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INFLUENCE OF TRACK STIFFNESS ON TRACK BEHAVIOUR UNDER VERTICAL LOAD

ABSTRACT

The analysis of track behaviour under vertical load is traditionally based on the presumption that the stresses and deformations in track elements can be determined by the application of the Winkler's Hypothesis. The rail is considered a beam on a continuous elastic foundation. The basis of the hypothesis is a presumption about the proportionality between the load and deflection. However, it is empirically known that the track and track foundation elements in a real environment during the railway exploitation behave neither linearly, nor completely elastically. Moreover, there is a problem with unevenness of the track stiffness along the track. This paper analyses the track stiffness from the aspect of its influence on the quality of the vertical track geometry. The paper analyses optimum stiffness. Optimum stiffness is conditioned by the single stiffness of all the elements of the superstructure and substructure as well as by their mutual compatibility.

KEYWORDS

railways, track, track stiffness, track geometry deterioration, maintenance

1. INTRODUCTION

The requirements for track stiffness as a complete system are an open point [1].

Track stiffness is a significant parameter from the aspect of designing, construction and maintenance of the railway superstructure and substructure.

This parameter represents the basis for calculating stresses in the elements of the track and track foundation. Stiffly leant rails have lesser elastic deflections and bending stresses in the rail, while the pressure force transmitted from the rail to the sleeper and further to the ballast and substructure is higher.

During the track exploitation, track stiffness influences considerably the following processes:

- track geometry deterioration
- rail fatigue, and
- deterioration of other components of the railway superstructure and substructure.

Therefore, understanding track stiffness, its correct definition and the choice of the optimum value while dimensioning superstructure and substructure are of the utmost importance for track geometry preservation within tolerance limits for as long as possible.

The objective of this paper is a critical review of the state-of-the-art in the domain of track stiffness, analysis of the conclusions and determination of the guidelines for further theoretical and experimental research.

2. DEFINING TRACK STIFFNESS

There are multiple ways to define track stiffness. The most common definition is that the stiffness (D) presents the proportion between vertical load (Q) and track deflection (y) at a given moment (t):

$$D(t) = \frac{Q(t)}{y(t)} \tag{1}$$

The analysis of the track behaviour under vertical load is traditionally based on the beam on an elastic foundation (BOEF) model and the Winkler's Hypothesis. Winkler's Hypothesis assumes that the elastic foundation is a system of identical, independent, closely spaced, discrete and linearly elastic springs. The basis of the hypothesis is a presumption about the proportionality between the load and deflection in every point of the contact surface.

The modern approach of stiffness definition includes inelastic and nonlinear behaviour of the superstructure and substructure elements, as well as the existence of the difference between the stiffness under static and under dynamic load.

The beam on an elastic foundation model is described in detail in numerous pieces of literature on the topic [2-5] and thus it will not be presented in this paper.

2.1 Nonlinearity of the load-deflection dependence

In reality, elements of the superstructure and substructure behave neither linearly nor completely elastically [6-10]. This can be explained on the example of ballast behaviour under the real conditions. In the majority of cases, leaning sleepers on the ballast is not ideal. There are voids beneath the sleepers which cause great deflections at small load intensity. Moreover, at great load intensities, nonlinearity and track stiffness increases are a consequence of ballast and substructure layers compaction. Load distribution through ballast is done through contact surfaces between ballast stones. As the load value increases, stone deformations lead to an increase of these contact surfaces and thus the ballast stiffness increases.

The load-deflection diagram in *Figure 1* presents the attempt to approximate nonlinearity with the bilinear curve [7]. Track stiffness is then defined as secant stiffness:

$$D_{s-t} = \frac{Q_t - Q_s}{y_t - y_s}, \qquad (2)$$

where:

- Q_s – boundary load after which there are no voids beneath the sleepers (kN),
- Q_t – maximum load (kN),
- y_s – boundary deflection after which there are no voids beneath the sleepers (cm), and
- y_t – total deflection (cm).

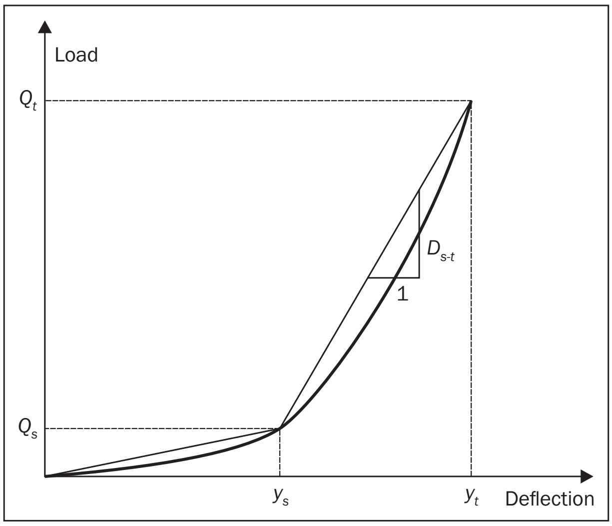

Figure 1 – Bilinear approximation of the load-deflection diagram [7]

Apart from the secant stiffness, tangent stiffness in a given point is also defined:

$$D_{tg,t} = \left| \frac{dQ(t)}{dy(t)} \right|_t . \qquad (3)$$

Measurements of the rail deflection under load point to continual nonlinear behaviour rather than bilinear behaviour. *Figure 2* shows the diagram derived based on the measurements on the Swedish Railways [11]. Gradual loading of the track up to 150kN was performed and the corresponding rail deflections at cross sections above sleepers were measured.

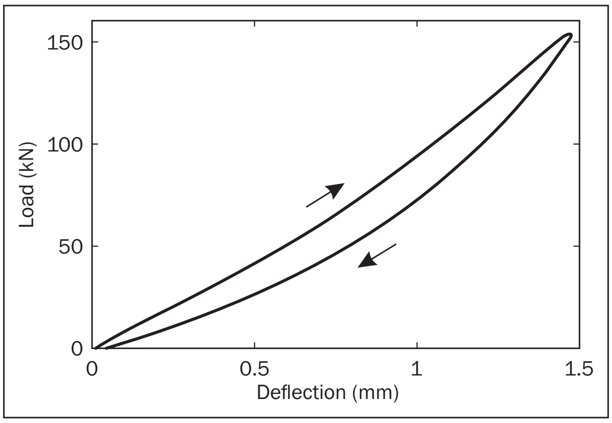

Figure 2 – Load-deflection diagram based on the Swedish Railways measurements [11]

Similar nonlinearity can also be distinguished in *Figure 3*, on the diagram derived based on the measurements performed in Australia [12]. The diagram shows three different lines which can be used to calculate different stiffness.

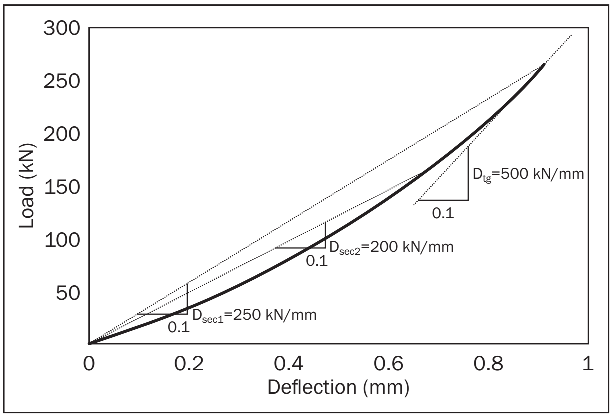

Figure 3 – Load-deflection diagram based on the measurements in Australia [12]

The absence of linearity of the load-deflection connection actually means that there is no unique value of the track stiffness. *Figure 4* shows the procedure for determining linearized stiffness as one of the possible procedures for determining numerical stiffness values in the construction calculations.

Linearization of the nonlinear load-deflection diagram is performed in the proper load range, which can be the range of dynamic load on the section for which the stiffness is being determined. Since there is a difference between the real and linearized stiffness (depending on the load value, the real stiffness can be lower or higher than the linearized one), it is necessary to keep in mind the error, which is the consequence of linearization, in the application of the calculation data in the calculation models [13].

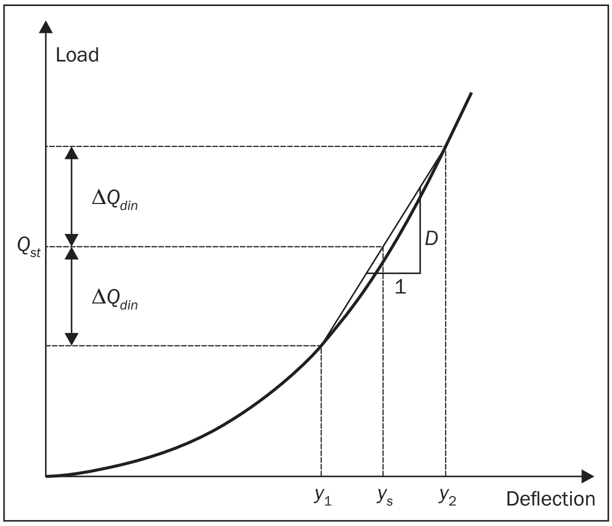

Figure 4 – Linearized track stiffness for the corresponding load range [13]

2.2 Dynamic stiffness

The term track stiffness, presented in the previous chapter, refers to static stiffness, i.e. stiffness under static load. However, it is necessary to consider the stiffness under dynamic load as well.

Except for the load value, track stiffness also depends on the excitation frequency (f) and thus a frequency-related definition of stiffness is necessary. The term receptance or dynamic flexibility (α) is introduced. It actually presents inverse dynamic stiffness and it is measured on the track under dynamic load:

$$\alpha(f) = \frac{y(f)}{Q(f)}. \quad (4)$$

Figure 5 shows dependence of receptance on excitation frequency, derived based on the measurements on the Swedish Railways [11]. Resonance is noted in the interval 5-7Hz and the track is dynamically "soft" in that area. Anti-resonant effect occurs at the frequency of 9Hz, after which relatively regular receptance decrease with frequency increase is distinguished. In other words, dynamic stiffness increases with the increase in frequency.

Dynamic stiffness is especially important for dimensioning elastic elements on the track (rail pads, under sleeper pads, ballast mats). For using numerical values of dynamic stiffness in the dimensioning process it is of great concern to know the behaviour

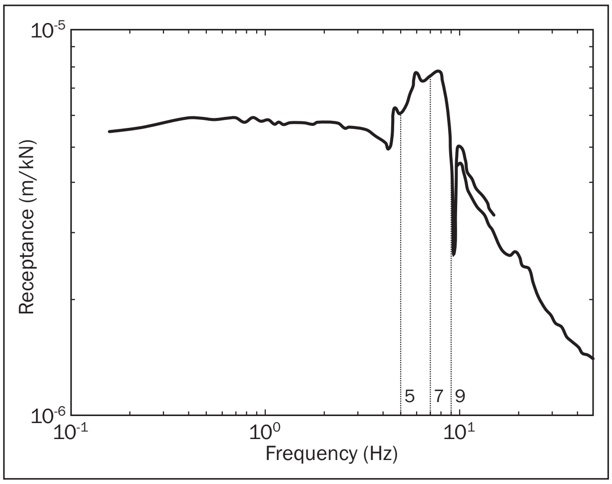

Figure 5 – Dependence of receptance on excitation frequency [11]

of this parameter, for which the following is important [14, 15]:

- nonlinearity of the load-deflection diagram,
- dependence on the frequency and temperature,
- stiffness increase due to aging and dependence on the installation method.

In all the experiments [14-17], lower static stiffness than dynamic stiffness was noted, where stiffness slightly increases with the increase of excitation frequency (*Figure 6*).

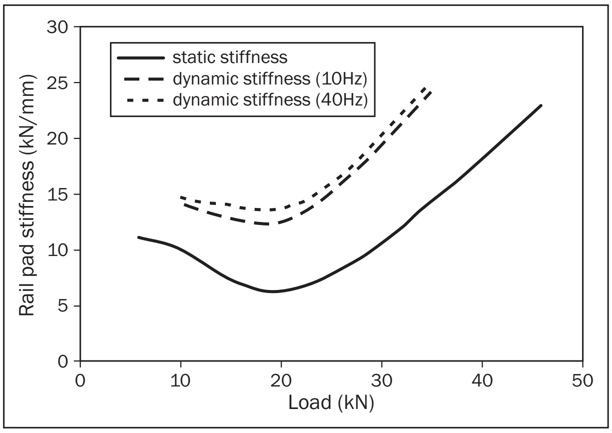

Figure 6 – Dynamic and static stiffness of the rail pad by Sylomer® [17]

3. INFLUENCE OF TRACK STIFFNESS ON TRACK GEOMETRY DETERIORATION

French author Prud'Homme was the first to point out the importance of track stiffness and its influence on vehicle dynamic loads. He suggested the following equation for calculating dynamic load [18]:

$$Q_{din} = 2\sqrt{\sigma^2(\Delta Q_S) + \sigma^2(\Delta Q_{NS})}, \quad (5)$$

where:

Q_{din} – dynamic wheel load (kN),

$\sigma(\Delta Q_S)$ – standard deviation of dynamic load due to sprung mass (kN), and

$\sigma(\Delta Q_{NS})$ – standard deviation of dynamic load due to unsprung mass (kN).

$$\sigma(\Delta Q_S) = \alpha Q_{st}, \quad (6)$$

$$\sigma(\Delta Q_{NS}) = bV\sqrt{m_n \cdot g \cdot D}, \quad (7)$$

where:

$\alpha \in [0.11, 0.16]$,

Q_{st} – static wheel load (kN).

V – vehicle speed (km/h),

b – parameter for the geometrical quality of track and wheels, $b \in [0.00042, 0.00084]$,

m_n – unsprung mass per wheel (t),

g – acceleration of gravity (m/s^2), and

D – track stiffness (kN/mm).

Since the standard deviation of dynamic load due to unsprung mass $\sigma(\Delta Q_{NS})$ is directly proportional to track stiffness (Equation 7), it is of practical interest to ensure the lowest possible stiffness value. Since dynamic load is one of the most important parameters that influence the track geometry deterioration [6, 19-25], the influence of track stiffness on the geometry deterioration is obvious.

One of the conclusions of the international research project EUROBALT II (European Research for an Optimized Ballasted Track), was that the low value of track stiffness directly influences the increase of ballast settlement [26]. This refers to the case when the low value of total track stiffness is consequence of insufficient compactness of ballast and platform. Increased settlement occurs as a logical consequence of increase of ballast deformation in conditions of insufficient stiffness.

Figure 7 shows the dependency of standard deviation of longitudinal level from the track stiffness, derived from the measurements within the mentioned project.

Therefore, the increase of compactness of the earthwork and trackbed contributes to minor settlements of these elements and thus to minor track settlements. However, an extreme increase of ballast compactness increases dynamic forces and load which is transmitted onto the earthwork. Platform compactness is also limited, so that dynamic load due to traffic would not crush the ballast stones and thus disturb the track geometry.

It is confirmed in many examples that the recommendation on increasing compactness of an earthwork and trackbed with the purpose of decreasing track settlement should be taken with reserve. When constructing the first high speed railways in Germany in 1991, the starting point was the presumption that the track geometry would be more tenable if better compactness of an earthwork and blanket layer was achieved and if the ballast was well compacted. Moreover, stiff rubber pads were used, whose spring constant is D_{gp} = 500kN/mm. In that way, track stiffness was increased manifold by comparison with the old and reconstructed lines. The decrease of elasticity of track and track foundation has lead to ballast crushing, appearance of so called "white stains" on the ballast surface and extreme track geometry deterioration.

Furthermore, at the end of 1990s the results of the research conducted in Germany indicated the importance of the vibration levels which are created under the traffic in the railway superstructure, especially in the ballast. Together with the stress in the ballast which is short-term activated, vibrations which are transmitted to the ballast are created under the wheel of the vehicle that runs at high speed. With the increase of the train speed, vibrations play a decisive role in the process of track geometry deterioration. In order to avoid quick deterioration of the ballast material and thus track geometry deterioration, maximum vibration speed in the ballast should not exceed 15 to 18mm/s [27]. Still, as shown in *Figure 8*, for the speeds of 250km/h the measured vibrations are almost 30mm/s. Also, it is shown that using pads of stiffness 20 to 60kN/mm can decrease the speed of vibrations in the ballast [28].

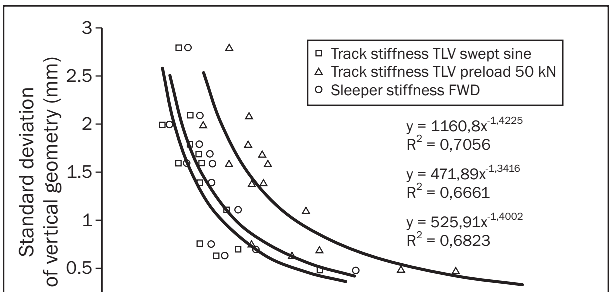

Figure 7 - Dependency of standard deviation of longitudinal level on track stiffness [26]

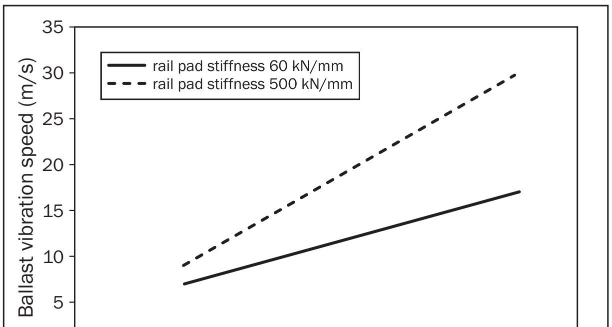

Figure 8 – Influence of rail pad stiffness on vibrations in the ballast (based on the measurements of ICE trains) [28]

4. OPTIMUM TRACK STIFFNESS

The necessity to find the optimum track stiffness is obvious. Too low a value would cause track settlement, with considerable stress increase in the rails. The value that exceeds the optimum would increase the dynamic load and thus accelerate track deterioration. The concept of optimal stiffness for the aspect of vertical track geometry deterioration is shown in *Figure 9*.

The following values of optimum track stiffness are quoted in the available literature:

- 80 –130kN/mm [10],
- 70 – 80kN/mm on high speed lines [29],
- 70 – 80kN/mm on freight-traffic lines [7].

The stated values must not be accepted as literal recommendations, since they are related to the specifics of the lines where the research was conducted.

Since total track stiffness depends on the stiffness of single construction elements, stiffness modifying can be achieved by installing the elastic elements.

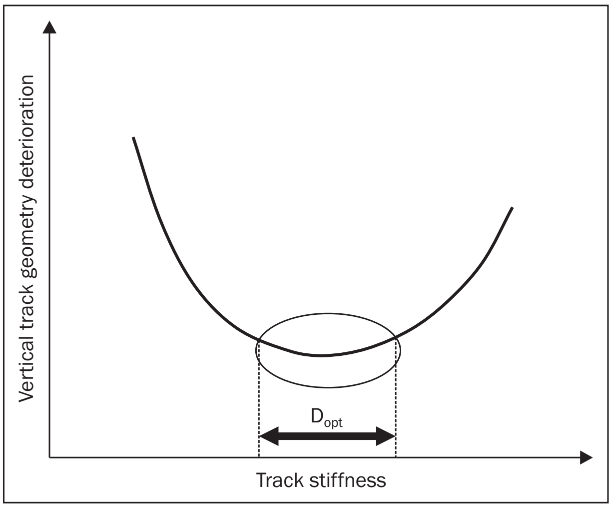

Figure 9 – Illustrative display of optimum track stiffness

Figure 10 shows the possibilities of installing the elastic elements and they relate to the following positions: elastic element beneath the rail foot (rail pads), elastic element beneath the concrete sleeper surface (under sleeper pads), and elastic mats beneath the ballast (ballast mats).

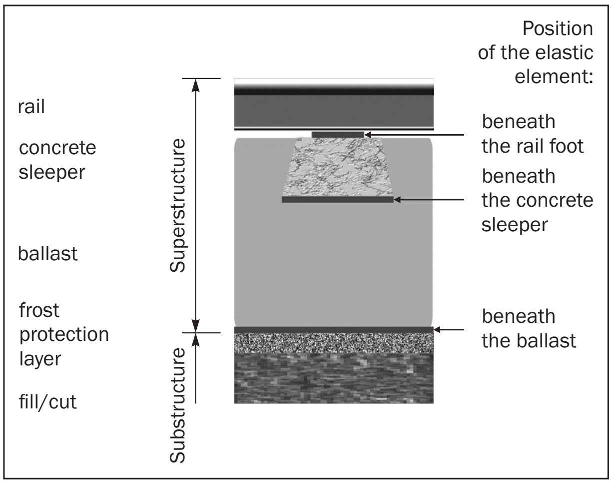

Figure 10 – Possible positions of elastic elements [24]

Elastic elements cannot be freely combined in the superstructure. For example, rail-fastening clip and pads of arbitrary stiffness cannot be combined, because pads which are too elastic under load can lead to loss of contact between the clip and the rail foot. Moreover, elasticity of rail foundation is being limited, so the tension stress in the rail foot, the stress in the rail head and rail deflection would not exceed their tolerable values. On the other hand, stiff elastic elements beneath the rail foot lead to load increase which is transferred to the lower parts of the construction, with the danger of exceeding the tolerable load of the lower layers.

Therefore, apart from determining the optimum total track stiffness, it is necessary while designing to determine optimum stiffness of every single superstructure and substructure element and to adjust them mutually.

5. SPATIALLY VARYING TRACK STIFFNESS

Spatially varying track stiffness is one of the basic causes of differential track settlement, which has primary influence on the track geometry deterioration [13, 19-23]. The basic causes of the occurrence of spatially varying track stiffness are the construction change of the superstructure and substructure along the line, variable ballast thickness, variable blanket layer thickness, characteristics of the material that the embankment was made of, moisture content, geological characteristics of the subsoil.

Uneven track stiffness along the track is the usual problem which has been explored within numerous research projects [28-35]. One of the conclusions of the so far conducted research is that the high degree of heterogeneity can be perceived even on the nearby sections.

Ballast cannot reduce the differences in track stiffness along the track. As consequence, uneven support of adjacent sleepers on the ballast occurs (*Figure 11*).

Numerical analysis that López Pita and Fonesco performed at the Technical University of Catalonia [29] clearly pointed that if two adjacent sections with considerably different track stiffness are being considered, the stress level on the ballast can be between 30 and 50% higher than the level corresponding to the hypothesis which assumes constant stiffness along the track (*Figure 12*). This additional stress accelerates the ballast deterioration process. That is why it is necessary to set the new reception quality criteria, which also considers track stiffness homogeneity together with the regulations related to

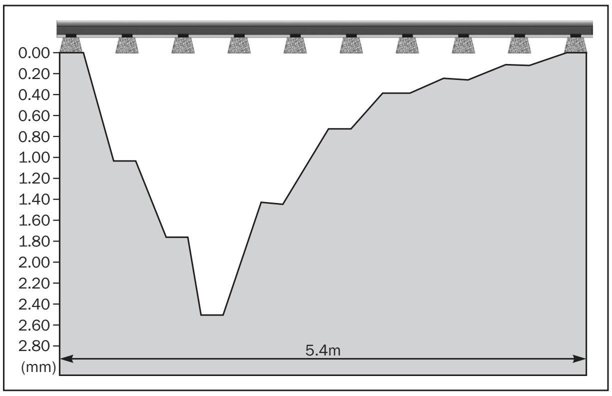

Figure 11 – Uneven support of sleepers on the ballast measured in October 1995 on the high speed line Hannover – Würzburg in Germany [36]

the quality of the track geometry, especially on high speed lines.

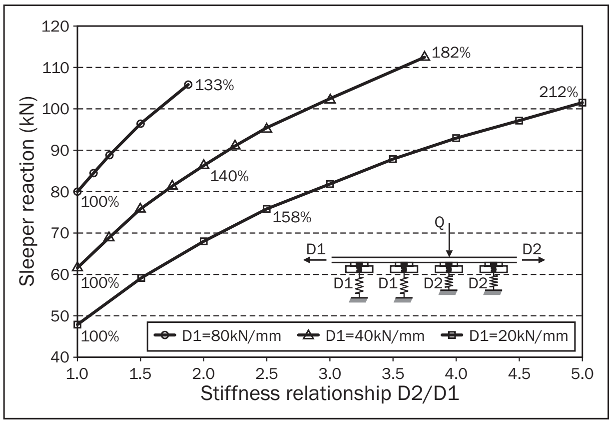

Figure 12 – Influence of uneven track stiffness on the reaction in the sleepers [29]

6. DISCUSSION

Considerations in this paper do not provide a solution for optimum track stiffness. This is still an open question for railways around the world. However, the current level of knowledge in this field gives the possibility of some practical applicability.

The realization of interoperability of the European railway network demands that every infrastructure manager has a maintenance plan for each conventional line for the infrastructure subsystem. Among other requirements, the maintenance plan is related to the track geometric quality and the limits on isolated defects. The existing EU legislation does not explicitly consider the optimum stiffness and does not include uniform stiffness along the track as criteria for acceptance of works. It is assumed that the track stiffness will be optimal if the other requirements of the trackbed construction, as defined in the UIC Code 719 [37], are also met. This code gives advice on the construction of earthworks for railway lines and considers also the special case of transitions to structures, in order to avoid abrupt changes in stiffness. Based on experiences, modern lines that are constructed in accordance with this code, demonstrate significantly lower maintenance requirement [10]. By far the more complex issue is that of modifying or improving suboptimal stiffness of the existing tracks in the Republic of Serbia.

Unfortunately, sporadic maintenance of the railway superstructure, often regardless of the state of the substructure, is applied in the Republic of Serbia. Errors in the vertical track geometry are corrected by tamping and adding the ballast material, without analyzing the causes of deterioration and the effectiveness of the applied measures. Such maintenance strategy, however, is a short-term solution that eventually leads to high maintenance costs. The condition of the substructure is generally poor on the Serbian Railways. Therefore, the superstructure maintenance management must include an analysis of the substructure. Data that indicate the state of all layers below the rails are obtained by measuring the track stiffness.

It is assumed that the track stiffness in the sections with high and stable geometric quality is close to the optimum. Continuous track stiffness measurements on the Serbian railway network and comparative analysis between the measured values and the supposed optimum track stiffness for the appropriate sections would provide valuable information for the maintenance management of the network.

7. CONCLUSION

Results of measurements of track geometry parameters offer data on sections where geometry deterioration is pronounced. However, the causes of such deterioration cannot be determined based on the measurement of geometry parameters. Knowing the causes of track geometry deterioration is of crucial importance for maintenance optimization. Track stiffness value data along the track help find the causes of deterioration in the majority of cases.

Some of the most significant factors that contribute to a decrease of track geometry deterioration and an extension of track and vehicle service life are mutual adjustment of stiffness of superstructure and substructure elements, determining optimum track stiffness and achieving homogenous stiffness along the track.

Research so far did not manage to solve the problem of quantification of the total track stiffness along the explored sections. Also, it is necessary to better understand the causes of stiffness non-homogeneity along the track. Although these causes are generally known, their influence is mostly qualitatively described. Quantification of the influence of the substructure layers moisture content on stiffness and on their general behaviour is particularly absent. One of the solutions would be determining methodology for measurement of track stiffness at cross sections above sleepers.

Although some railway managements practise track stiffness measuring, scientific and professional literature only points to the existence of a correlation between vertical track geometry deterioration and track stiffness without closer, practically applicable quantification of their relationship on conventional mixed traffic lines. Values stated in the literature must not be adopted as literal recommendations since they are related to specifics of the lines where the research was conducted.

Experimental research on the field in conditions specific for the Serbian Railways would contribute to consideration of the vertical track geometry deterioration process and defining the optimum track stiffness. This would create a basis for domestic law regulations and standards to be in line with directives and standards of the European Union in the area of maintenance. The objective of adjusting regulations with the EU regulations is defining a unique approach for the evaluation of track geometry quality and creation of the railway infrastructure maintenance plan.

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SAŽETAK

UTICAJ KRUTOSTI ŠINSKE PODLOGE NA PONAŠANJE KOLOSEKA POD VERTIKALNIM OPTEREĆENJEM

Analiza ponašanja koloseka pod vertikalnim opterećenjem tradicionalno se vrši na osnovu pretpostavke da se naponi i deformacije u osloncima šina mogu odrediti primenom Vinklerove hipoteze. Šina se posmatra kao greda na kontinualno elastičnoj podlozi. Osnovu hipoteze čini pretpostavka o proporcionalnosti između pritiska i ugiba. Međutim, iskustveno je poznato da se elementi koloseka i kolosečne podloge u realnom okruženju tokom ekspolatacije železničke pruge ne ponašaju ni linearno, ni potpuno elastično. Pored toga, postoji problem i sa neravnomernošću krutosti šinske podloge duž koloseka. U ovom radu je krutost šinske podloge analizirana sa aspekta njenog uticaja na kvalitet vertikalne geometrije koloseka. U radu se razmatra optimalna krutost. Optimalna krutost je uslovljena pojedinačnim krutostima svih elemenata konstrukcije gornjeg i donjeg stroja, kao i njihovom međusobnom usklađenošću.

KLJUČNE REČI

železničke pruge, kolosek, krutost šinske podloge, propadanje geometrije koloseka, održavanje

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