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DYNAMIC APPROACH IN GENERALISED MODELLING OF TRAFFIC PROCESS

1. NEED FOR GENERALISED FORMAL DESCRIPTION OF TRAFFIC PROCESS

Fundamental understanding of traffic and traffic technologies are closely related with deep and unified research of traffic system and its generic process. Traffic science cannot emerge from "embryonic state" if it hasn't unified and formalised treatment of traffic system behaviour relevant for different transportation subsystems (highway transportation, air transportation, railway, water transportation, postal or courier services, etc.) and telecommunication subsystems (telephony, data communications, ISDN, GSM, etc.). Long before traffic system knowledge can be synthesized, some small contributions can increase our fundamental understanding of traffic and traffic system behaviour.

During the years, a mathematical (quatitative) approach based on practical traffic observations and experiments, has been applied to various but specific traffic and transportation problems [1], [2]. Quantitative traffic models are based on stochastic and statistic theory, queuing theory, different elements of physical theories, some elements of economic theory, etc. However, generalised traffic (system) theory has not been developed although some "fundamental relations of traffic flow" and traffic equations for partial problems are well established in literature [3], [4]. The terms "system", "traffic system", "traffic activity", "process", etc., have been used for a long time but without precise explanation and formalisation of generic (fundamental) traits of traffic system.

A general system approach and methodology is necessary background for unified traffic system (process) description with precise formalisations. According to General systems theory, the fundamental traits of system studied by "experimental branches" of science are:

- the set of quantities,
- the resolution level,
- the time-invariant relations between quantities,
- the properties that determine the relations.

In the developing generalised traffic system description, it will be necessary to introduce a collection of concepts and definitions related to the fundamental traits. "Space-Time" specification must be considered for fundamental traffic quantities. The frequency and accuracy with which we record the chosen quantities is "resolution level". By observing the activity of the system, we must examine fundamental relations for specified time interval and conditions.

A minimal formal definition of traffic system can be one of the basic definitions associated with the fundamental systematic traits.

Definition 1. A traffic system TS is a triple (X,t,L) where:
$X = \{x_1, x_2, \ldots x_n\}$ is the set of external quantities,
t is time,
$L = \{X_1, X_2, \ldots X_n, T\}$ is the resolution level.

Traffic system is also defined by ([5]):

- system activity,
- real UC-structure,
- permanent behaviour,
- real ST-structure.

Whatever is added to the basic definitions must reflect some aspect or performance. Non-permanent traits (time-varying) must be shifted to the area of "secondary traits". The relationships which exist between permanent and non-permanent traits are investigated in several doctoral theses, but it is not a solved problem.

This paper is concerned only with dynamic approach as a part of generalised formal description of traffic system. The main thesis is that we can identify some generic traffic relations which can be relevant for different transportation and telecommunication systems. Classic traffic flow theory (for road traffic) and teletraffic theory, may provide good testing ground for our investigations. Traffic flow can be measured and the relevant parameters (quantities) can be identified at some resolution level.

We can start with relatively simple situations which can possibly then be expanded. In generalised model or "metamodel" (model of models), traffic network and entities are considered only in terms of their generic characteristics, rather than in terms of their specific technology or technical design. In general, traffic denotes the aggregate flow of traffic entities through a transport or telecommunication system. Traffic is associated with arriving (input) and outgoing (output) process which change the system state. Level (grade) of services is predetermined by system capacity, offered traffic load and traffic management.

Before the introducing concept of dynamic description, we must give summarised review of various type of traffic models and relevant cornerstones from classical traffic flow theory and teletraffic theory.

2. REVIEW OF TRAFFIC MODELS

Various types of models (or approaches) have been used in analyses and synthesis of traffic system or subsystems (see Fig. 1). System state descriptions are compared according to levels of accuracy with scale 1 to 5.

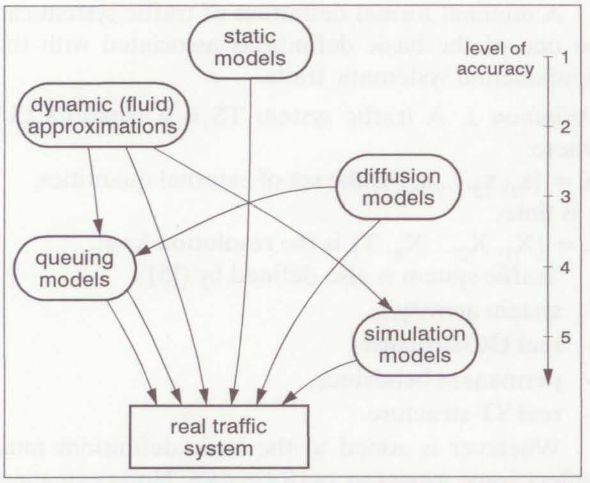

Figure 1 - Different type of models in traffic description

The most accurate description of traffic system behaviour specify the full particulars of all events occurring in the system. For the most real traffic systems and subsystems, such model cannot be constructed.

Stochastic behaviour (properties) of traffic system can be investigated with different type of models with different levels of accuracy in description. The most accurate analytical description is possible with queuing models if we now the probability distributions of all events. Less accurate description is possible with diffusion models which deal with averages and variances. Dynamic (flow) models can be used as an approximation of queuing model and as an independent description of a real traffic system. These models are based on hydrodynamic analogy or fluid approximations and they utilise time-dependent average quantities. Static (flow) models describe the traffic behaviour by means of long-term averages only (level of accuracy : 1).

Alternative to analytical or numerical description are simulation models. The simulation is useful not only for the case that the analytic solution is not obtained, but also in cases where numerical computation is difficult. The model in the simulation should abstract the essence of the system, and be as simple as possible to facilitate the simulation. Traffic simulation is classified as a discrete-event simulation which can be event-oriented, process-oriented and activity-oriented modelling. Detailed simulation models can describe system behaviour with high level of accuracy (level 5 in Fig. 1).

In systematic treatment of traffic process we must apply fundamental concepts of system and define a traffic system with one of the basic definitions [5]. The fundamental traits of generalised system are the set of quantities, resolution level, the time-invariant relations between quantities and the properties that determine the relations.

Observation 1. For generalised model (metamodel) of traffic system (according to Definition 1) we must recognise basic representative of traffic phenomena (quantity or measure values) which can be associated with main attributes of transportation and telecommunication system.

From the view of generalised traffic modelling, we introduce "instantaneous number of traffic entities in system" as a basic quantity. This quantity can be observed and measured with different space-time specifications. Elaborating of this approach and generalised traffic system concept is the subject of another paper. The basic idea is illustrated in Figures 2 and 3.

Observation 2. Traffic system can be formally described by arrivals (input process) and outlets (output process) which change the system state x(t).

The average number of traffic entities in the system during the period T from arrival of 1, A_1, to departure of the last, D_N, is given by:

$$\bar{x} = \frac{1}{D_N - A_1} \cdot \sum_{i=1}^{N} (D_i - A_i) \qquad (1)$$

The main attributes, i.e. the average time in the system (T_q) and the average waiting time (T_w) can be calculated by:

$$T_q = \frac{1}{N} \cdot \sum_{i=1}^{N} (D_i - A_i) \qquad (2)$$

$$T_w = \frac{1}{N} \cdot \sum_{i=1}^{N} (E_i - A_i) = \frac{1}{N} \cdot \sum_{i=1}^{N} (D_i - A_i - t_{s_i}) \qquad (3)$$

where:

A_i – moment of arrival of unit (entity) i,

D_i – moment of departure of unit (entity) i,

T_w – average waiting time,

T_q – average time in system (or delay time),

t_{s_i} – time required to serve unit i without waiting,

E_i – moment unit i enters server.

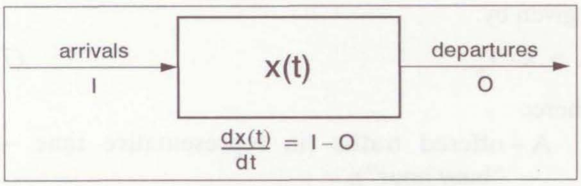

Figure 2 - Basic traffic description

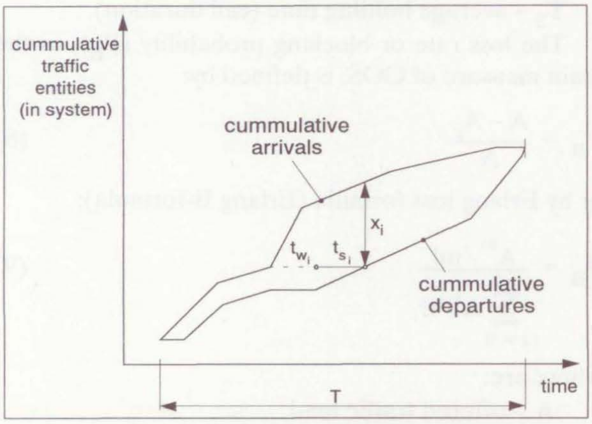

Figure 3 - Description of traffic system behaviour

Traditionally, part of traffic problem has been treated in models of stochastic service systems (birth-death process) and fluid approximations. In this context, the distinctive features and benefits of dynamic approach are:

- unified approach to analysis and synthesis of control in different types of systems (networks),
- analysis of stationary and non-stationary network behaviour,
- joint treatment of congestion phenomena and traffic control rules,
- detailed knowledge of time-dependent probabilities is not necessary.

3. RELEVANT CORNERSTONES IN CLASSICAL TRAFFIC FLOW THEORY AND TELETRAFFIC THEORY

"Classical" traffic flow theory has been focused on road traffic. Generalised model of a vehicular stream for the simple case of identically scheduled vehicles on an exclusive right of way is formulated and this model is extended to the various cases of highway traffic [1]. The behaviour of the traffic systems has been expressed with the set of "fundamental relations" between the variables: mean speed (v), flow or volume (q), and concentration or density (k).

Traffic equations and fundamental diagram of traffic are considered in several theoretical forms [2]. In traffic flow theory q-k relations have a central position, but for practical research speed-flow relations are more important. Theoretical fundamental relations based on time independent homogenous solutions of equations simulating the dynamics of traffic flow, are proposed by few authors [7].

For the purpose of this paper, we use a variant of phase-space description involving both the positions and the velocities of cars. We may assume that there exists at a given time (t) and a given point of speed (v) and road space (s), some distribution function f(s,v,t). The number of traffic entities (the numbers of cars dX) that at given time t are in the road segment $s+\Delta v$ is given by:

$$dX = f(s, v, t) \cdot ds \cdot dv \tag{4}$$

When the velocity distribution function is known, we may derive basic quantities, such as local concentration k(s,t):

$$k(s, t) = \int_0^\infty dv \cdot f(s, v, t) \tag{5}$$

and local flow q:

$$q = k \cdot \bar{v}(s, t) = \int_0^\infty dv \cdot v \cdot f(s, v, t) \tag{6}$$

To any uniform stretch of the road the flow $q = k \cdot \bar{v}$ is a function of the concentration k. It is a key (starting) point of the hydrodynamic theory of traffic. The relations between flow and concentration and between speed and concentration are illustrated with well known diagrams (Fig. 4). For low concentration the flow grows approximately linearly with the concentration; after that it begins to grow less rapidly; then it goes through a maximum, and finally falls to zero ("jam" concentration). In the case of highway traffic, drivers make their own decisions relating to the trade-off between safety and speed.

Observation 3. We can assume that traffic equations based on average values do not necessarily provide accurate estimates because:

- the traffic flow is not in a steady state,
- average flow rate and concentration are defined as simple mean values,
- equation of state is not linear.

Therefore, we need more dynamic description of traffic behaviour which can be a part of generalised traffic theory derived on systems principles.

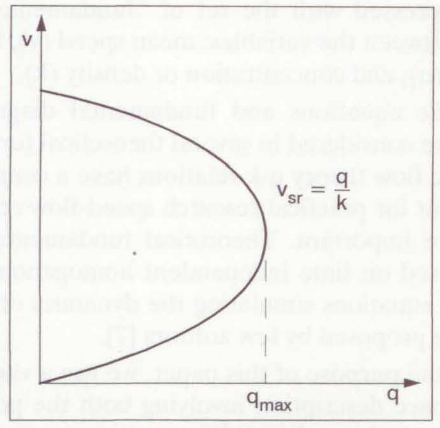

Figure 4a - Relationship between traffic flow and speed

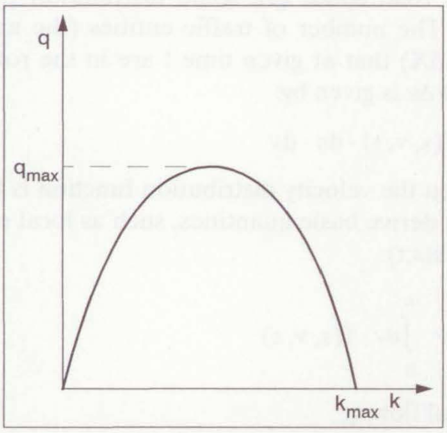

Figure 4b - Relationship between traffic flow and concentration

Teletraffic theory and performance evaluation of telecom networks and system, have a long tradition. The basic cornerstones are:

- Birth and Death analysis of loss and delay systems (Erlang 1917, Engset 1918)
- Traffic analysis of systems with general service times (Pollaczek 1930)
- Imbedded Markov Chain analysis (Kendall 1953)
- Queuing networks (Jackson 1954 - 1970, BCMP 1975)
- Queuing network algorithms (Buzen 1972, Kabayashi 1976)
- Equivalent Random Traffic Theory (Bretschneider, Wilkinson, Riordan 1955-1960)
- Simulation techniques
- Dynamic Flow Models (Filipiah 1988)
- Teletraffic analysis with Non-Markovian Models (Akimaru, Kawashima 1993)

Classical teletraffic models may be treated as static long-term average descriptions for stationary network behaviour ("statistic equilibrium"). Typical traffic engineering problem was how many shared resources must be provided to ensure adequate user performance (GOS) in "busy hour" (BH). Traffic demand for connections services is described by two components:

- arrival rate (how often users arrive or request resources),
- holding time (how long they use resources before releasing them).

Traffic intensity or offered load in stationary state is given by:

$$A = \lambda \cdot T_s \tag{7}$$

where:

A – offered traffic (in representative time → "busy hour"),

λ – arrival rate,

T_S – average holding time (call duration).

The loss rate or blocking probability (P_B), as the main measure of GOS, is defined by:

$$P_B = \frac{A - A_c}{A} \tag{8}$$

or by Erlang loss formula (Erlang B-formula):

$$P_B = \frac{A^m / m!}{\sum_{i=0}^{m} A^i / i!} \tag{9}$$

where are:

A – offered traffic load,

m – number of channels or lines (servers).

The carried load $A_C = A \cdot (1\text{-}P_B)$ is the expectation of the number of calls ($\bar{x}$) existing in the stationary state.

While telephone system operates as a loss system with "lost-call-cleared" (LCC) discipline, data networks and another delay subsystem include queues (line) and wait for service. The key measure of performance (GOS) is average delay time T_q. Delay is defined as time interval between the instant at which network terminal (station) seeks access to a transmission channel to transmit a message, and the instant that the network completes delivery of the message.

For Markovian delay system with Poisson input, exponential service time and infinite waiting room, the waiting probability in steady state is given by:

$$P(t_w > 0) = \sum_{r=m}^{\infty} P_r = \frac{a^m}{m!} \cdot \frac{m}{m-a} \cdot P_0 =$$

$$= \frac{\frac{a^m}{m!} \cdot \frac{m}{m-a}}{\sum_{r=0}^{m-1} \frac{a^r}{r!} + \frac{m}{m-a}} \tag{10}$$

where:

P_r – is a steady state probability that r entities (messages) present in the system, a – is offered traffic load,

m – is number of servers (channels),

P_0 – is the probability that system is empty.

The average waiting time (T_w) can be calculated from:

$$T_w = P(t_w > 0) \cdot \frac{T_s}{m-a} \qquad (11)$$

where T_s is average service (transmission) time and traffic load a is given by:

$$a = \lambda \cdot \frac{B}{C} \qquad (12)$$

where:

B – is average message length (in bits),

C – is line capacity or max throughput (bits per second),

λ – is arrival rate.

In evaluating the waiting time for different systems and queue discipline (FCFS, LCFS, etc.) several tools are used [9].

Figure 5 shows typical numerical example for one data link (m=1) with C=9600 b/s and B=430 bits, where we use Pollaczek-Khintchine formula to obtain average waiting time:

$$T_w = \frac{a}{1-a} \cdot \frac{1+C_s^2}{2} \cdot T_s \qquad (13)$$

where $C_s^2 = \sigma_s^2 / T_s^2$ is the squared coefficient of variation (SCV) of the service time with its variance σ_s^2.

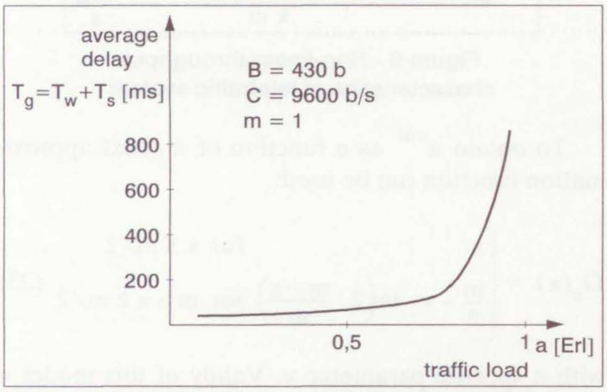

Figure 5 - Delay phenomena in data traffic

Observation 4. New challenges for teletraffic analysis and synthesis come with the introduction of telecommunications network with "service integration" (ISDN, BISDN) and variable bit rate (VBR). To solving these problems advanced traffic engineering capabilities with more dynamic approach are necessary.

4. DYNAMIC DESCRIPTION

To introduce dynamic description we must identify generic traffic parameters (quantities) that can be measured or at least estimated directly from traffic measurement during the time period (0, T).

Let Λ (t) be the cummulative number of arrivals of traffic entities to time t, $t \in (0,T)$: and $\Gamma(t)$ the cummulative number of entities serviced during time (0,t), t<T. With X (t) we denote the number of entities in the system. These variables are related in accordance to the "conservation principle":

$$X(t) = \Lambda(t) - \Gamma(t) + X(0) \qquad (14)$$

We assume that the time behaviours of Λ (t), Γ (t) and X (t) have been measured or observed N times over the time interval (0,T) and we can calculate the averages $\bar{\Lambda}$ (t), $\bar{\Gamma}$ (t) and $\bar{X}$ (t). If these averages are continuous functions of time (differentiable in (0,T), we can write:

$$\frac{d\bar{X}(t)}{dt} = \frac{d\bar{\Lambda}(t)}{dt} - \frac{d\bar{\Gamma}(t)}{dt} \qquad (15)$$

with the initial condition $X(0) = X_0$.

With usual notation we have:

$$\lambda(t) = d\bar{\Lambda}/dt \qquad (16)$$

where $\lambda(t)$ denotes the average rate of arrivals.

The service intensity is in close relation with outgoing flow. We assume that the intensity $d\bar{\Gamma}/dt$ of an outgoing flow can be approximated by a function $G(\cdot)$ of the system state x(t), then follows:

$$\frac{d\bar{\Gamma}(t)}{dt} = \beta \cdot G[x(t)] \qquad (17)$$

where β is the service capacity defined as the number of entities which can be served per unit of time. Every traffic entity has to stay in the system some time (time t_q), so we may expect that the incoming traffic does not influence the outgoing traffic. We can accept that G(x) is approximation to the instantaneous system utilisation factor or traffic concentration in space-time segment as a service system.

With substituting (17) into (16) it follows that evolution of the mean number of traffic entities in the system can be described by the following non-linear differential equation:

$$\frac{dx(t)}{dt} = -\beta \cdot G[x(t)] + \lambda(t) \qquad (18)$$

with the initial condition $x(0) = x_0$.

For analytic modelling, it is convenient to consider a "normalisation equation". For this purpose we choose time units in such a way that the serving capacity β is equal to unity (β=1). With time-scaling transformation, from (18) we have:

$$\dot{x}(t) = -G[x(t)] + a(t) \qquad (19)$$

where $a(t) = \lambda(t/\beta)/\beta$.

For a constant intensity input ($a(t)=a_0$) the system will asymptotically approach the state $\bar{x}$ given by:

$$\dot{x}(t) = -G(\bar{x}) + a_0 = 0 \quad (20)$$

and for stationary state we have $\bar{x} = a_0/(1-a_0)$.

Thus, for stationary state $G(\bar{x}) = a_0$; $\dot{x}(t) = 0$, and $G(\cdot)$ must represent the steady-state utilisation factor as a function of mean number of traffic entities in the system.

Described model can be viewed as a tool for approximation of transient solution of birth-and-death equations. Differential equation model has more advantage in solving different "network of queues" [9].

For the stationary state, the function $G(\cdot)$ is given by:

$$G[x(t)] = \frac{x(t)}{1+x(t)} \quad (21)$$

because we have $\bar{x} = a_0/(1-a_0)$ and $\dot{x}(t) = 0$.

With substitution of (21) into (18) we have:

$$\dot{x}(t) = -\beta \cdot \frac{x(t)}{1+x(t)} + \lambda(t) \quad (22)$$

or, in normalised form:

$$\dot{x}(t) = -\frac{x(t)}{1+x(t)} + a(t) \quad (23)$$

with the initial condition x_0. Bottom approximation was proposed in [7].

In the non-stationary conditions we must introduce higher order description of the system and try to reduce the difference $\delta x(t)$ between the observed state of a real system and model.

5. APPLICATIONS OF DYNAMIC MODELLING IN TELETRAFFIC AND TRANSPORTATION SUBSYSTEM

Dynamic modelling is useful and powerful apparatus especially for:

- modelling non-linear systems with queue,
- modelling non-stationary behaviours,
- join treatment of traffic congestion, delay and control rules,
- investigation of routing rules with time-dependent average quantities (adaptive routing), etc.

We will consider some possible applications of dynamic modelling in solving teletraffic and road traffic problems, without pretension that these are the most representative examples.

Traditionally, the queuing model M/M/m(0) is used to describe the performance of a telephone system (circuit switching), while model M/M/1(r) is used to data communications with buffered data link and packet switching. For telephone concentrator with Poisson input process and an exponential distribution of service time, for stationary state we can note:

$$\dot{x} = -G(x) + a^{in}$$

$$a^B = G(x) - \bar{x} \quad (24)$$

$$a^c = \bar{x}$$

where:

- $\bar{x}$ – is the average number of telephone calls served at time t, and it is also the intensity of carried traffic $a^c(x) = \bar{x}$.
- a^{in} – is the intensity of offered traffic (in Erlangs).
- a^B – is the intensity of rejected (blocked) traffic.

The real throughput characteristics of teletraffic system are not linear, since we describe the system with the non-linear differential equation (see Figure 6):

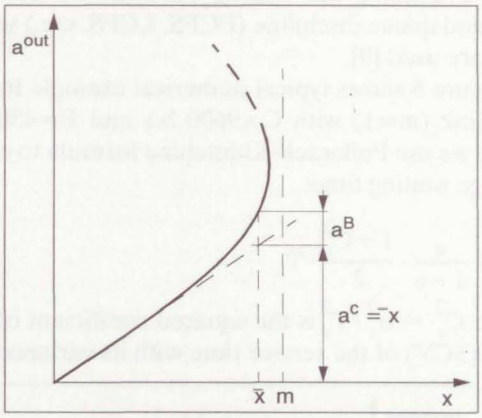

Figure 6 - Non-linear throughput characteristics of teletraffic system

To obtain $\bar{a}^{out}$ as a function of $\bar{x}$, next approximation function can be used:

$$G_a(x) = \begin{cases} x & \text{for } x \le m/2 \\ \frac{m}{2} - \nu \cdot \ln\left(2 \cdot \frac{m-x}{m}\right) & \text{for } m > x \ge m/2 \end{cases} \quad (25)$$

with a specific parameter ν. Validy of this model is tested by simulation experiments which show good results [7].

Non-linear systems with queue (packet switching) will be illustrated next. In data communication link is equipped with a buffer memory of finite capacity. Hence, we use model M/M/1(r) for this system (with max r entities in system). Queuing theory provides the following formula for the average number of entities in the system [9]:

$$\bar{x} = \sum_{r=1}^{R} r \cdot \frac{1-\rho}{1-\rho^{r+1}} \cdot \rho^r \quad (26)$$

where ρ is average traffic intensity.

Example: For dynamic flow model for R=4 and $0 \le \rho \le 1$ we can write:

$$\dot{x}(t) = -G[x(t)] + \rho(t)$$

with the approximation polynomial:

$$\rho = F(x) = d_0x^5 + d_1x^4 + d_2x^3 + d_3x^2 + d_4x + d_5$$

The fifth-order polynomial has been chosen because it is sufficient to represent the original (system) curve over the range of loads considered.

Several dynamic models, supported by adequate algorithms and computer programs, have been developed for particular road traffic, railway or air transportation problems [10]. We will illustrate one concrete model of dynamic programming which is used to optimum control of traffic signals at a single intersection at which two conflicting traffic streams compete for the same road space.

An "optimum program" is defined as the set of the signal change times which minimises total (average) delay at the intersection over a specified period. At any time (t) the state of the intersection is defined by the queues on arms A and B, whether the signals are green on A or B. The optimum program after time (t) depends on the state at (t) and on the future vehicle arrivals, but it is independent of events before time (t). This conclusion suggest a method of finding the optimum program by working backwards from the end of the period (T) to time t=0.

If the minimum delay time and optimum program are known for all states at time (t), we can calculate the minimum delay and optimum program at time (t-1). Procedure is simple and it is implemented in several computer programs (DYPIC, etc.). In comparison with conventional type of the intersection control methods (fixed time, optimised fixed time, etc.) this dynamic model gives substantially less delay (see Fig. 7).

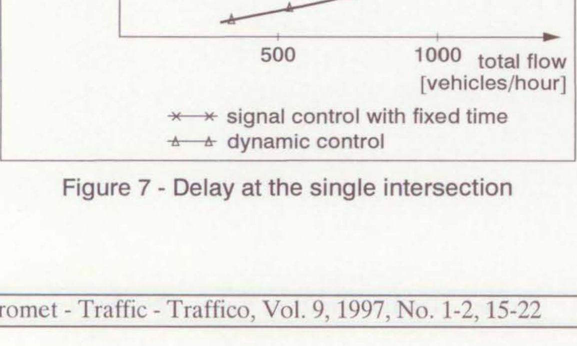

Figure 7 - Delay at the single intersection

6. CONCLUSION AND AREA FOR FURTHER STUDY

Traditionally, traffic system and flows (process) have been analysed by the deterministic and stochastic models developed for particular problems. Stochastic traffic evaluations are based on queuing (theory) models, diffusion approximation and various numerical techniques for computing or approximating time-dependent probabilities in Markovian and Non-Markovian systems. In dynamic (deterministic) description, traffic system has been considered by its state variables and "fundamental equations", but without precise explanation and formalisation according to principles of general system theory.

In this paper, dynamic approach or group of investigations were considered as a part of generalised traffic (system) theory. We assume that generic traffic system laws manifest analogy or isomorphy of laws that are formally identical but pertain to quite different practical observations. With the identification of some generic isomorphy between different transportation and telecommunication system we gain ground for unified approach to analysis and synthesis. Areas for further study are associated with further formalisation and axiomatisation of traffic system knowledge. Human factor, safety aspects and technological considerations must be added to generic model in adequate manner.

SUMMARY

Traffic science cannot emerge from "embryonic state" if it hasn't common methodology and unified (formal) treatment of traffic, relevant for different transportation and (tele)communication subsystems. Generalised model (metamodel) of traffic process or traffic (system) theory, must ensure consistency and provide common platforms for large scale of traffic engineering (technologies) problems.

This paper is concentrated only to consider dynamic approach in unified formal (mathematical) description of traffic process. Identification of some generic traffic relations and isomorphy are associated with the relevant contributions from "classical" traffic flow theory and teletraffic references. Dynamic flow models are considered as an approximation of queuing models and as an independent deterministic and stochastic description of traffic process. Further "more-integrated" contributions and development of generalised traffic (system) theory are suggested.

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