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KINEMATIC CHANGES AND THEIR INFLUENCE ON THE DURABILITY OF UNIVERSAL JOINTS

ABSTRACT

Various theoretical models for determining the durability of universal joints are proposed in literature, so that their evaluation and comparison with test results helps to acquire knowledge of the universal joint operation principles.

The paper considers, therefore, the connection between the dynamic behaviour of universal joint and the damage that occurs. Since the proper performance of universal joints is directly related to the performance of universal joint journal bearings, monitoring and analysing the damage initiation and development are necessary for the evaluation of safety and durability.

KEY WORDS

universal joints, kinematic changes, bearings, durability, safety, monitoring, analysis

1. INTRODUCTION

In order to determine the acting of dynamic load values in driving mechanisms with universal joints, it is necessary to determine the differential equations of motion considering the values of transmission and flexible elements. Describing the dynamic phenomena in such mechanisms is significantly simplified by the use of Mathieu's differential equations, which follow from the Lagrange equation of motion of the second order. Friction forces are neglected (which act in a stabilising manner) as well as the rigidity of the universal joints which is practically much smaller compared to the shafts.

During operation, universal joint has to function satisfactorily for some time, that is, it has to meet the requirements within a range of applications. In order to determine its optimum performance, it is necessary to analyse the basic characteristics compared to the specific ones that are the result of changes in the operating conditions. In such a comparison, it is necessary to consider the limitations regarding the torque and safety in the operation of the universal mechanism. These limitations are determined by the analysis of exploitation or laboratory testing, i.e. by calculation methods using appropriate mathematical models. The best results are obtained by the comparison of calculation parameters and values from the laboratory or exploitation testing.

Studying the process of damaging the universal joint bearings, and especially the journal bearings (cross joints) gives insight into the possibility of increasing the durability of the complete joint mechanism. The most efficient device for laboratory testing of the four samples (joints) simultaneously, which operate in a closed circle was used also for these testing in the Laboratory for Machine Elements at the Faculty of Mechanical Engineering and Naval Architecture (FSB) - Zagreb.

2. DETERMINING VALUES OF CHANGES IN THE DYNAMIC MOMENTUM AND TURN ANGLE AT PARTS OF TESTING SYSTEM OF UNIVERSAL JOINTS

Dynamic changes in the given testing conditions are determined by means of Mathieu's equation of motion.

$$\frac{d^2 y}{dz^2}+(a-2q\cos 2z)y=$$
$$=\frac{M_3}{I_3\omega_1^2}-\frac{M_1}{I_1\omega_1^2}+a\sqrt{A^2+B^2}\sin 2(\varphi_1+a_e)$$

for the presented dynamic system according to Figure 1.

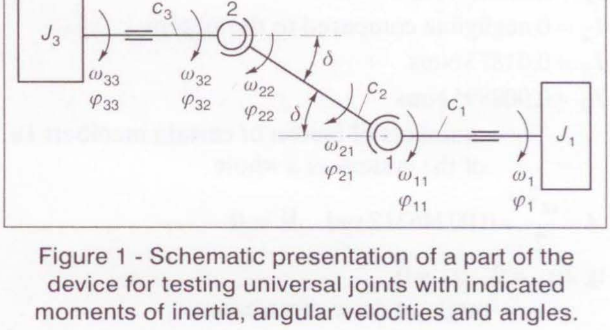

Figure 1 - Schematic presentation of a part of the device for testing universal joints with indicated moments of inertia, angular velocities and angles.

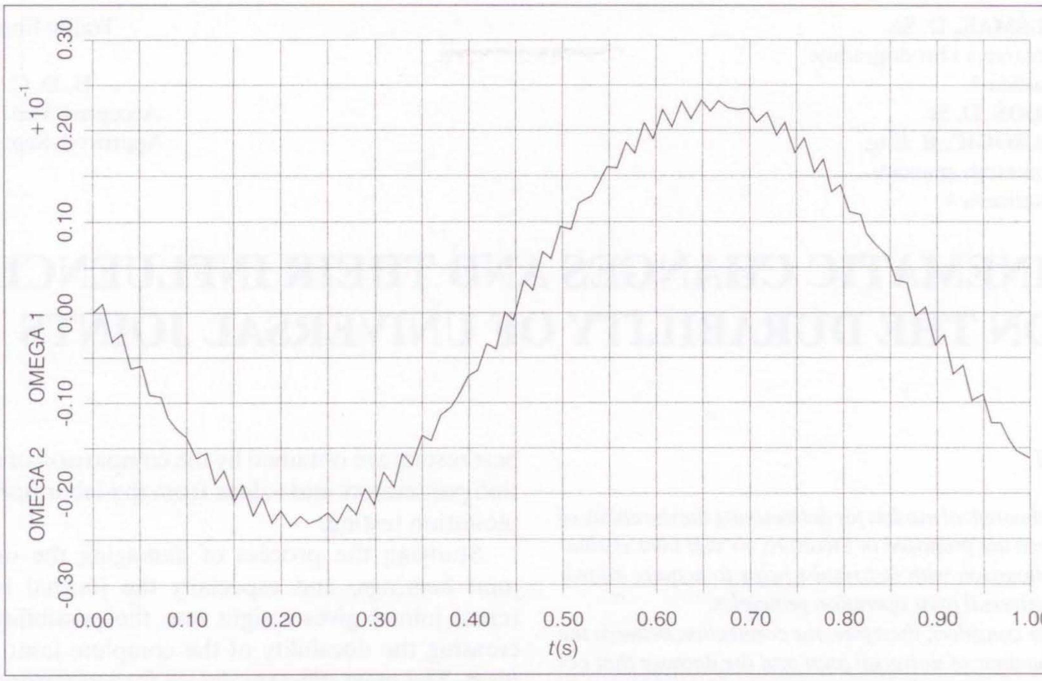

Figure 2 - Functional dependence of the difference of angular velocities ω_2-ω_1 on the change in time t

where the set values are determined by character of testing or calculated out of the parameters of the testing device and amount to:

$n = 1400 \dfrac{o}{\min}$ $\omega = 146.607$ s^{-1}

– number of revolutions, that is, angular testing velocity

$M_1 = M_2 = 60$ Nm

– the set preloading moment

$M_3 = ...$ Nm

– output moment if mass 3 is not bound within the preload device

$\alpha = 7° = 0.122173$ rad

– set angle of installing universal joints

$c_{11} = c_{12} = c_{13} = 0.00373156 \left(\dfrac{\text{Nm}}{\text{rad}}\right)$

– coefficients of rigidity of the drive members

$c = 18742.452 \left(\dfrac{\text{Nm}}{\text{rad}}\right)$

– total coefficient of the drive rigidity

$I_1 = 0.01694$ Nms

$I_2 = 0$ negligible compared to the system

$I_3 = 0.01873$ Nms

$I_0 = 0.008895$ Nms

– moments of inertia of certain members i.e. of the system as a whole

$A = \dfrac{\alpha^2}{2} = 0.00746312$ rad B = 0

$\text{tg } 2\alpha_e = 0 \quad \alpha_e = 0$

– angle of drive disturbance

$z = \varphi_1 + \alpha_e = \varphi_1$

– auxiliary coefficients for equations of motion

$a = \dfrac{c}{I_1 \omega_1^2} = 51.597$

$2q = 1{,}71$

By introducing the system parameters from the above mentioned, follows the differential equation of the fluctuation angles difference in joint 2:

$$\frac{d^2(\varphi_2 - \varphi_1)}{d\varphi_1^2} + (51.597 - 1.71 \cos\varphi_1) \cdot (\varphi_3 - \varphi_1) = $$
$$= -0.16479 + 0.38062 \sin 2\varphi_1$$

i.e. in case that mass M is not further bound in the system, the difference of the shaft fluctuation angles at input and output with variable values:

$$a = 97.94896$$
$$2q = 1.73688$$

$$\frac{d^2(\varphi_2 - \varphi_1)}{d\varphi_1^2} + (97.949 - 1.73688 \cos\varphi_1) \cdot (\varphi_3 - \varphi_1) = $$
$$= 0.01575 + 0.731 \sin 2(\varphi_1 + 0.12217)$$

If we use $\varphi_1 = \omega_1 t$ after introducing into the mentioned expressions, graphical (plotter) presentations of the changes in angular velocities are obtained by means of the double integration carried out by a computer.

In the system which considers friction, periodic conditions of fluctuations of velocities and acceleration which can be seen in the harmonic phenomena within normal change in the flow of these two parame-

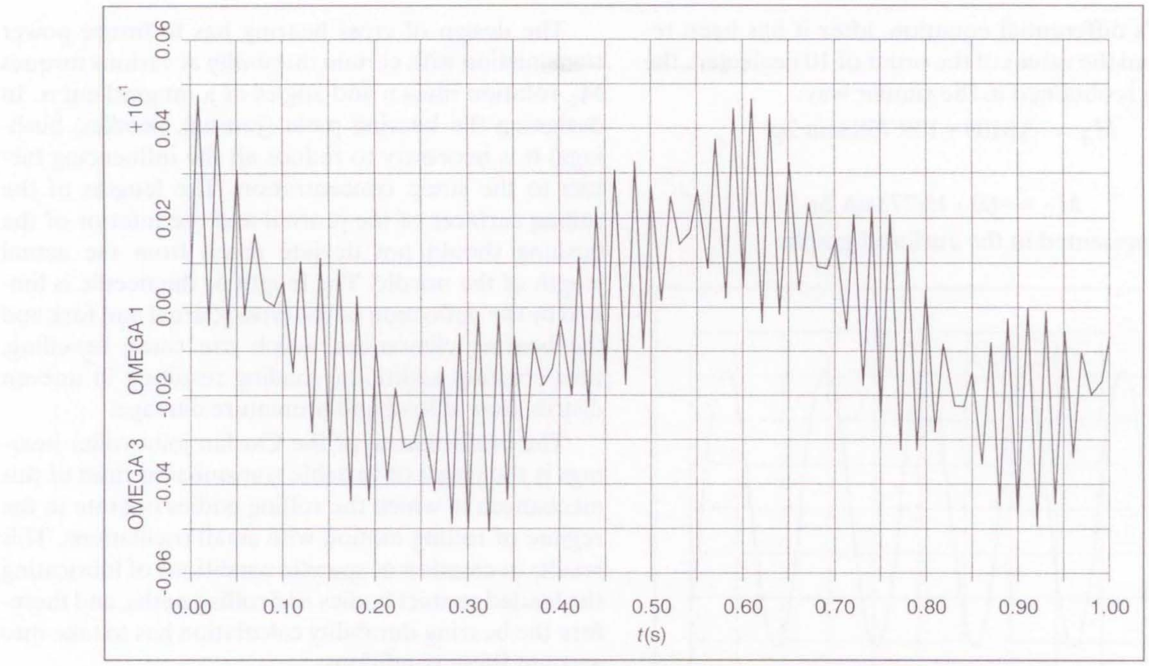

Figure 3 - Difference of angular velocities ω_3-ω_1 as function of time t

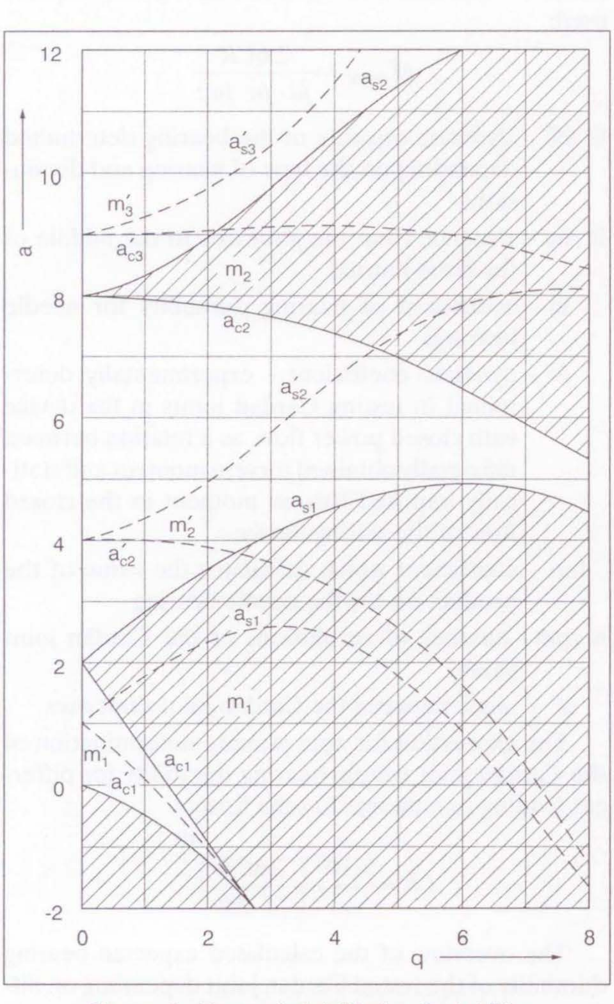

Figure 4 - Zone of stability i.e. instability of the elastic testing system

ters. It follows that we are discussing a stable, that is, unstable system, if similar phenomena occur.

By transforming Mathieu's equation of motion into the form which is most often used in solving practical problems with universal joints, where one can state with enough precision that f(φ)=2n=const., we get:

$$y'' + 2ny + f_z(\varphi)y = 0$$

Further solving and simplification of this expression yields:

$$y = \sum_{k=1,3,5}^{\infty} (A_k \sin k\omega t + B_k \cos k\omega t)$$

The coefficients of trigonometric functions obtained from the conditions presented by these two equations, form infinite system of linear differential equations whose solution is presented in Figure 5 both for the case when friction is not considered (broken curves) and for the case when it is taken into account (full curves).

If the obtained values a and q for the considered system are analysed, one may notice that they are within the zone of instability, that is, outside the hatched fields m_1 and m_2 for the case with lubrication (i.e. m_1' and m_2' for the case of unlubricated surfaces). It follows that the shaft with universal joints will behave as a flexible system within which excitations will occur due to proper rigidity in the form of disturbances in differences of angular velocities, which is presented in Figures 3 and 4, and was described earlier.

From the expression for the change of momentum $M_3 = c_3(\varphi_3 - \varphi_{3,2})$, and using the developed

Mathieu's differential equation, after it has been rewritten and the values of the order of 10 neglected, the following is obtained in the similar way:

$$M_3 = -3.0109 + 139.7583 \sin 2\varphi$$

that is

$$M_2 = -60 + 15.778 \sin 2\varphi$$

which is presented in the enclosed graphs

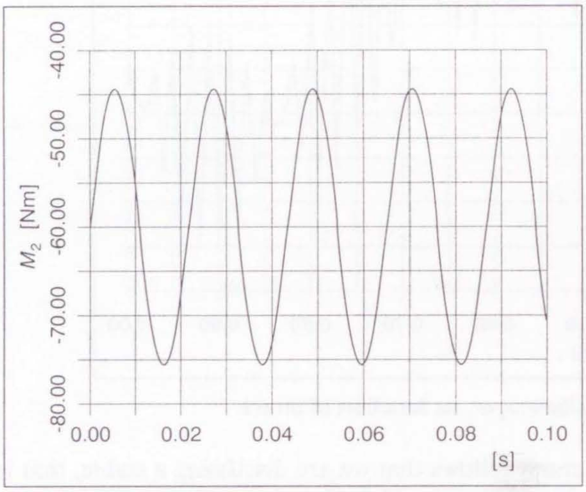

Figure 5 - Change of momentum M_2 in time t

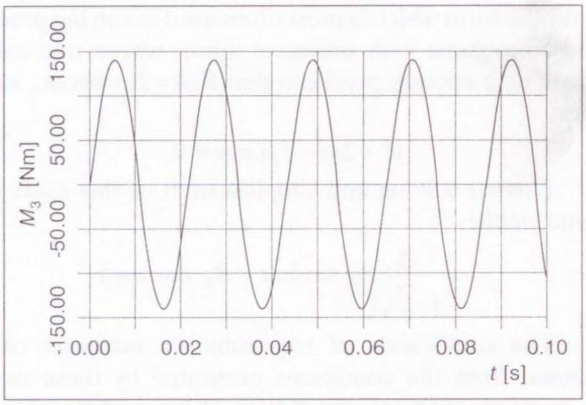

Figure 6 - Change of momentum M_3 depending on time t

3. ANALYTICAL METHOD OF DETERMINING THE DURABILITY OF UNIVERSAL JOINT BEARINGS

In literature, mostly by foreign authors, several mathematical models for determining the durability of universal joints have been suggested, which are used for comparison analysis with the laboratory testing of the Cardan joint with needle roller bearings on journals. The tested Cardan joints are used in transmission of power on the machine tools and along with small dimensions provide high level of efficiency and loading capability.

The design of cross bearing has to insure power transmission with certain durability at various torques M_t, rotation rates n and angles of joint gradient α. In designing the bearing parts (journal, needles, bushings) it is necessary to reduce all the influencing factors to the stress concentration. The lengths of the rolling surfaces of the journal and the interior of the bushing should not deviate much from the actual length of the needle. The length of the needle is limited by the deflection of the cross journal and fork and the bearing clearances, which can cause bevelling, thus creating additional loading resulting in uneven distribution of load and premature damage.

The performance of the Cardan joint roller bearings is the result of variable transmission ratio of this mechanism in which the rolling bodies operate in the regime of rolling motion with small oscillations. This results in creation of specific conditions of lubricating the loaded contact bodies and rolling paths, and therefore the bearing durability calculation has to take into account these conditions.

For the calculation of Cardan joint bearing durability, the expression for the maximum given torque is used:

$$M_{\max} = \frac{2.6CR}{fd \cdot fn \cdot fn\alpha}$$

C /N/ – dynamic capacity of the bearing determined depending on the type of bearing and dimensions

R /m/ – distance from the joint axis to the middle of the cross journal

fn – coefficient of bearing durability for needle bearings

fd – dynamic coefficient - experimentally determined in testing Cardan joints in the device with closed power flow, as a relation between maximally obtained torsion moment and statically imposed torsion moment in the closed flow of the testing device.

fnα – coefficient which calculates the value of the product nα for the needle bearing

n rpm – number of revolutions of the Cardan joint shaft

α^* – angle between the Cardan joint shaft axes

The expression for approximate determination of the Cardan joint needle bearing durability for different loading parameters has the form:

$$Lh = \frac{5 \cdot 10^5}{n\alpha} \left(\frac{2.6CR}{fd \cdot Mt} \right)^{\frac{10}{3}}$$

The overview of the calculated expected bearing durability of the tested Cardan joint depending on different loading conditions Lh = f (Mt, nα), is given in the diagram in Figure 8.

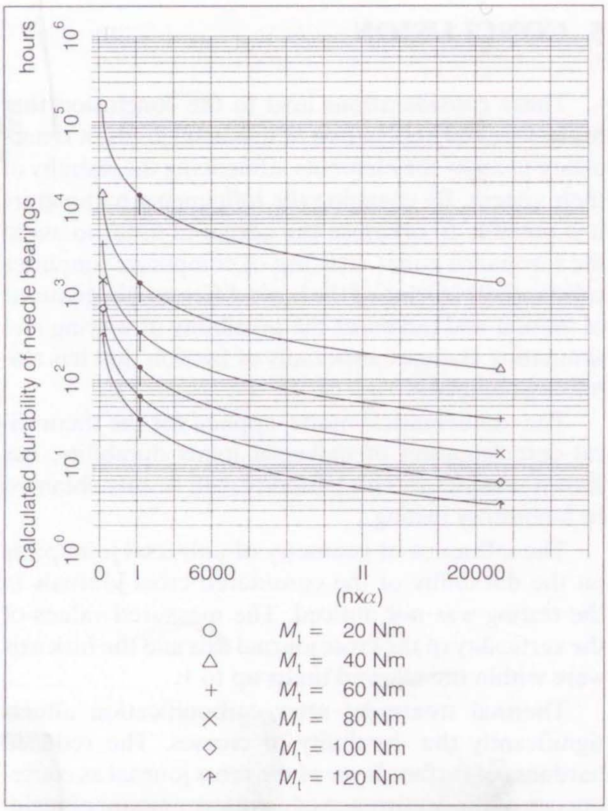

Figure 7 - Calculated bearing durability as function of operating conditions

The diagram in Figure 7 was used to determine the testing conditions and for comparison with the experimental data on Cardan joint needle bearing durability.

In variable torque, the operating conditions have been selected at I (n=710 rpm, α=3, nα=2130) and II (n=1400 rpm, α=10, nα=14000).

The expected bearing life can be determined, i.e. assessed, only after the laboratory testing, with determined loading parameter relations.

According to the Miner's theory, the total assumed durability of the Cardan joint bearing is determined from the relation:

$$L_{hekv} = \left(\sum_{j=1}^{m} \frac{N_j}{L_j} \right)^{-1} = \frac{1}{\frac{N_1}{L_1} + \frac{N_2}{L_2} + \ldots + \frac{N_m}{L_m}}$$

$L_1, L_2 \ldots L_m$ – calculated bearing durability in a certain operating regime

$N_1, N_2 \ldots N_m$ – operating time in a certain loading regime in relation to the total operating time of the joint in percents.

After testing (causing sudden increase in temperature and stronger shaft vibrations), Cardan joints were disassembled and degreased. In order to determine the level of damage on the journal surface, the depth of the grooves that appeared at the cross journals were measured.

The results of measuring the groove depths and the actually obtained durability of certain cross specimens were compared to theoretical values presented in Table - Figure 8. The table also presents the values of calculated maximum contact stresses (Hertz's pressures) for the measurement operating conditions.

experim.	1			2	3	4					
	w.cond.			P_{Nmax}	Lh	gutter depth on cross					
	n	α	M	N/mm²=10³	calc. durat.	work u.co.		L_h		▲	↕
	min⁻¹		Nm	4 3 2 1		h	%	eq h	re h		
I	710	3	20		12796	1.32	2.5	160	189.2	0.14	0.07
			40		1269	-	-				
			60		328.5	50.7	96				
			80		126	38.6	73				
			100		59.8	8.72	16.5				
			120		32.6	0.66	1.25				
II	1400	10	20		1946	2.2	2.5	98.5	114	0.2	0.13
			40		193	7.1	81				
			60		50	24.5	28				
			80		19	1.75	1				
			100		9.1	0.17	0.25				
			120		5	0.22	0.16				

Figure 8 - Comparison of testing results with the calculated values for the selected operating conditions

4. ANALYSIS OF THE PROCESS OF DAMAGING THE CONTACT SURFACES OF CARDAN JOINT CROSS JOURNALS

The durability of the needle bearing - journal couple is determined by the durability of the cross journal surface based on the characteristic form of wear that shows formation of bevelled grooves.

The grooves are laid at an angle of approximately 296° to the journal axis, which corresponds to the angle of twist of needles in the bearing. The formation of bevelled grooves obtained during the above mentioned operating conditions may be explained as the consequence of:

- operating of rolling bodies at high contact pressures,
- intensive flow of corrosion oxidation,
- possible plastic deformation of contact surfaces,
- insufficient lubrication
- variable relation of the transmission of motion.

The damaging process of the journal surface based on the observed phenomena during testing can be divided into two phases. At the beginning of the first phase, which includes running-in of the surfaces, the indents do not occur yet. At the end of the first phase, mechanical damages start to appear (abrasive wear, as consequence of cutting or tearing activity of rigid particles of the carried away material, visible on the journal surfaces as separate cracks). The intensity of the damage longitudinally and along the journal flange is not even, which can be explained by inaccuracies in the geometry of journals and needles, as well as different magnitude of stress in certain parts of the contact surface.

In the second phase, indents appear and their further propagation may be noticed.

In order to explain the process of damaging the cross journals, the character of structural changes created in the surface layer during the first phase of the joint operation needs to be explained. Therefore, surfaces with formed indents were tested. By measuring the hardness on these surfaces, a decrease in hardness was noticed as compared to the initial values. It is assumed that this decrease in hardness is the result of high local temperatures, causing yield, and thus also reduction in surface hardness. This reduction in hardness was not proportional longitudinally with the cross journal flange. This occurrence in the initial period may be regarded as thermal damage of the journal. The measured values of hardness at certain parts of journal surface were about 100 HV less than the initial hardness. The reduction of surface hardness affects significantly the deformation of cross journal surfaces.

After the focus of indent has been formed, the kinematics of needle motion changes significantly. The increased radial clearance due to the journal surface wear, depending on the position of the indent focus, makes the twist of the needles possible and keeps them thus twisted. The reduction of the contact surface creates greater edge pressures and leads to further more intensive development of indents, making the needles even more fixed in that position. The increase in the depth of such grooves causes knocking in the bearing and increase of the joint shaft vibrations.

After the completed measurement, the hardness on the journal surface was controlled, so that the hardness in the grooves and at the joining parts of the surface between the grooves was measured. This may be explained by the fact that the first phase of surface damaging resulted in yielding of the surface layer which hardened at points of indent foci, that is, the material hardens due to the microplastic deformations in contact between the surface needle layers and the cross journals. Such hardening did not take place between the grooves, and therefore such difference in hardness. These hardened groove bottom surfaces would probably become brittle after a certain number of stress cycles and would result in pitting within the grooves.

5. CONCLUSION

These considerations lead to the conclusion that for testing and application of universal joints, it is necessary to know the elements influencing the stability of their system. By changing the influencing parameters it is possible to optimise the design in order to avoid the resonance zones. The use of computers simplifies substantially solving of the basic differential equations of motion and increases the possibility of varying and simulating changes, especially of friction that has stabilising character.

The mathematical model applied for the theoretical determination of universal joints durability, has shown satisfactory compatibility with results obtained by laboratory testing.

The influence of geometry of universal joint parts on the durability of the considered cross journals in the testing was not noticed. The measured values of the verticality of the cross journal axis and the fork axis were within the allowed limits up to 1.

Thermal treatment after carbonification affects significantly the durability of crosses. The reduced hardness of surface layer of the cross journal as consequence of the occurrence of a greater amount of residual austenite is one of the basic reasons for the premature damage of the contact surfaces. Higher values of Hertz's pressures at rolling surfaces of contact between the needles and the cross journals increase the probability of such damages. The values of measured depth of bevelled grooves, characteristic occurrence in the universal joints cross journals damage, indicate the significant influence of hardness of the rolling journal surfaces. Every decrease in hardness gave rise to the possibility of creating the foci for development of indents which turn into characteristic bevelled grooves due to further impact loads.

By expanding the experimental research and by quality analysis of the insufficiently studied observed phenomena, especially the pitting, greater reliability and increased durability of universal joints may be provided.

SAŽETAK

KINEMATSKE PROMJENE I NJIHOV UTJECAJ NA TRAJNOST SFERNIH ZGLOBOVA

U literaturi su predloženi različiti teorijski modeli za određivanje trajnosti, pa je njihovo vrednovanje i usporedba s eksperimentalnim rezultatima jedan od putova saznanja o zakonitostima rada sfernih zglobova.

U sklopu rada razmatra se utjecaj veze između dinamičkog ponašanja sfernog zgloba i pojava oštećenja. Budući da je ispravan rad sfernog zgloba izravno vezan uz rad ležaja na rukavcima križa, praćenje i analiza nastajanja i daljnjih tokova oštećenja nužni su za ocjenu veličina sigurnosti i trajnosti.

LITERATURE

[1] **F. Duditza**, *Kardangelenkgertriebe und ihre Anwendungen*, VDI-Verlag GmbH, Dusseldorf, 1973

[2] **K. Fujita, A. Yoshida**, *Surface fatigue failure of cas-hardened nickel-chromium steel rollers under pure rolling and sliding - rolling contacts*, 2(1978) 365-374

[3] **M.J. Hartnett**, *The analysis of contact stresses in rolling element bearings*, Trans. ASME., J. Lubric Technol, 1(1979) 105-109

[4] **S.N. Koževnikov** and **P.D. Perifiljev**, *Kardanie peradači*, Tehnika, Kiev, 1978

[5] **O. Muftić, K. Drača**, *Uvod u teoriju mehanizama*, SN Liber, Zagreb, 1974

[6] **I.R. Vagner** et al., *Universal joint and driveshaft design manual*, Society of Automotive Engineers Inc., 1979