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INVENTORY MANAGEMENT MODELS IN THE SUPPLY SYSTEM OF SPARE PARTS FOR MILITARY VEHICLES

ABSTRACT

The term inventory implies various material means which are for a certain period of time excluded from the production process or transport with the aim of being used at a later point in time, as the need might arise.

The work analyses an example of the operation method of one system according to the model of rejecting unsatisfied demands, with the task of determining the optimal inventory volume and the ordering levels in order to minimise the annual costs.

KEY WORDS

operations research, inventory, inventory management, Williams formula

1. INTRODUCTION

In order to protect the economy and the society against the disturbance due to the lack of goods, methods and instruments should be found that insure regular supply to customers even at times of substantially increased consumption in relation to the production. Creating of inventories that would be controlled and managed so that they insure the autonomy of consumers regarding production over a certain period of time, and at the same time not representing great financial burden, is the only right way under the conditions of stochastic, that is, non-deterministic market environment

The inventory management theory has developed on the basis of production problems, and in particular on the problem of insuring it by the necessary material resources. In order to satisfy the stochastic demand, it is necessary to have inventories in the supply system. The problem of creating inventories has been noticed not only in the production but also in a number of other activities.

The combat power of every army depends on the structure, size and reliability of inventories. In logistic chains inventories represent an important element. Modern flexible production cannot be organised without inventories.

By the birth of civilisation and its transformation into the technological civilisation, human production capabilities have outgrown the regular needs so that hyperproduction of some goods occurs occasionally, resulting in market disturbances, price fall and collapse of industrial plants and whole groups. War, as special form of endangering civilisation has brought about the creation of armies that during their combat activities represent extremely big consumers of special types of goods that cannot be supplied by regular, not even by expanded production.

The general military practice is known, that three characteristic levels of inventories are determined, and these are usually called minimal, signal, and maximal inventories. Maximal inventories are often called also "war reserves" and these inventories are not to be consumed in peacetime operating conditions. When the current inventories reach the level of minimal "war reserves" inventories, then the system under peacetime conditions operates as if there was an inventory deficit. Compared to certain models, minimal inventories correspond to guaranteeing inventories, that is to the condition in stock inventories when the delivery of goods from the supplier is expected.

The developed mathematical models can be relatively easily applied in concrete operating practice of certain military units, overhaul support brigades, various shopfloors and other operating facilities using some kind of a storage system.

In the general sense, **inventories** are defined as types and amounts of goods that are stored at the times of reduced demand with the purpose of avoiding standstill in the operation of the system during the period of reduced supply or increase of demand beyond the regular capabilities of the means resources.

The term "inventory" itself covers a wider area and is more comprehensive than it would seem at first. Inventories may imply the status of goods in production or sales, workforce engaged in performing a certain task, amount of the insurance company capital, transport means capacity, company production capacity, etc. This means that the implementation of the inventory management theory is also not limited only to the area of production and storage operations.

The operation of military institutions can be divided into two seemingly completely different sections. Operation in peacetime period and operation in wartime period. However, war operations, as final task and purpose of the existence of all the military institutions diminish these distinctions. In applying the theory of inventory management at military institutions, either peacetime or wartime period, some terms should be explained, such as: "order", "delivery", "delivery period", "demand", "satisfied demand", "unsatisfied demand", "acceptance of unsatisfied demands", "rejection of unsatisfied requirements", "costs" etc.

Order is the synonym for the quantity of goods that are ordered from the external supplier or producer so as to replenish the inventory on stock.

Delivery represents the quantity of goods at the moment it arrives at the storage in order to replenish the inventory.

Generally some time has to pass between submitting the order to the producer and the delivery of goods to the inventory stock. This period is called the **delivery period** τ.

Demand is the reason for the existence of inventory. It is submitted by the inventory user, i.e. consumer. The demand mainly depends on the consumer and represents a random value. However, sometimes the demand is balanced, as when the consumer has a serial production. Then we speak of deterministic demand, that is about the deterministic management models.

Satisfied demand is a demand which has been fully satisfied.

Unsatisfied demand means that the demand has not been fully satisfied.

To accept an unsatisfied demand means accepting the obligation of full satisfaction after the delivery.

To reject an unsatisfied demand means no obligation of fully satisfying it after the delivery.

The dilemma regarding unsatisfied demands is whether:

- to fully satisfy the demand after the delivery, or
- to satisfy the demand only partly.

Satisfying the demand after the delivery is known as **acceptance of unsatisfied demands**, and only partly satisfaction of the demand is known as rejection of the unsatisfied demand.

Cost refers to a single unit of goods. Since inventory management is a complex process, we speak of the inventory management system.

Signal inventories represent the level of inventory when replenishment is ordered which in developed models corresponds to the level of ordering.

The following factors have significant influence on the increase of inventories:

1. Limitations of delivery, that cannot be consumed at once, when they arrive from the producer to the inventory stock.
2. Stochastic character:
   a) of the demands between two deliveries;
   b) of the delivery volume;
   c) of the time interval between two deliveries.
3. Assumed changes in market conditions:
   a) seasonal production;
   b) seasonal demand;
   c) expected increase in prices.

There is also a number of factors that require a reduction in inventories, such as:

1) Costs of physical storage and holding of inventory;
2) Unrealised income regarding frozen capital;
3) Qualitative and quantitative reduction of the value of the stored goods, etc.

Reduction of value of the goods depends proportionally on the total value of the goods on the market. **The inventory holding cost coefficient** I indicates this proportional relation. The following is true:

$$0 \leq I \leq 1$$

I is usually expressed in percentage.

The theory of inventory management should provide answers to two basic questions that arise in practice and which are:

1) At what inventory status should the order be issued for inventory replenishment?
2) What goods and what quantity are to be ordered?

The answer to these questions does not basically depend on the complexity of the supply system. In solving these problems, it is assumed also that the supply system cannot manage the demand, but the supply can be and has to be managed.

This article deals with an example of how a system operates according to the model of rejecting unsatisfied demands.

2. DETERMINISTIC MANAGEMENT MODEL WITHOUT INVENTORY DEFICIT

Inventory deficit is a status of inventory when the demand cannot be satisfied, although its satisfaction has been accepted.

The inventory management system without inventory deficit can be schematically represented (Figure 1).

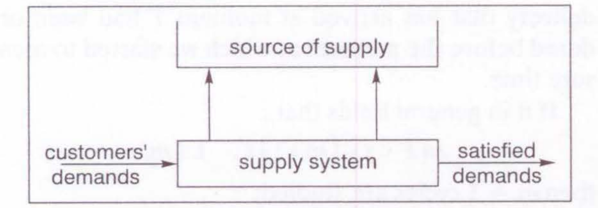


Figure 1 - Management system without inventory deficit

The mathematical model requires that the considered values be defined. The volumes of order, delivery, and demand are measured in units of goods, whereas all the time values are measured in parts of the year (1 day = 1/365 year).

The intensity of the annual demand is λ units of goods. It is assumed to be constant and known. The task consists in determining the moment and volume of issuing the order for replenishment. Let the delivery period τ (time between the moment of issuing the order to the moment of delivery of the ordered goods) be constant and not depending on the intensity of demand λ nor **quantity of one order** Q. It is assumed that by issuing an order always the same quantity of goods Q is ordered and the level of present inventory at the moment of delivery is always the same. It is assumed that only one type of goods is on stock. Since the demand is not random and the delivery period is constant, there cannot be inventory deficit in the system, nor can the customer demand be left unsatisfied regarding inventory deficit.

Let us denote:

$$T=\frac{Q}{\lambda}.$$

T is the interval between two subsequent orders or between two subsequent deliveries, that is the duration time of one cycle in the system.

The system behaviour is observed in an arbitrary time interval of l duration.

Let v be the biggest integer, less or equal l/T. Then v is the number of cycles in the time period of l length. The number of orders given in that period of time is v or v+1 and it equals $\frac{l}{T}+\varepsilon$, where $-1<\varepsilon<1$, that is $\frac{\lambda l}{Q}+\varepsilon$.

If C is the price of a unit of goods on stock and A is the price of issuing an order, then the costs of issuing orders over the considered period of l duration are determined by:

$$\left(\frac{\lambda l}{Q}+\varepsilon\right)A$$

and the value of the ordered goods is:

$$\left(\frac{\lambda l}{Q}+\varepsilon\right)QC.$$

The moment of delivery is the moment when the goods arrive to the inventory storehouse.

If the current inventories at the moment of delivery are denoted with s, then immediately upon delivery they equal $Q+S$.

After a period of time of t duration has passed from the moment of delivery, the inventory status amounts to:

$$Z(t)=Q+s-\lambda t.$$

If I is the inventory holding cost coefficient, then the inventory holding costs in one cycle are as follows:

$$\begin{aligned} &IC\int_0^T(Q+s-\lambda t)\,dt=\\ &=IC\left(QT+sT-\frac{\lambda T^2}{2}\right)=\\ &=ICT\left(Q+s-\frac{\lambda T}{2}\right)=\\ &=ICT\left(\frac{Q}{2}+s\right)\end{aligned}$$

If the number of cycles over the period of time of l duration equals

$$v=\frac{l}{T}-\hat{\varepsilon}$$

where $0\leq\hat{\varepsilon}<1$, then inventory holding costs over the whole period of l duration are determined by

$$ICT\left(\frac{Q}{2}+s\right)\left(\frac{l}{T}-\hat{\varepsilon}\right)+\eta,$$

where η denotes inventory holding costs in the part of the cycle of l - v T duration. Since $\hat{\varepsilon}<1$, also η is less than inventory holding costs over one cycle.

The total variable costs for the whole period of l duration are composed by the sum of the values of the ordered goods for the considered period of time, costs of issuing the orders for the considered period, inventory holding costs during the considered period and amount to:

$$\begin{aligned}&\left(\frac{\lambda l}{Q}+\varepsilon\right)QC+\left(\frac{\lambda l}{Q}+\varepsilon\right)A+\\ &+ICT\left(\frac{l}{T}-\hat{\varepsilon}\right)\left(\frac{Q}{2}+s\right)+\eta\end{aligned}$$

Average costs γ_l for the period of l duration are the total costs divided by l:

$$\gamma_l = \lambda C + \frac{\varepsilon Q C}{l} + \frac{\lambda A}{Q} + \frac{\varepsilon A}{l} + I C\left(\frac{Q}{2}+s\right) - $$
$$- \frac{\hat{\varepsilon}}{l} I C T\left(\frac{Q}{2}+s\right) + \frac{\eta}{l}.$$

By increasing the duration l of the arbitrary time interval, the share of the inventory holding costs for the time within the time interval, and outside the cycle will be shorter and shorter compared to the inventory holding costs within time intervals that belong to the cycles. Therefore, average annual costs γ are calculated as:

$$\gamma = \lim_{l \to \infty} \gamma_l.$$

The following is obtained:

$$\gamma = \lambda C + \frac{\lambda A}{Q} + I C\left(\frac{Q}{2}+s\right)$$

If dependence γ is considered only on variable s, it is obvious that γ is the least when $s = 0$. The task is to determine the level of inventory at which an order has to be issued, so that at the moment of delivery s = 0. This level in inventories depends on the delivery period τ, and is called **signal demand** r.

Generally, the delivery period τ can be less or greater than the cycle duration time T.

Let $\tau < T$. At the moment t the inventory is considered, set by:

$$Z(t) = Q + s - \lambda t.$$

If $s = 0$, Figure 1 represents a function

$$Z(t) = Q - \lambda t.$$

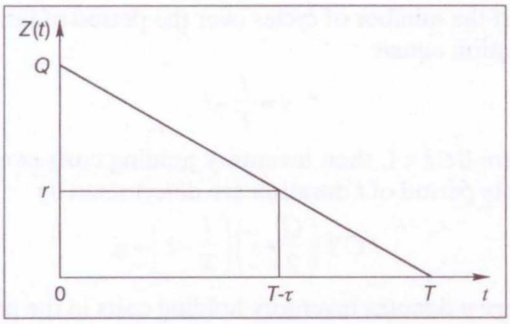


Figure 1

Signal demand r equals:

$$r = Z(T-\tau)$$

It follows:

$$r = Q - \lambda(T-\tau) = \lambda \tau.$$

Let $\tau > T$.

If $T < \tau < 2T$ two cycles are considered (Fig.2).

Now, it is:

$$r = Z(2T-\tau) = Q - \lambda(2T-\tau) =$$
$$= Q - 2T\lambda + \lambda\tau = \lambda\tau - Q.$$

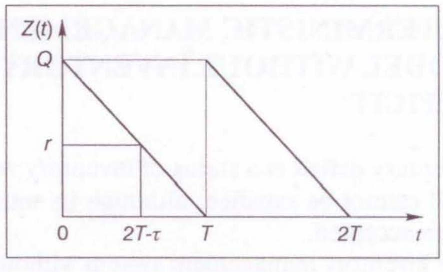


Figure 2

Note. $[0, T]$ represents one isolated interval. The delivery that has arrived at moment T had been ordered before the moment at which we started to measure time.

If it in general holds that :

$$mT < \tau < (m+1)T, \quad 1 \le m,$$

then $m + 1$ cycles are studied:

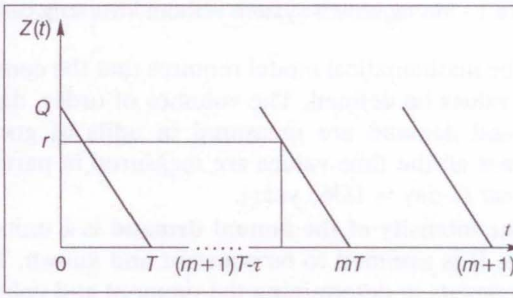


Figure 3

It follows that:

$$r = Z[(m+1)T-\tau] = Q - \lambda[(m+1)T-\tau] = \lambda\tau - mQ.$$

From the inequality:

$$mT < \tau < (m+1)T$$

follows:

$$m < \frac{\tau}{T} < m+1,$$

so that m is the biggest integer that is less than τ/T.

If $\mu = \lambda\tau$ denotes the volume of demand during the delivery period, then signal demands equal:

$$r = \mu - mQ$$

Inventory management means that every time when the current inventory in the system reaches the level r, an order of quantity Q of units of goods is issued.

Average annual costs γ depend only on variable $Q(s=0)$, so that:

$$\gamma = \lambda C + \frac{\lambda A}{Q} + \frac{I C Q}{2}$$

If it is assumed that Q is a continuous variable, the optimal value $Q_{opt} = Q^*, \quad 0 < Q^* < \infty$, must satisfy the equation:

$$\frac{d\gamma}{dQ} = 0.$$

It follows:

$$-\frac{\lambda A}{Q^2}+\frac{IC}{2}=0$$

By solving the above equation the Williams formula is obtained:

$$Q^* = Q_W = \sqrt{\frac{2\lambda A}{IC}}.$$

A deterministic model of inventory management has been obtained free of inventory deficit. It was assumed that the demand and ordering quantities were continuous values. If in practice only a whole number of goods units is ordered then Q_W is rounded to the closer integer or to the greater integer.

3. MANAGEMENT IN RANDOM DEMAND

The mathematical management model in case of **random demand** is more complex. There may occur demand deficit since demand is a random variable.

If the delivery period is also a random variable, then the inventory at the moment of delivery cannot be accurately determined, since in that case this current inventory is also a random variable. Therefore, **pure** and **fictitious** inventories are introduced. **Pure** inventory equals the current inventory reduced by the volume of received and accepted orders, and **fictitious** inventory equals pure inventory increased by the volume of the order. **Expected pure inventory** in case of accepting unsatisfied demands and **expected current inventory** in rejecting unsatisfied demands are called **guaranteed inventory**. They are designated by s.

In the system which includes accepting of unsatisfied demands the value of s can be positive, negative, or it can equal zero, and in the system which includes rejecting of unsatisfied demands the value of s can only be non-negative.

Since from the moment of issuing an order to the moment of delivery some time has to pass $\tau \geq 0$, the existence of positive guaranteed inventory $(s \geq 0)$ is justified, at the moment of delivery (not insisting on $s = 0$). Due to the stochastic character of the demand, during the time interval τ the inventory deficit may (but does not have to) occur.

The annual number of cycles due to rejection of unsatisfied demands is not λ/Q, but is rather less and amounts to:

$$\frac{\lambda}{Q+\lambda \overline{T}},$$

where $\overline{T}$ is the average time during the cycle when there are no stocks in the system.. In practice $\overline{T}$ is a very short part of the cycle duration. In analysis $\overline{T}$ is usually neglected since it would cause a number of problems. Thus, this model as well, takes the number of cycles per year as equal to λ/Q.

It is assumed that:

1. Price C of the unit of goods does not depend on the order quantity Q.
2. There may be only one unfulfilled order in the system.
3. The costs of the information processing system do not depend on Q and r.
4. Signal inventory r is positive.

Losses as result of rejecting the unsatisfied demands affect the income. Therefore, we introduce **fixed cost of rejecting one unsatisfied demand** Π.

The level of current inventory at the moment of delivery equals guaranteed inventory s, and immediately upon delivery it is $Q + s$. If it is assumed that the level of inventory within a cycle changes from $Q + s$ to s, then the average of the current inventory equals:

$$\frac{Q}{2}+s.$$

The average inventory holding costs are then:

$$IC\left(\frac{Q}{2}+s\right).$$

Let the continuous random variable X denote the demand during delivery with the probability density function $h(x)$. If signal inventory equals r, then current inventory at the moment of delivery is determined by:

$$\varepsilon(x,r)=\begin{cases} r-x & \text{if } r \geq x \\ 0 & \text{if } r < x \end{cases}$$

Since $x>0$ is the expected value of guaranteed inventory at the moment of delivery:

$$\bar{s}(r)=\int_0^\infty \varepsilon(x,r)\cdot h(x)\,dx=\int_0^r (r-x)\cdot h(x)\,dx$$

because:

$$\int_r^\infty \varepsilon(x,r)\cdot h(x)\,dx=\int_r^r 0\,h(x)\,dx=0.$$

From the equation:

$$\int_0^\infty (r-x)h(x)\,dx=\int_0^r (r-x)h(x)\,dx+\int_r^\infty (r-x)h(x)\,dx$$

it follows that:

$$s=\int_0^\infty r\,h(x)\,dx-\int_0^\infty x\,h(x)\,dx-\int_r^\infty r\,h(x)\,dx-\int_r^\infty x\,h(x)\,dx$$

Mathematical expectation μ of the random variable X which is assigned positive values, equals:

$$\mu=\int_0^\infty x\,h(x)\,dx.$$

Let:

$$H(x)=\int_x^\infty h(t)\,dt=1-\int_0^x h(t)\,dt.$$

It follows:

$$s=r-\mu-r\,H(r)+\int_r^{\infty} x\,h(x)\,\mathrm{d}x.$$

By inserting the obtained s in the expression for the average inventory holding costs within one time interval T (cycle), the following is obtained:

$$I\,C\left(\frac{Q}{2}+s\right)=I\,C\left(\frac{Q}{2}+r-\mu\right)+I\,C\left[\int_r^{\infty} x\,h(x)\,\mathrm{d}x-r\,H(r)\right]$$

The costs due to the rejection of unsatisfied demands need to be calculated now.

Let $\eta(x,r)$ denote the number of rejected unsatisfied demands. It follows then:

$$\eta(x,r)=\begin{cases} r-x, & \text{if } x\geq r \\ 0, & \text{if } x<r \end{cases}$$

The number of rejected unsatisfied demands, according to the above formula depends also on the size of demand and on the status of signal inventory. The size of demand cannot be influenced, but the status of signal inventory can.

If we assume that the demand distribution is known, the **expected number of rejected unsatisfied demands** $\bar{\eta}$ within one time interval can be calculated as a value that will depend only on r as parameter that can be influenced.

$$\bar{\eta}(r)=\int_0^{\infty}\eta(x,r)\,h(x)\,\mathrm{d}x=\int_0^{r}0\,h(x)\,\mathrm{d}x+\int_r^{\infty}(x-r)\,h(x)\,\mathrm{d}x=$$

$$=\int_r^{\infty} x\,h(x)\,\mathrm{d}x-r\,H(r)$$

Because of the assumption that the cost of rejecting one unsatisfied demand is fixed and equals Π, the average costs caused by the rejection of unsatisfied demands in one cycle equal:

$$\Pi\cdot\bar{\eta}(r)=\Pi\left[\int_r^{\infty} x\,h(x)\,\mathrm{d}x-r\,H(r)\right].$$

Annually this amounts to:

$$\frac{\lambda}{Q}\Pi\cdot\bar{\eta}(r)=\frac{\Pi\,\lambda}{Q}\cdot\left[\int_r^{\infty} x\,h(x)\,\mathrm{d}x-r\,H(r)\right].$$

The total average annual costs including costs of orders, inventory holding costs and costs due to the rejection of unsatisfied demands amount to:

$$\gamma=\frac{\lambda\,A}{Q}+I\,C\left(\frac{Q}{2}+r-\mu\right)+$$

$$+\left(I\,C+\frac{\Pi\,\lambda}{Q}\right)\cdot\left[\int_r^{\infty} x\,h(x)\,\mathrm{d}x-r\,H(r)\right] \qquad (1)$$

Optimal values Q^* and r^* of Q and r need to be determined. They minimise the costs γ. If:

$$0<Q<\infty \quad \text{and} \quad 0<r<\infty$$

then Q^* and r^* have to be solutions of the system of equations:

$$\frac{\partial\gamma}{\partial Q}=0$$

$$\frac{\partial\gamma}{\partial r}=0$$

By inserting:

$$\bar{\eta}(r)=\int_r^{\infty} x\,h(x)\,\mathrm{d}x-r\,H(r) \qquad (2)$$

into the equation

$$\frac{\partial\gamma}{\partial Q}=0$$

the following is obtained:

$$-\frac{\lambda\,A}{Q^2}+\frac{I\,C}{2}-\frac{\Pi\,\lambda}{Q^2}\cdot\bar{\eta}(r)=0.$$

It follows:

$$\frac{I\,C}{2}=\frac{\lambda}{Q^2}\left(A+\Pi\,\bar{\eta}(r)\right)$$

and

$$Q^2=\frac{2\lambda}{IC}\left(A+\Pi\,\bar{\eta}(r)\right)$$

that is

$$Q=Q^*=\sqrt{\frac{2\lambda}{IC}\left(A+\Pi\,\bar{\eta}(r)\right)}.$$

Since:

$$\int_r^{\infty} x\,h(x)\,\mathrm{d}x=\mu-\int_0^{r} x\,h(x)\,\mathrm{d}x$$

it follows that:

$$\frac{\mathrm{d}}{\mathrm{d}r}\int_r^{\infty} x\,h(x)\,\mathrm{d}x=-\frac{\mathrm{d}}{\mathrm{d}r}\int_0^{r} x\,h(x)\,\mathrm{d}x=-r\,h(r)$$

Now:

$$\frac{\partial\gamma}{\partial r}=I\,C+\left(I\,C+\frac{\Pi\,\lambda}{Q}\right)\left[\frac{\mathrm{d}}{\mathrm{d}r}\int_r^{\infty} x\,h(x)\,\mathrm{d}x-\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,H(r)\right)\right]=$$

$$=I\,C+\left(I\,C+\frac{\Pi\,\lambda}{Q}\right)\left(-r\,h(r)-H(r)-r\,H'(r)\right)$$

The function:

$$H(r)=\int_r^{\infty} h(x)\,\mathrm{d}x=1-\int_0^{r} h(x)\,\mathrm{d}x$$

so that

$$H'(r)=h(r)$$

The equation

$$\frac{\partial\gamma}{\partial r}=0$$

now reads:

$$I\,C-\left(I\,C+\frac{\Pi\,\lambda}{Q}\right)H(r)=0.$$

It is then

$$H(r)=H(r^*)=\frac{QIC}{\Pi\lambda+QIC}.$$

If it is assumed that the demand X has normal distribution $X \approx N(\mu, \sigma)$, with μ denoting mathematical expectation, and σ standard deviation of the random variable X, then its probability density function is:

$$h(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},$$

and distribution function:

$$F(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}\,dt.$$

Using the standard normal variable with $\mu=0$, $\sigma=1$, with the probability density function

$$\varphi(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$$

and distribution function

$$\Phi(z)=\int_{-\infty}^{z}\varphi(t)\,dt$$

it is

$$h(x)=\frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right),$$

$$F(x)=\Phi\left(\frac{x-\mu}{\sigma}\right).$$

Now

$$H(a)=\frac{1}{\sigma\sqrt{2\pi}}\int_{a}^{\infty}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\,dx=1-F(a)$$

that is,

$$H(a)=1-\Phi\left(\frac{a-\mu}{\sigma}\right). \tag{3}$$

Assuming normal distribution for the demand, the total average annual costs (1) and optimal values Q^*, r^* that minimise the costs are looked for.

The following is calculated:

$$\int_{r}^{\infty}x\,h(x)\,dx=\frac{1}{\sigma\sqrt{2\pi}}\int_{r}^{\infty}x\,e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\,dx.$$

By substituting $\frac{x-\mu}{\sigma}=t$, the integral is transformed into:

$$\int_{r}^{\infty}x\,h(x)\,dx=\frac{1}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^{\infty}(t\sigma+\mu)e^{-\frac{1}{2}t^2}\,dt=$$

$$=\frac{\mu}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^{\infty}e^{-\frac{1}{2}t^2}\,dt+\frac{\sigma}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^{\infty}t\,e^{-\frac{1}{2}t^2}\,dt$$

Since

$$\frac{1}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^{\infty}e^{-\frac{1}{2}t^2}\,dt=1-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}}e^{-\frac{1}{2}t^2}\,dt=$$

$$=1-\Phi\left(\frac{r-\mu}{\sigma}\right)$$

and

$$\int t\,e^{-\frac{1}{2}t^2}\,dt=-e^{\frac{1}{2}t^2}$$

the following is obtained:

$$\int_{r}^{\infty}x\,h(x)\,dx=$$

$$=\mu\left(1-\Phi\left(\frac{r-\mu}{\sigma}\right)\right)+\frac{\sigma}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}=$$

$$=\mu\left(1-\Phi\left(\frac{r-\mu}{\sigma}\right)\right)+\sigma\varphi\left(\frac{r-\mu}{\sigma}\right)$$

Furthermore:

$$r\,h(r)=r\left(1-\Phi\left(\frac{r-\mu}{\sigma}\right)\right). \tag{5}$$

By inserting (4) and (5) into (2) the following is obtained:

$$\bar{\eta}(r)=(\mu-r)\left(1-\Phi\left(\frac{r-\mu}{\sigma}\right)\right)+\sigma\varphi\left(\frac{r-\mu}{\sigma}\right)$$

The total average annual costs are obtained by inserting (4) and (5) into (1) and amount to:

$$\gamma=\frac{\lambda A}{Q}+IC\left(\frac{Q}{2}+r-\mu\right)+\left(IC+\frac{\Pi\lambda}{Q}\right)\cdot\left[(\mu-r)\Phi\left(\frac{r-\mu}{\sigma}\right)+\sigma\varphi\left(\frac{r-\mu}{\sigma}\right)\right]$$

The minimum costs for:

$$Q^*=\sqrt{\frac{2\lambda(A+\Pi\bar{\eta}(r))}{IC}}$$

and

$$H(r^*)=\frac{QIC}{\Pi\lambda+QIC}.$$

4. NUMERICAL EXAMPLE

The operation of one system is considered, according to the model of rejecting the unsatisfied demands. It is assumed that one type of goods is held as inventory. The following is known:

- annual demand for the goods;
- λ = 3200 items;

- price of the unit of goods;
- C = 50 monetary units;
- price of issuing one order
- A = 500 monetary units;
- inventory holding cost coefficient;
- I = 0,1 ;
- costs of rejecting unsatisfied demands;
- Π = 5000 monetary units;
- demand during delivery has normal distribution;
- mathematical expectation;
- μ = 600 items,
  standard deviation;
  σ = 50 items.

The task is to determine the optimal volume of ordering and the level of ordering, so as to keep the annual costs minimal.

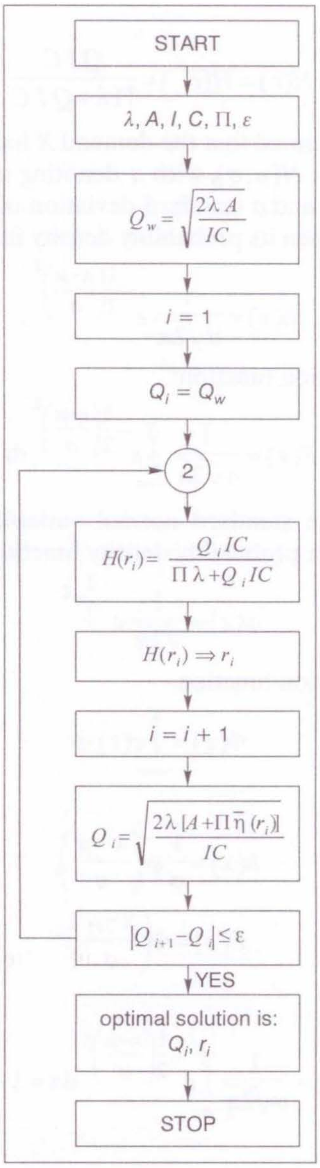
Optimal values of the ordering quantity Q^* and level of ordering r^* are determined by an iterative procedure. The initial value for the ordering quantity is determined by means of the Williams formula:

$$Q_1 = Q_W = \sqrt{\frac{2\lambda A}{IC}}.$$

After short calculation we obtain:

$$Q_1 = Q_W = \sqrt{\frac{2 \cdot 3200 \cdot 500}{0.1 \cdot 50}} = 800.$$

Optimal values Q^*, r^* are obtained in the iterative manner according to the flowchart.



5. THE FLOWCHART FOR NUMERICAL DETERMINATION OF Q^* AND r^* IN THE MODEL OF REJECTING UNSATISFIED DEMANDS

r_1 is found from the equation:

$$H(r) = 1 - \Phi\left(\frac{r_1 - 600}{50}\right)$$

therefore:

$$1 - \Phi\left(\frac{r_1 - 600}{50}\right) = \frac{Q_1 I C}{\Pi \lambda + Q_1 I C} =$$

$$\frac{800 \cdot 0.1 \cdot 50}{5000 \cdot 3200 + 800 \cdot 0.1 \cdot 50} = 0.0002499$$

The normal distribution tables show that:

$$\frac{r_1 - 600}{50} = 3.48$$

that is:

$$r_1 = 744$$

In order to calculate the value Q_2, one should calculate $\bar{\eta}(r_1)$:

$$\bar{\eta}(r_1) = (\mu - r_1)\left(1 - \Phi\left(\frac{r_1 - \mu}{\sigma}\right)\right) + \sigma\varphi\left(\frac{r_1 - \mu}{\sigma}\right) =$$

$$= -174 \cdot 0.0002499 + 50 \cdot 0.00094 = 0.00352$$

Therefore;

$$Q_2 = \sqrt{\frac{2\lambda(A + \Pi\bar{\eta}(r))}{IC}} =$$

$$= \sqrt{\frac{3 \cdot 3200 \cdot (500 + 5000 \cdot 0.00352)}{0.1 \cdot 50}} = 814$$

The value r_2 is obtained:

$$\Phi\left(\frac{r_2 - \mu}{\sigma}\right) =$$

$$= \frac{814 \cdot 0.1 \cdot 50}{5000 \cdot 3200 + 814 \cdot 0.1 \cdot 50} = 0.0002543$$

that is:

$$\frac{r_2 - 600}{50} = 3.476$$

and finally:

$$r_2 = 773{,}8.$$

For calculating the value Q_3, $\overline{\eta}(r_2)$ is calculated:

$$\overline{\eta}(r_2) = -173.8 \cdot 0.0002543 + 50 \cdot 0.000965 = 0.00413$$

and then:

$$Q_3 = \sqrt{\frac{2 \cdot 3200 \cdot (500 + 50000 \cdot 0.00413)}{0.1 \cdot 50}} = 815.3$$

Then the value r_3 is calculated:

$$1 - \Phi\left(\frac{r_3 - 600}{50}\right) =$$

$$= \frac{815.3 \cdot 0.1 \cdot 50}{5000 \cdot 3200 + 815.3 \cdot 0.1 \cdot 50} = 0.0002547$$

$$\frac{r_3 - 600}{50} = 3.474$$

that is,

$$r_3 = 773.7.$$

Since between r_3 and r_2, that is between Q_3 and Q_2, there are no significant differences, the iterative procedure can be completed and the following assumed $Q^* = 815$ pcs. or $r^* = 773$ pcs. The guaranteed inventory is $S^* = r^* - \mu = 173$ pcs. The average duration of a cycle is 93 days, and during a year 3.9 orders are submitted.

The average annual costs are 4868 monetary units. If it were a determined model with the same parameters then the average annual costs would be only 4000 monetary units.

Let us assume that the costs of rejecting an unsatisfied demand are only 100 monetary units, instead of the previous 5000. Then $Q^* = 817$ and $r^* = 712$ pcs., and $\gamma = 5388$ monetary units. This means that due to the changes in costs of rejecting an unsatisfied demand of 5000 to 100 monetary units, the value of Q^* has changed only by 2 pcs., and the value r^* by 61 pcs., and the costs have changed by 520 monetary units or by 10.7% (and the value π has changed by 50 times). This means that the costs, order quantity and ordering level are relatively insensitive to changes in the rejection costs (or acceptance) of unsatisfied demand and do not have to be expressed absolutely accurately.

Let us assume now that the demand for delivery period of 1000 pieces of goods, and that all the other parameters have remained the same as for the first case. It is then $Q^* = 975$ pcs., $r^* = 1137$ pcs., and $\gamma = 5573$ monetary units. This shows at the same time that the increase of expected demand during the delivery period from 600 to 1000 pieces, increased the level of order by 160 pcs., level of ordering by 364 pcs., and costs by 705 monetary units, that is, that the behaviour of the system is relatively sensitive to changes in the expected demand during the delivery period.

In the above mentioned examples the costs of rejecting unsatisfied demands are a hundred times greater than the price of the product, and this can be justified by a very high risk when during the war, for example, the unsatisfied demand is rejected.

In case the cost of rejecting an unsatisfied demand is the same as the price of the product, after the above mentioned iterative procedure, with other conditions from the beginning, we obtain:

$Q^* = 818$ pcs.,

$r^* = 698$ pcs.,

It may be noted that the demand during delivery period is high compared to the annual demand. Let us consider a case where this is not so:

$\lambda = 32\,000$ pcs

$C = 50$ mon. units

$I = 10\%$

$\Pi = 50$ mon. units

$A = 500$ mon. units

$\mu = 600$ pcs

$\sigma = 50$ pcs

In that case, after several iterations one obtains:

$Q^* = 2547$ pcs.,

$r^* = 721$ pcs.,

which shows that in this case relatively more goods have been ordered than the expected demand.

Let us consider now a case of still higher annual demand, high costs of issuing an order, low costs of rejecting unsatisfied demand, low inventory holding coefficient cost and low value of random demand:

$\lambda = 320\,000$ pcs

$C = 50$ mon. units

$I = 1\%$

$\Pi = 10$ mon. units

$A = 5000$ mon. units

$\mu = 60$ pcs

$\sigma = 5$ pcs

In this case we get:

$Q^* = 80\,004$ pcs.,

$r^* = 82$ pcs.,

This means that it is worth having goods as inventory.

6. CONCLUSION

Today we can say that inventory and inventory management represent an element of increasing the system reliability, which is a crucial condition for rendering the system economic. The average level of inventory has direct influence on the economic characteristic of the system since inventory "freezes" substantial capital thus reducing the amount of free capital on the market. The quantity and turnover of free capital on the market are directly related to the economical quality of the system.

Although army does not represent an economic category of the society in the sense of income, the inventory management in a military organisation has great significance due to the requirement of high combat readiness and limited material possibilities of a country.

The implementation of the inventory management theory, especially in military institutions, requires monitoring of the status and movements of the system, and the latest data have to be used for decision-making, and the respective parameters changed as needed. Although experiments in military practice are very expensive or impossible, still various exercises provide the most necessary parameters.

Since our army is very young, and has originated from the conditions of the Croatian War of Independence, now, in peace, there is bound to be interest in advanced methods of inventory management, especially in the supply system of spare parts for the motor vehicles within the military programme. The task of the inventory management theory is to enable in practice the implementation of scientific methods instead of intuitive inventory management.

SYMBOLS

Q – quantity of goods ordered by one order

r – quantity of goods on stock at the moment of issuing the order

C – price of the unit of goods

A – fixed costs of issuing one order

I – inventory holding cost coefficient

π – fixed costs of rejecting an unsatisfied demand

X – demand, random value

μ – mathematical expectation of demand

λ – annual demand intensity

T – duration time of one cycle

γ – average annual costs

τ – delivery period

SAŽETAK

MODELI UPRAVLJANJA ZALIHAMA U SUSTAVU OPSKRBE PRIČUVNIM DIJELOVIMA ZA VOJNA MOTORNA VOZILA

Pod pojmom zaliha podrazumijevamo različita materijalna sredstva koja su određeno vrijeme isključena iz procesa proizvodnje ili prometa, s ciljem da se kasnije, kada se ukaže potreba, iskoriste.

U radu je obrađen primjer rada jednog sustava po modelu odbijanja nezadovoljenih zahtjeva, sa zadatkom određivanja optimalnog obujma zaliha i razine naručivanja s ciljem da godišnji troškovi budu minimalni.

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