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AN OPTIMAL CAPACITY PLANNING MODEL FOR GENERAL CARGO SEAPORT

ABSTRACT

In this paper the application of the queuing theory in optimal capacity planning for general cargo seaport is presented. The seaport as a queuing system is defined and thus, on the basis of the arrival and serviced number of ships in an observed time unit, the appropriate operating indicators of a port system are calculated. Using the model of total port costs, the number of berths and cranes on the berth can be determined whereby the optimal port system functioning is achieved.

KEY WORDS

queuing theory, port system, port capacity

1. INTRODUCTION

Optimal capacity planning is one of the conditions for efficient functioning of a port system, as well as for a general cargo port.

The general cargo port capacity is the capability of the port to accept a certain number of ships simultaneously, i.e. the cargo quantity (accommodative capacity), or to load and unload a certain number of ships, i.e. the cargo quantity in an observed time unit, e.g. during the year (traffic capacity). Optimal port capacity must satisfy the given limits and the optimisation criterion and so provide the users of port services with as few delays as possible and with profitable port operations.

The aim of this paper is to show the possibility of optimal port capacity dimensioning, using the queuing theory.

The general cargo port can be defined as a queuing system for which the appropriate operating indicators are calculated on the basis of arrival and serviced number of ships. Some of these indicators are for example: the probability of unoccupied berths, the average number of ships in queue, the average time of the ship's time in port, etc.

However, by the introduction of value indicators, i.e. by means of the costs, the number of berths and cranes on the berth, for which the sum of the total port costs is minimum in an observed time unit can be determined thus realising the optimal functioning of the port system.

2. PORT FUNCTIONING AS A QUEUING SYSTEM

The port system generally and the general cargo port also can be defined as a queuing system. The basic parameters of the port system are the average number of ships arriving at the port in an observed time unit and the average number of ships that can be serviced in the same time unit. The port system functioning is observed by the indicators on the basis of which it is possible to determine the optimal number of berths and cranes.

2.1. Definition of the port as a queuing system

The port can be defined as a queuing system with the following structure [8]: entrance units are ships forming (or not) a queue (depending on the immediate situation) to be serviced (loading or unloading of cargo) onto the berths of the seaport (servicing channels), and leave the system when the service has been performed.

The number of ship arrivals and the servicing time can be taken as random variables and empirical distributions of these variables approximated to the corresponding probability distributions [3], [4]. In this case, an analytical approach to the operating port indicators can be applied, i. e. the formulae established by the queuing theory.

From the queuing theory viewpoint, the port system has the following characteristics:

- The port system is an open system, as the sources of the entrance flow of ships are not a component of the system.
- The port system is a single or multi-channel system (depending on the number of berths), where the ships queues for particular berths are formed at anchorage.
- The number of ship arrivals as well as the duration of servicing time, i.e. the ship's time on the berth are

distributed according to certain probability distributions, most often to Poisson's distribution or Erlang's distribution of the k-th order, where k is a natural number. The ship's time in a port consists of the ship's time spent queuing on the berth and of its servicing time.
- The ships are "patient customers" because they wait patiently to be serviced, although it happens in practice that the ships for various reasons leave the queue.
- As for the queuing discipline, the port is a system in which the servicing is in most cases carried out according to the FIFO rule (first come–first served), but the existence of priority service ships is also possible (depending on the type of cargo or contract with shipowners).

2.2. Indicators of the port system functioning

The basic parameters for the port system analysis as a queuing system are:

λ – is the ship arrival rate, i.e. the average number of ships arriving at the port in the course of time observed (during the year, month, day),

μ – is the service rate, i.e. the average number of ships that can be serviced in the same time unit as parameter λ.

The relation between the arrival rate and service rate is the utilisation factor or berth occupancy ρ:

$$\rho = \lambda / \mu .$$

If $\lambda > \mu$ one berth is not sufficient, and in this case the number of berths has to be increased until the stability condition $\rho/S < 1$ is satisfied.

With the basic parameters of the port system, the operating indicators can be calculated, for example: the probability of unoccupied berths, average number of ships in queue and in a port system, average ship's time in queue and in port, etc.

A change in the number of berths has an influence on the increase or reduction of the values of the particular port system indicators: with an increase in the number of berths, the number of ships in queue and in port is reduced, as well as the waiting time of the ship spent in queue and in port, but the berth unoccupancy is increased.

On the basis of the operating port indicators, the question may arise how to determine the number of berths so as to reduce the waiting time of the ships and berths to a minimum.

In order to make a decision on the optimum number of berths in the port system, a criterion for decision-making has to be introduced, e.g.: the percentage of berth capacity utilisation, the time spent in queue or the number of queuing ships. The criterion which is considered as the most important for the efficient functioning of the port system should be chosen. Port efficiency can best be provided by the introduction of value indicators, i.e. by means of costs, as in practice, waiting time of the ships has to be paid for, and the berth unoccupancy can also be expressed in terms of value.

3. MODEL FOR DETERMINING OPTIMAL CAPACITY OF GENERAL CARGO PORT

In order to eliminate the waiting which occurs at the general cargo port as a queuing system, either a great number of berths would have to be built (so that the ships would not have to wait at all), or only that number of berths which would be permanently occupied (so that the berths are not unoccupied). These extreme solutions are, of course, not rational, as any elimination of waiting for one participant leads to maximal waiting for the other participant in the queuing system. The optimal solution is the one for which the loss, as a result of the ships waiting time and berth unoccupancy, as well as other costs, leads to a minimum amount.

3.1. Model of total port costs formulation

The model of total port costs is presented by a function which contains these costs [1],[2],[7]: the total costs of berths (C_b), total costs of port cranes (C_d), total costs of warehousing (stacking area) (C_{wh}), total labour costs (C_l), total ship costs (C_W) and total cargo costs (C_Q). Therefore, the function of the total port costs is as follows:

$$C = C_b + C_d + C_{wh} + C_l + C_w + C_q, \tag{1}$$

where C is the sign for the total port costs expressed in currency units per observed time unit, e.g. USD/hr.

The amount of each kind of costs is calculated by means of the appropriate formulae:

$$C_b = S \cdot c_b \tag{2}$$

$$C_d = S \cdot d \cdot c_d \tag{3}$$

$$C_{wh} = k_{wh} \cdot a \cdot c_{wh} \tag{4}$$

$$C_l = \lambda \cdot d \cdot t_1 \cdot c_1 \tag{5}$$

$$C_W = \lambda \cdot W \cdot c_W \tag{6}$$

$$C_Q = \lambda \cdot W \cdot Q \cdot c_Q, \tag{7}$$

where:

S – number of berths in seaport

c_b – costs per berth (USD/hr); if c_b is not given, it can be calculated, taking into consideration the capital recovery factor and the costs of the berth maintenance:

$$c_b = \left[B_0 \cdot \frac{i(1+i)^{N_b}}{(1+i)^{N_b} - 1} + M_b \right] \cdot \frac{1}{365 \cdot 24}, \tag{8}$$

where:

B_0 – initial berth value (USD)

N_b – economic lifetime of berth (years)

i – interest rate

M_b – annual costs of berth maintenance (USD).

d – number of cranes per berth, i.e. the number of crane operators in formula (5)

c_d – costs per crane (USD/hr) calculated analogous with c_b on the basis of the initial crane value, the economic lifetime and the annual costs of crane maintenance

k_{wh} – required capacity of warehouses (stacking areas)

$$k_{wh} = \lambda \cdot x \cdot t_{wh} \cdot f_s, \quad (9)$$

λ – ship arrival rate (ships/hr)

x – number of unit cargo loading/unloading per ship (unit cargo/ship)

t_{wh} – average dwell time of cargo in warehouse (hr/unit cargo)

f_s – safety factor ($0 < f_s < 1$)

a – size of the storage area per unit cargo (m²/unit cargo)

c_{wh} – costs per unit of the storage area (USD/m²/hr) calculated analogous with the unit costs of berths c_b and the unit costs of port cranes c_d

c_l – labour cost per crane operator (USD/crane/hr)

t_l – paid labour time per crane operator (hr/crane/ship)

t – transfer time of cargo, i.e. the trans-loading time of ship (hr/ship), which is given as follows:

$$t = x \cdot y / d^f, \quad (10)$$

where:

y – crane cycle time (hr/unit cargo)

f – crane interference exponent determined empirically ($0 < f < 1$).

However, if t is less than the minimum shift duration, e.g. 4 hours, then the port pays the amount of minimal number of hours and not the real value of t. This means, that t_l must be taken:

$$t_l = \max(t, t_{min}), \quad (11)$$

where t_{min} signifies the minimum shift duration.

c_W – costs of ship's time in port (USD/hr) calculated analogous with the costs c_b and c_d on the basis of the initial ship's value, the economic lifetime of a ship and the annual costs amount of ship maintenance

W – average ship's time in port, i.e. the ship's time spent in a queue and the ship's servicing time on the berth (hr/ship).

A W value is one of the port operating indicators given by the queuing theory model, but the manner of calculation depends on the queuing problem type, taking into consideration the elements which determine the kind of queuing problem: the distribution of ship's arrival, distribution of ship servicing time, servicing discipline and number of berths. In this paper the queuing problem type $M/M/S/\infty$ is chosen because this type of queuing problems is the most common for port systems.

In a port with one berth (μ-λ), and in a port with more berths:

$$W = W_Q + 1/\mu, \quad (12)$$

$$W_Q = \frac{\rho^{S+1}}{\lambda(S-1)!(S-\rho)^2}\left[\sum_{n=0}^{S-1}\frac{\rho^n}{n!} + \frac{\rho^S}{S!(1-\frac{\rho}{S})}\right]^{-1} \quad (13)$$

where $\rho = \lambda/\mu$.

μ – service rate (ships/hr) calculated according to formula:

$$\mu = \frac{1}{t_l + t_m} \quad (14)$$

However, in calculating the ship costs in a port, the maximal value t_l is not taken (as in the case for c_l), but the value of t which is given on the basis of the real data according to formula (10).

t_m – manoeuvring (docking and undocking) time (hr/ship)

Q – cargo quantity, i.e. the number of unit cargo (number of unit cargo/ship),

c_Q – cost of unit cargo (USD/hr).

The costs from (2) to (7) are the kinds of costs with different behaviour depending on the number of berths and cranes:

- costs of warehousing (C_{wh}) are constant and independent of the number of berths and the number of cranes on a berth,
- costs of berths (C_b) are proportional to the number of berths but they do not depend on the number of cranes on a berth,
- costs of cranes (C_d) depend on the number of cranes and on the number of berths also because d is the sign for the number of cranes on a berth,
- labour costs (C_l) depend on the number of cranes on a berth, because the number of crane operators according to the number of cranes is determined, but these costs do not depend directly on the number of berths,
- costs of ships in a port (C_W) depend on the number of berths and the number of cranes on a berth; the same conclusion is related to cargo costs C_Q.

By means of a computer program, which has been developed for the function of total costs, it is possible to calculate as follows: the amount for each kind of costs, costs for a selected number of berths, costs for a selected number of cranes, total costs depending on the number of berths and cranes and finally, to determine the number of berths and cranes on a berth for which the total costs are minimum.

With these results the fixed aim of this paper is achieved.

3.2. Numerical example

The presented model of total port costs for general cargo seaport has been tested on the selected numerical example [2]: [1]

Number of berths	$1 \le S \le 5$
Initial berth value	$B_0 =$ 10 000 000 US$
Interest rate	$i = 0.12$
Economic lifetime of berth	$N_b =$ 40 years
Annual costs of berth maintenance	$M_b =$ 1 000 000 US$
Number of cranes per berth	$1 \le d \le 5$
Initial port crane value	$D_0 =$ 2 000 000 US$
Economic lifetime of port crane	$N_d =$ 20 years
Annual costs of port crane maintenance	$M_d =$ 100,000 US$
Ship arrival rate [2]	$0.006 \le \lambda \le 0.25$ ships/hr
Number of unit cargo per ship	x = 800 unit cargo
Average dwell time of cargo in warehouses	t_{wh} = 400 hr/unit

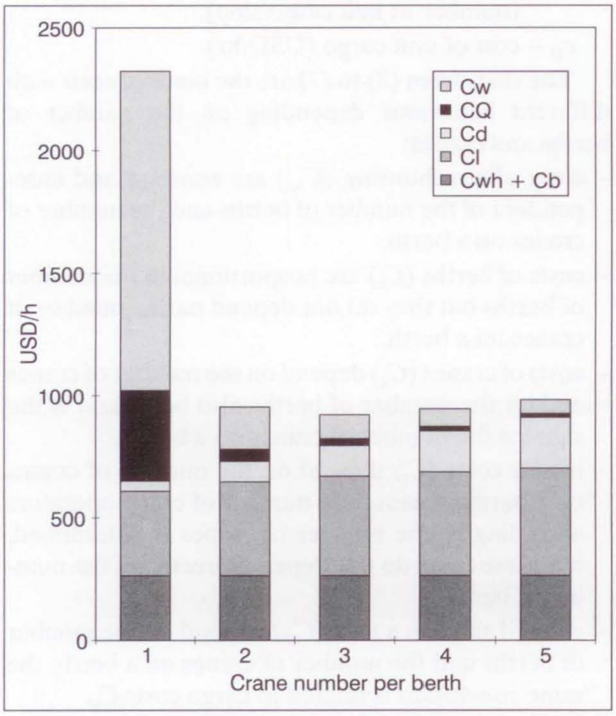

Figure 1: Structure of the total port costs depending on the kind of costs and the number of cranes per berth

Safety factor	$f_s = 1$
Storage area per unit cargo	a = 1.32 m²/unit
Initial value of warehouse	K_0 = 50 US$/m²
Economic lifetime of warehouse	N_{wh} = 40 years
Annual costs of warehouse maintenance	M_{wh} = 5 US$/m²
Crane cycle time	y = 0.045 hr/unit
Crane interference exponent	f = 0.85
Minimum shift duration	t_{min} = 7 hr
Labour costs per crane operator	c_l = 400 US$/hr
Manoeuvring time	t_m = 1 hr
Initial ship value	B_0 = 6 850 000 US$
Economic lifetime of ship	N_b = 30 years
Annual costs of ship maintenance	M_b = 685 000 US$
Average number of unit cargo [3]	Q = 50 unit
Costs of unit cargo waiting for loading/unloading	c_Q = 1 US$/unit/hr .

Calculation of each kind of costs according to formulae from (1) to (14) is as follows:

$$C_b = S \cdot 252,$$
$$C_d = S \cdot d \cdot 42,$$
$$C_{wh} = \lambda \cdot 528,$$
$$C_l = \lambda \cdot d \cdot t_1 \cdot 400,$$
$$C_W = \lambda \cdot W \cdot 175,$$
$$C_Q = \lambda \cdot W \cdot 50,$$

where:
$1 \le S \le 5$, $1 \le d \le 5$; $0.06 \le \lambda \le 0.25$; $t_l = \max(36/d^{0.85}, 7)$, $\mu = 1/(t_l + t_m)$, $W = 1/(\mu-\lambda)$ for $S=1$, or according to (12) and (13) for $S>1$.

Assuming the arrival ship rate is on the average 4 ships weekly, i.e. $\lambda = 0.0238$ ships/hour, the required number of berths depends on the service rate μ which is calculated on the basis of the transloading time according to (10) and (11), also on the basis of the shiphandling time by (14) and for the selected example the results are presented in Table 1.

Table 1: Ship service rate depending on the number of cranes per berth (λ = 0.0238 ships/hr)

Cranes per berth	Time of transloading (hr/ship)	Service rate (ship/berth/hr)	Berth utilisation factor
d	t_l	μ	ρ
1	36.000	0.0270	0.88
2	20.000	0.0476	0.50
3	14.170	0.0659	0.36
4	11.080	0.0820	0.29
5	9.167	0.0983	0.24

Table 2: Total port costs depending on the kind of costs and the number of cranes per berth in US$/hr

Cranes per berth	Warehouse	Berths	Labour	Port cranes	Cargo	Ships	Total costs
d	C_{wh}	C_b	C_l	C_d	C_Q	C_W	C
1	12.57	252	342.72	42	371.88	1301.56	2322.73
2	12.57	252	380.80	84	49.98	174.93	954.28
3	12.57	252	404.70	126	28.26	98.92	922.45
4	12.57	252	421.93	168	20.44	71.55	946.49
5	12.57	252	436.35	210	15.97	55.89	982.78

According to the values of berth utilisation factor ρ ($\rho<1$) the queuing system is stable with one berth and the optimal solution with the number of cranes on a berth for which the amount of the total port costs is minimum.

Calculation of total port costs has been done in Table 2 and presented in Figure 1.

For the selected example on the basis of the results from Table 2 and Figure 1, optimal port capacity is determined as follows: one berth with three port cranes on a berth.

4. CONCLUSION

Port capacity planning is one of the conditions for efficient functioning of general cargo seaports.

This paper presents the queuing theory model of the total port costs, which can be applied in planning development and exploitation of the port system capacity.

The model of the total port costs is presented by a function which consists of the following costs: costs of berths, costs of port cranes, costs of warehousing, labour costs, ship and cargo costs. The optimal solution is the number of berths and port cranes on a berth for which the sum of the total port costs is minimal.

The presented model is applicable not only in the case of the observed port changes, but also for different kinds of seaports.

SAŽETAK

U ovom je radu prikazana primjena teorije redova čekanja u planiranju optimalnog kapaciteta luke za opći teret. Luka se definira kao sustav usluživanja za koji se, na temelju broja brodova koji pristižu u luku i koji se mogu uslužiti u promatranoj jedinici vremena, izračunavaju odgovarajući pokazatelji funkcioniranja sustava. Primjenom modela ukupnih lučkih troškova može se odrediti broj pristana i dizalica po pristanu kojim se postiže optimalno funkcioniranje lučkog sustava.

NOTES

1. In order to test the shown model the authors have used data from papers listed in the literature, plans of port development, shipping companies and discussions with experts, while in some cases these were the author's estimation.
2. For analysing the influence of the number of ship arrivals on the optimal solution of the problem, different values are used, for example: from one ship per week (0.006 ship/hour) to 42 ships per week (0.25 ship/hour).
3. Q is a parameter which represents the number of cargo units, for example: the number of containers if container terminal is considered.

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