the location the model can be required to meet several functions of objective, e.g. that the lowest sum of the distances from the hubs to the customers is minimal and at the same time that the time of last delivery i.e. maximal time from hub to customer is minimal, and that at the same time the transport costs are minimal and that the environmental requirements are maximally met which leads to multi-criteria programming. In fact, all the real problems are of multi-criteria character but for the sake of simplicity of solving the problem they are often reduced to single criterion ones.

#### 5. MATHEMATICAL MODEL OF SELECTING HUB FACILITY LOCATIONS

The problem of selecting the location of urban logistic terminals in big cities with the aim of minimizing the total costs requires a number of analyses for every potential location:

- demand for “heavy” infrastructure – capabilities of suppliers, distributors, traffic networks, telecommunication networks, water installations, sources of electrical and other energy, facilities intended for public needs, etc.
- demand for “soft” infrastructure – availability of labour.

One of the possible mathematical models can be formulated as a combined linear problem (with integer

and continuous variables) for the selection of the location of urban logistic terminals with the aim of minimizing the overall costs [7], [8].

Let:

- $n$  – number of nodes on the logistic network;
- $p$  – number of hubs;
- $f_i$  – annual fixed costs of hubs on  $i$ -th node;
- $d_{ij}$  – distance between  $i$ -th and  $j$ -th nodes;
- $D_i$  – annual demand on  $j$ -th node;
- $K_i$  – potential capacities of hub on  $i$ -th location.

Decision-making variables:

- $y_i = 1$ , if hub is located on  $i$ -th location;  
otherwise 0,
- $x_{ij}$  = quantity of cargo transported from  $i$ -th to  $j$ -th node.

Mathematical model of selection of the hub object can be written as:

$$\min \left( \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} \right) \tag{1}$$

with restrictions:

$$\sum_{i=1}^n y_i = p \tag{2}$$

$$\sum_{i=1}^n x_{ij} = D_j, (j = 1, 2, \dots, p) \tag{3}$$

$$\sum_{j=1}^m x_{ij} \leq K_i y_i, (i = 1, 2, \dots, n) \tag{4}$$

$$y_i \in \{0, 1\}, (i = 1, 2, \dots, n) \tag{5}$$

$$x_{ij} \geq 0 \tag{6}$$

The function of objective (1) minimizes the total costs. Restriction (2) defines the number of hubs. Restriction (3) insures that the demand in nodes is met. The restriction of capacities is expressed with (4). Restriction (5) insures that the variables that refer to the location of the hubs are binary. Restriction (5) that the variable  $x_{ij}$  is a non-negative real number.

*Numerical example:* Let on the logistic network with five nodes three nodes be selected where the distribution centres are to be located. The data about the nodes are given in Table 1.

The numerical mathematical model:

$$\min(70y_1 + 25y_2 + 40y_3 + 32y_4 + 22y_5 + 2x_{12} + 3x_{13} +$$

$$+ 3x_{14} + 6x_{15} + 2x_{21} + 2x_{23} + 4x_{24} + 4x_{25} +$$

$$+ 3x_{31} + 2x_{32} + 2x_{34} + 3x_{35} + 3x_{41} + 4x_{42} +$$

$$+ 2x_{43} + 5x_{45} + 6x_{51} + 4x_{52} + 3x_{53} + 5x_{54})$$

$$y_1 + y_2 + y_3 + y_4 + y_5 = 1$$

$$x_{21} + x_{31} + x_{41} + x_{51} = 3$$

$$x_{12} + x_{32} + x_{42} + x_{52} = 2$$

$$x_{13} + x_{23} + x_{43} + x_{53} = 2$$

$$x_{14} + x_{24} + x_{34} + x_{54} = 3$$

$$x_{15} + x_{25} + x_{35} + x_{45} = 5$$

$$x_{12} + x_{13} + x_{14} + x_{15} \leq 16y_1$$

$$x_{21} + x_{23} + x_{24} + x_{25} \leq 12y_2$$

$$x_{31} + x_{32} + x_{34} + x_{35} \leq 15y_3$$

$$x_{41} + x_{42} + x_{43} + x_{45} \leq 15y_4$$

$$x_{51} + x_{52} + x_{53} + x_{54} \leq 13y_5$$

$$y_i \in \{0, 1\}, x_{ij} \geq 0 \text{ continuous}$$

Using the software package *lp\_solve* specialized for combined variables the following solutions are obtained:

$$y_1 = y_4 = 0; y_2 = y_3 = y_5 = 1; x_{21} = 3,$$

$$x_{23} = x_{32} = 2, x_{34} = 3, x_{35} = 5, \text{ Figure 3}$$

In [2] the mathematical model of capacitated single allocation hub location problem is given, which is often used in postal networks of fast delivery.

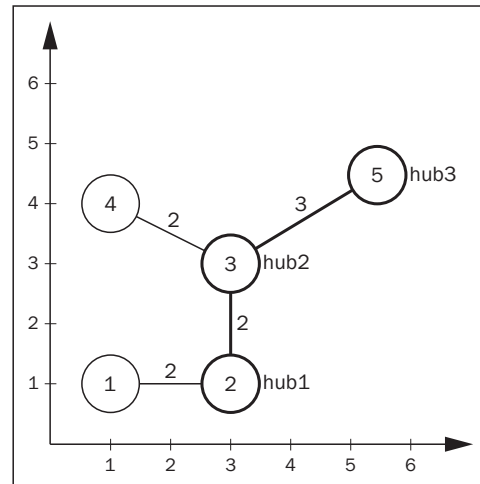


Figure 3 – Logistic network with five nodes and three hubs

Table 1

Nodes, potential distribution centres	Node coordinates	Matrix of distances					Potential capacity of nodes $K_i$	Annual fixed costs of $i$ -th node, $f_i$	Annual demand in $i$ -th node $D_i$
		1	2	3	4	5			
1	(1,1)	0	2	3	3	6	16	70	3
2	(3,1)	2	0	2	4	4	12	25	3
3	(3,3)	3	2	0	2	3	15	40	2
4	(1,4)	3	4	2	0	5	15	32	3
5	(5,5,4,5)	6	4	3	5	0	13	22	5



It should be mentioned that the model provides the solution about the cargo quantity in the domain of real numbers. In real logistic processes the quantity of cargo is measured by integers since it is impossible to take out a certain quantity of cargo from its packaging, and the result needs to be rounded up to an integer which will not affect the correctness of the solution.

## 6. DIFFERENT RESEARCH DIRECTIONS OF HUB LOCATION PROBLEMS

The theory of graphs enables different studies on networks particularly on the transport networks and logistic chains. The location problems on the network are especially interesting to different scientists in solving both theoretical problems and the practical problems of allocation. For the study of location problems related to this paper, the works by the following have been analyzed: logistic experts [1], [9], [12], [13] operation researchers [3], [7], [15], [16]; computer science experts [4], [10], [14]; mathematicians [2], [8]. The analysis of the mentioned works indicates possible further research of hub location problems:

- a/ Formulation of new, more complex hub location models where the number of hubs has not been determined in advance, but rather the optimal number of hubs and their locations on the network need to be determined [2], [9]. Models in which hubs are selected from a pre-defined subset of the network nodes.
- b/ Minimization of the maximum time in collecting and delivery of fast shipments on logistic chains. Because of the globalization processes fast deliveries are gaining more and more significance so that the models of fast deliveries are the subject of research of many scientists.
- c/ Study of the model with variable lower transport costs between hubs (for instance, in dependence on the quantity of carried cargo). These models have an application in air cargo transport.
- d/ Models with increased concentration of the flow on the transport network when the prices are lower on single branches if the transport quantity exceeds a certain threshold. These situations are encountered in practice [10].
- e/ Models in which the transport from the supplier to the customer can be performed via hubs and directly, e.g. in the transport of oil products the transport is performed from the refinery to the warehouse, from warehouse to warehouse, from warehouse to retail centres and directly from refineries to retail centres.
- f/ The study of the routing problems from supplier to hub and from hub to customer. Optimization of the lengths of logistic chains.

Determining of the number and locations of hubs on the transport network attracts attention of many researchers of different scientific fields since this can minimize the transport costs or the delivery times in "door-to-door" logistic chains. These studies are useful in designing of networks, especially of cargo traffic, of each traffic branch.

In the research of hub location problems special place belongs to the capacitated hub location problems. In [3] the models with limited quantity of cargo have been described, with cargo arriving and/or passing through the hub node, models with limited flow between hubs as well as/or between hubs and assigned users/suppliers and the capacitated models of non-hub nodes.

Hub location problems have a relatively simple formulation but due to an enormous number of combinations, relations and limitations their solution is more difficult compared to classical discrete location problems. If the network is full of nodes the resolution time on the computer is non-polynomial and therefore these problems belong to non-polynomial hard problems, the so-called NP-problems. Still there are cases of hub location problems that can be solved by classical exact methods, e.g. determining of hub nodes between the Croatian airports [11].

## 7. SOLVING OF HUB LOCATION PROBLEMS

Hub location problems of smaller dimensions can be solved by different exact mathematical methods. For the solution of problems of big dimensions with a large number of possible relations various heuristic methods have to be used, and sometimes a combination of exact mathematical and heuristic methods is possible. In the project "TAYLOR MODE" the solving of the transport problem – Optimization of parameters of the transport of raw materials and products of the INA oil refinery Sisak, (Institute of Traffic and Communications in Zagreb, 2003, Zagreb) where a daily plan of the transport of oil products (about 80 products) had to be done from the warehouses of two refineries Rijeka and Sisak to retail centres (about 500 pumps) the making of an optimal plan by exact mathematical methods would take several hours, which was unacceptable, and so heuristic algorithms were used which provided an acceptable plan in several minutes.

### 7.1 Exact solving of hub location problems

For well structured problems the mathematical models that are solved by exact methods can be defined. These methods always start from a precisely formulated mathematical model which reflects sufficiently well the nature of the problem being solved.

The majority of real problems are poorly structured i.e. they can have a complex structure with a large number of diverse restrictions that often cannot be fully mathematically formulated, so that therefore a precise mathematical model that would present this structure in a satisfactory manner cannot be formulated. Besides, in some well structured problems it may happen as well, in case of large dimensions, that exact methods cannot find an optimal solution within a reasonable time. Such as the mentioned transport schedule of oil products from several warehouses.

The mathematical model of hub object location selection on the transport network mentioned in the 5th section is a Mixed Integer Linear Programming problem. Different methods for solving of the mixed integer linear problems have been developed. One of the basic methods of solving mixed integer linear problems is the search of the space of solving i.e. studying each network node. However, with the increase of the problem dimension, full search of the solution space is usually not feasible, not even with the assistance of most powerful computers. Therefore methods of intelligent searches have been developed, and by means of estimate, they direct search in a certain direction and eliminate useless solutions e.g. they eliminate some network nodes that regarding a criterion are not logical to be hub nodes. Two such methods, Branch and Bound (BnB) and Branch and Cut (BnC) methods are intensely used to solve the hub location problems [7], [12]. The branch and bound method is based on the idea that it is not necessary to analyze all the nodes as hub nodes, since they cannot be that because of different reasons so that the problem dimension is reduced. There are various variants of branch and bound methods that are adjusted to specific structure of the problem to be solved.

Exact algorithms, if implemented correctly, guarantee an optimal solution whereas the implementation of heuristic algorithms yields solutions that are close to the optimal one or are even optimal ones although there is no proof for this [5].

## 7.2 Heuristic methods

The exact methods that have been proposed in literature cannot provide a solution in case of hub location problems of big dimensions. Therefore, various heuristic approaches have been developed that for real dimensions of the problem provide good results within a reasonable time of computer operation.

Heuristics, as a general notion today is usually defined in the following way: *Heuristics is a technique which tries to find some "good" solutions of the problem (i.e. such permitted solutions of the problem that are sufficiently close to their optimum) within a reasonable time, not guaranteeing that the found solu-*

*tion will be optimal, nor can their closeness to the optimal solution be determined* [7].

Heuristic methods allow that in a very free way, by using sound logics, intuition and past practices in problem solving, most different intelligent rules are formalized, that, implemented in the solving process, can ensure that they are on the average sufficiently close to the optimum.

Heuristic algorithms need to have another important characteristic – that they operate within a reasonable time, i.e. to be computer-efficient.

A heuristic approach to problem  $p$  – non-capacitated single allocation hub centres is described in [13]. The authors propose four different strategies, based on the shortest route method, for the selection of the set of  $p$  hubs. The proposed method has been tested on the problem of  $n = 25$  nodes and  $p = 5$  hubs, for different combinations of initial hub location strategies.

For the case of multiple allocations the method of finding the optimal pair of hubs is usually used, then gradually adding hubs, until there are exactly  $p$  hubs. The heuristics further performs the replacement of hubs and the assigned non-hub nodes until the solutions are improved [14], [15], [16].

The doctoral dissertation "Genetic Algorithm for Solving of some NP-hard Hub Location Problems" [2], developed at the Mathematical Faculty in Belgrade, emphasized the genetic algorithms for the solution of complex hub location problems. The genetic algorithms imitate the process of natural selection of entities in the biological systems.

Determining of locations of urban logistic terminals as hub location problems in big cities certainly belongs to problems of big dimensions. Therefore, the algorithms for practical solution of the problems need to be found in the domain of heuristic methods adjusted for solving of city logistics specific problems.

## 8. CONCLUSION

This paper describes the location problems of city terminals as hub problems that can be solved in practice by heuristic methods or a combination of heuristic and exact methods.

The location problems considered in this paper, although easily understood and formulated, are difficult to be solved since this includes a large number of combinations. Regarding computing solving time they belong to hard NP-problems.

Based on the performed numerical analysis the following can be concluded: the supply of customers via one or several hubs leads to the reduction of transportation costs, since the goods at hubs are consolidated and bundled, which results in higher level of usage of the vehicle cargo space, as well as to generally better exploitation of the transport capacities. By introducing

the hub logistic centre in city logistics the goods are distributed by means of smaller delivery vehicles thus avoiding the transport within the very centre of the city using big trucks and reducing the traffic congestion in the city network. It has been shown that the multiple allocation schemes allow greater flexibility in the method of delivery organization which increases the efficiency of distribution by better usage of infrastructural capacities and greater possibility in the selection of transport solutions.

The methods used for solving the issues of city terminal locations described in this paper can be applied to design the transport networks of interurban and urban delivery of parcels and documents, as well as for the optimization of the already established transport networks which could lead to improvement in the segment of transportation costs and usage of resources.

The models mentioned in this paper are statistical; in practice there is usually the need to monitor the changes in time, and therefore, dynamic location models are used more. The most successful method in solving the problem of this type has proven to be the genetic algorithms, which can be applied both for statistical and for dynamic models of determining the location of hubs in the network.

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## SAŽETAK

### LOKACIJA GRADSKIH LOGISTIČKIH TERMINALA KAO HAB LOKACIJSKI PROBLEM

U ovom radu problem lokacije gradskih logističkih terminala promatra se kao hab (hub) lokacijski problemi koji zbog velikog broja potencijalnih čvorova u velikim gradovima spadaju u teške nepolinomske probleme tzv. NP-probleme. Hab lokacijski problemi nalaze veliku primjenu kod prostornog planiranja transportnih i telekomunikacijskih sustava, poštanskih sustava, posebno sustava brze isporuke, mreže logističkih distribucijskih centara i terminala teretnog prometa velikih gradova i drugo. U radu su definirane jednostruke i višestruke alokacije te obrađeni numerički primjeri. Istraživani su hab lokacijski problemi ograničenih kapaciteta s jednostrukom alokacijom te je za njih dan matematički model izbora lokacije habova na mreži. Pokazane su razlike u mogućnostima primjene egzaktnih i heurističkih metoda za rješavanje realnih lokacijskih problema velikih dimenzija tj. hab problema velikih gradova.

## KLJUČNE RIJEČI

city logistika, hab lokacijski problemi

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