

ZORAN RADMILOVIĆ, Ph.D.  
E-mail: z.radmilovic@sf.bg.ac.yu  
VLADISLAV MARAŠ, M.Sc.  
E-mail: marasv@inffo.net  
University of Belgrade,  
Faculty of Traffic and Transport Engineering  
Vojvode Stepe 305, 11000 Belgrade, Republic of Serbia  
SAŠA JOVANOVIĆ, B.Eng.  
E-mail: sjovanovic73@yahoo.co.uk  
Technical University of Catalonia  
Department of Nautical Science and Engineering  
Pla del Palau 18, 08003 Barcelona, Kingdom of Spain

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## SHIP LOCK AS GENERAL QUEUING SYSTEM WITH BATCH ARRIVALS AND BATCH SERVICE

### ABSTRACT

The real lock operations with ships and barge convoys are considered dependent on the transport technologies applied, or more precisely, the kinds of ships/convoys requiring the lockage. The fleet can be divided as follows: (1) groups of single ships, (2) pushed and pulled tows of barges and (3) different combinations of previous systems (1) and (2). The groups of ships and tows passing through the lock have extremely stochastic characteristics thus forming various arrivals and service time patterns. It means that uniform navigation or strong scheduling between locks and lock operations are not possible even though highly sophisticated equipment is at disposal. Therefore, in this paper an analytical method was developed using bulk queuing systems for the analysis and planning of lock requirements supporting it with numerical example. The developed methodology can be applied to determine the mean queue length of ships – convoys at lock anchorage, without blocking behaviour between upstream and downstream navigation for single-lane traffic.

### KEYWORDS

monitoring of traffic infrastructures, ship lock management, bulk queuing systems, inland navigation

### 1. INTRODUCTION

The total ship/convoy lockage time includes as follows: the times of opening and closing lower and upper gates, the enter/exit times of ships/convoys to and from the lock chamber, the preparation times with manoeuvres for berthing and deberthing to and from lock bollards (walls in lock chamber) and the times of water filling/discharge in lock chamber.

In other words, passing times of ships/convoys through the lock depend on numerous variables. The

states of the lock chamber change as result of the following basic operations:

- I. Upstream direction
  - (1) Closing of the gates at the lower lock head
  - (2) Filling the chamber with water
  - (3) Opening of the gates at the upper lock head
- II. Downstream direction
  - (4) Closing of the gates at the upper lock head
  - (5) Discharge of water in the chamber
  - (6) Opening of the gates at the lower lock head.

The lockman controls these operations, as well as the signalling system ahead of the lock and within the chamber itself, thus being able to halt the ship/convoys ahead of the lock, for example, due to lock/chamber occupancy or within the chamber due to failures of the lock elements and similarly.

In defining the lock queuing systems, the following is assumed:

- (1) The queuing systems are the systems where the sources of arrival patterns are not an integral part of a lock system.
- (2) The ship arrivals may be single (motor ships and self-propelled barges) and in batches as convoys or barge tows (convoys of single ships, pushed and pulled barge tows), random arrivals follow arbitrarily batch arrival probability distribution ( $GI$  – general input).
- (3) All batches of ships or convoys wait until served at the lock (at the lock anchorage and/or at the lock chamber).
- (4) The service channels are an independent single lock chamber and two parallel lock chambers as it is case at lock operations on Upper and Middle Danube. Two different service policies are in use. The ships are serviced as single ships and in

batches of variable size with a minimum size ( $a$ ) when lock starts the service and a maximum convoy size or maximum capacity of ship lock chamber ( $b$ ). In the case of the minimum batch (MB), a minimum number ( $a$ ) of ships,  $a < b$ , is sufficient to start the service of the batch. If there are more than ( $b$ ) waiting ships, only ( $b$ ) ships are collected into a batch and serviced together. In the case of full batch policy (FB) with batch size ( $b$ ), the service of the batch is started only after ( $b$ ) ships of the batch have arrived. The batch service times in lock are independently distributed according to arbitrary distribution ( $G$  – general probability distribution). In some cases, the negative exponential distribution and other distributions may be preferred as a useful first approximation and a base for derivation of the parameters resulting from another distribution [1], [2], and [3].

- (5) The batch/convoy size is a random variable. However, the queuing systems here are systems in which the number of ships or barges in convoy/tow has uniformly distributed batch sizes of the arriving ships/barge tows.
- (6) The queue discipline is first-come-first-served (FCFS) by batch/convoys and random within the convoys. It is assumed that the lock system does not offer priorities for the service of ships/convoys.

Nevertheless, the queuing systems discussed here are the systems in which enter/exit times of batch/convoy to/from the lock chamber are assumed as random and time-homogenous, which means that lock may fail not only when it works, but it may fail even when it is idle. Also, it is assumed that the times of water filling/discharging of the lock chamber are randomly distributed [1].

## 2. LITERATURE REVIEW

The real lock operations with ships and barge tows are considered as the queuing system with batch arrivals and batch service. The basic objective of the application of the queuing theory is the determination of explicit values for different lock performance measures such as: the average number of ships/convoys in lock system ( $L$ ), the mean queue length of ships/barges ( $L_q$ ), the average time of ships/convoys in lock system ( $W$ ), the average time of ships/convoys in the queue ( $W_q$ ), and the average waiting time/average service time ratio ( $\gamma$ ) depending on the utilization factor or lock occupancy ( $\rho$ ), as well as other ship lock operating parameters [1].

Some studies have been conducted in relation to the ship lock operation to date [4], [5], [6], [7], [8]. All these studies referred to ship locks as different queuing systems and networks, heuristic and simulation

models. However, none of these authors dealt with ship and lock operations as a common and independent service system.

The ship/convoy – ship lock link have been considered as  $M^X / M^{a,b}(RB) / M(R) / 1$  delayed system [1]. In extended Kendall notation these symbols have the following meanings:  $M$  – Poisson distribution of ship/convoy arrivals to the lock system and batch service (lockage) time with negative exponential distribution;  $X$  – random variable of batch/convoy size/number of barges in tow;  $a$  – quorum for batch service or minimum convoy size for service commencement;  $b$  – maximum capacity for batch service or maximum number of ships in lock chamber;  $RB$  – enter/exit and preparation times for ships/convoys and random breakdowns in lockage processes;  $M(R)$  – times of water filling/discharging of lock chamber are random variables with exponential probability distributions; and 1 – independent lock with one chamber in the inland waterway.

By using mathematical derivations referred to in [2], [3], [9] and [10], the explicit results for performance measures of the ship/convoy – ship lock link as single queuing system with batch arrivals and batch service have been developed. All results can serve for different analyses and estimations, for example, the estimate of total amount of cargo and number of ships and convoys passing through the lock during the considered time periods depends on the acceptable level of average waiting time / average service time ratio, the analysis of ship/convoy lockage time spent in the lock system depending on the lock occupancy, mean arrival and mean service rates, preparation times and time of water filling/discharging of lock chamber.

## 3. SHIP/CONVOY – SHIP LOCK LINK AS $GI^X / G^{b,b} / c$ AND $GI^X / G^{a,b} / c$ QUEUING SYSTEMS

### 3.1 Approximations for the mean queue length of ships/convoys in the $GI^X / G^{b,b} / c$ and $GI^X / G^{a,b} / c$ queues

From the literature [11], [12] it is known that the queuing systems with general batch arrivals and batch service patterns cannot be treated analytically. For these reasons, for approximating the performance measures of the  $GI / G / c$  queue ( $GI$  – single random ship arrivals follow arbitrary probability distribution,  $G$  – single ship service time with arbitrary probability distribution,  $c$  – number of parallel ship locks or chambers), as well as,  $GI^X / G^{b,b} / c$  queue ( $GI^X$  – bulk random ship arrivals with arbitrary probability distribution and  $G^{b,b}$  – batch service time with arbitrary probability distribution), the Allen/Cunneen and Kraemer/

/Langenbach-Belz formulae are applied for the mean queue length, which only depends on the first two moments of the inter-arrival and service time distributions [11], [12].

Firstly, the general definition for the coefficient of variation as the normalized standard deviation ( $\sigma_x$ ) can be written as:

$$C_x = \frac{\sigma_x}{\bar{X}} \quad (1)$$

where:

$\bar{X}$  – mean value or expected value.

The service rule of ship lock is  $(b, b)$  or full batch service policy (FB), i. e. the batch size service must be exactly  $b$  ships and if the number present, when one of the ship chambers or lock becomes idle, is less than  $(b)$  ships, the ship chamber/lock waits until  $(b)$  ships accumulates. The service time ( $S$ ) needed to process a batch of ships, is arbitrarily distributed with mean  $E(S) < \infty$  and squared coefficient of variation of batch service time  $C_B^2 < \infty$ . The lock's anchorage capacity is supposed to be unlimited.

The coefficient of variation  $C_{A^b}^2$  of the inter-arrival time of batches with batch size  $(b)$  ships can be derived (see [11] for further details):

$$\frac{C_{A^b}^2}{2} = \frac{\bar{X}}{b} (C_X^2 + C_{A^X}^2) \quad (2)$$

with the mean value of the size of the arriving batches/convoys of ships  $\bar{X}$ , the coefficient of variation of the size of the arriving batches of ships  $C_X^2$  and the coefficient of variation of the inter-arrival time of arriving batches of ships  $C_{A^X}^2$ . By using the Allen/Cunneen formula found in [11], [12] for the mean waiting time ( $W_q$ ) in a  $GI/G/c$  FCFS system and Little's law, the mean queue length ( $L'_q$ ) of the batches of ships, in a  $GI/G^{b,b}/c$  queuing lock system, can be obtained as:

$$L'_q \approx \frac{\rho P_c}{1-\rho} \cdot \frac{C_{A^b}^2 + C_B^2}{2} + \frac{b-1}{2} \quad (3)$$

where  $P_c$  is the probability of blocking.  $P_c$  can be approximated by Erlang's formula for the  $M/M/c$  queue [12] or:

$$P_c = \frac{\frac{(c\rho)^c}{c!} \frac{1}{1-\rho}}{\sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!} \frac{1}{1-\rho}} \quad (4)$$

By using the Kraemer/Langenbach-Belz formula, which produces more accurate results, the approximate queue length of the batches of ships [11] can be obtained as:

$$L'_q \approx \frac{\rho P_c}{1-\rho} \cdot \frac{C_{A^b}^2 + C_B^2}{2} G_{KLB} + \frac{b-1}{2} \quad (5)$$

where

$$G_{KLB} = \begin{cases} \exp \left[ -\frac{2}{3} \frac{1-\rho}{P_c} \frac{(1-C_{A^b}^2)^2}{C_{A^b}^2 + C_B^2} \right] & \text{for } C_{A^b}^2 \leq 1 \\ \exp \left[ -(1-\rho) \frac{C_{A^b}^2 - 1}{C_{A^b}^2 + 4C_B^2} \right] & \text{for } C_{A^b}^2 > 1 \end{cases} \quad (6)$$

Now, the mean queue length of the individual ships is approximately given by [11]:

$$L_q \approx bL'_q + \frac{b-1}{2} \quad (7)$$

The first term in this formula is due to the ships in the  $(L'_q)$  batches of the queue, and the second term is the mean number of ships that are still not combined into a batch/convoy. The accuracy of Eq. (7) depends mainly on the accuracy of the approximation formula for  $(L'_q)$  [11].

The mean queue length of the individual ships can now be calculated by replacing the mean queue length  $(L'_q)$  of the batches of ships (Eq. (3) in Eq. (7) and Eq. (5) in Eq. (7)):

$$L_{qAC} \approx b \frac{\rho P_c}{1-\rho} \cdot \frac{C_{A^b}^2 + C_B^2}{2} + \frac{b-1}{2} \quad (8)$$

and

$$L_{qKLB} \approx b \frac{\rho P_c}{1-\rho} \cdot \frac{C_{A^b}^2 + C_B^2}{2} G_{KLB} + \frac{b-1}{2} \quad (9)$$

On the other side, the accuracy of Eq. (8) and especially of Eq. (9) is very high for  $\bar{X} \leq b$  [11]. In the case of  $\bar{X} > b$ , Eq. (9) gives a lower bound for the mean number of ships at the lock anchorage.

As appointed by [12] good results are achieved if

$$0.25 < C_{A^b}^2 \leq 1.25$$

and

$$0.25 < C_B^2 \leq 1.25.$$

In view of Eq. (2), it is therefore to be expected that this approach will fail if  $(\bar{X})$  is substantially larger than  $(b)$ . It is seen that accuracy decreases with increasing coefficient of variation of inter-arrival times. If  $\bar{X}/b \geq 1$ , an application of the Eqs. (8) and (9) is meaningful only for moderate values  $C_X^2$  and  $C_{A^X}^2$  [12].

In this case the service rule of ship lock is  $(a, b)$  or minimum batch service policy (MB), i. e. minimum number  $a (< b)$  of ships is sufficient to start the lockage of the ship's convoy. The ship lock capacity is maximum  $(b)$  ships/convoys per one lockage, until more than  $(b)$  ships must wait in lock anchorage for the next lockage.

By using the Kraemer/Langenbach-Belz formula we try to determine the approximate values of the

mean queue length of ships/convoys in  $GI^X / G^{a,b} / c$  queuing systems as heuristic extension [11]. The mean queue length of ships can be determined as:

$$L_q \approx b \frac{\rho P_c}{1-\rho} \cdot \frac{C_A^2 + C_B^2}{2} G_{KLB} + P_c \frac{b-1}{2} + (1-P_c) \frac{a-1}{2} \quad (10)$$

### 3.2 Analytical approximate results of the mean queue length of ships/convoys at ship locks on the Danube river

The ship/convoys – ship lock link is examined as the delayed, queuing systems. Under state assumptions, the  $GI^X / G^{b,b} / c$  and  $GI^X / G^{a,b} / c$  queuing

systems are used for the operational analysis of the real lock operations with ships and barge tows during the lockage time. For the present, analytical performance measures cannot be obtained in the queuing theory. However, for approximating the performance values of the  $GI / G / c$  queuing system or, more precisely, the mean queue length of ships/convoys at lock anchorage Allen/Cunneen and Kraemer/Langenbach-Belz approximation formulae are used, which only depend on the first two moments of the inter-arrival and service time pattern distribution. The approach is based on a three-part classification of delay: batching time, waiting time and service time of ships/convoys.

The barge tows and motor cargo ships are homogeneous customers since integrated, pushed and pulled

**Table 1 - Mean queue length of ships,  $L_q$ , for a  $GI^X / G^{b,b} / 1$  ship lock queuing system with uniform distributed batch sizes of the arriving ships with  $P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = 0.25$**

Lock occupancy $\rho$	Coefficients of variations		Approximations for ship lock queuing system			
	$C_{A^X}$	$C_B$	$GI^X / G^{1,1} / 2$	$GI^X / G^{2,2} / 2$	$GI^X / G^{3,3} / 2$	$GI^X / G^{4,4} / 2$
			<i>KLB</i>	<i>KLB</i>	<i>AC</i>	<i>AC</i>
0.2	1.2	1.2	0.1009	0.6488	1.1980	1.7340
0.2	1.0	1.0	0.0727	0.6083	1.1375	1.6625
0.2	1.2	0	0.0505	0.5631	1.0889	1.5900
0.2	1.1	0.7	0.0635	0.5890	1.1124	1.6246
0.2	0.7	1.1	0.0591	0.5732	1.1214	1.6516
0.2	0	1.2	0.0056	0.5113	1.1080	1.6440
0.2	0.7	0.7	0.0405	0.5383	1.0674	1.5796
0.2	0.5	0.5	0.0143	0.5030	1.0344	1.5406
0.5	1.2	1.2	1.0793	2.0365	2.9799	3.8400
0.5	1.0	1.0	0.7796	1.5985	2.3750	3.1250
0.5	1.2	0	0.6272	1.2206	1.8990	2.4000
0.5	1.1	0.7	0.6488	1.4296	2.1237	2.7463
0.5	0.7	1.1	0.4172	1.3625	2.2137	3.0163
0.5	0	1.2	0.2266	0.6130	2.0800	2.9400
0.5	0.7	0.7	0.4139	0.9718	1.6737	2.2963
0.5	0.5	0.5	0.1965	0.6077	1.3437	1.9063
0.7	1.2	1.2	3.7508	5.6265	7.4677	9.1438
0.7	1.0	1.0	2.6670	4.1227	5.4916	6.8082
0.7	1.2	0	2.3672	3.0729	3.9399	4.4399
0.7	1.1	0.7	2.5413	3.6291	4.6708	5.5710
0.7	0.7	1.1	1.9694	3.4073	4.9649	6.4530
0.7	0	1.2	0.9643	2.4286	4.5280	6.2039
0.7	0.7	0.7	1.3711	2.2319	3.2008	4.1010
0.7	0.5	0.5	0.6838	1.2270	2.1229	2.8270

AC – Allen/Cunneen approximation formula, *KLB* – Kraemer/Langenbach-Belz approximation formula

barge tows or motor vessels must be standardized according to the current navigable conditions and especially according to lock chamber dimensions in view of the main particulars: length, breadth, draught and carrying capacity. For instance, the corresponding pushed barge tows on the Danube are the tows with two and four barges of the types Europe I and II in the German sector in the formation of P+1+1, P+2, and P+2+2 (P – pushboat), in the Austrian and Slovakian sector – P+2+2, P+3+3, and P+2+2+2, and in Serbian-Romanian sector – P+3+3+3 according to the dimensions of lock chambers.

In the first case for  $GI^X / G^{b,b} / c$  queue, the numerical results and graphs for the ship/convoy – ship lock link for  $GI^X / G^{1,1} / 1$ ,  $GI^X / G^{2,2} / 1$ ,  $GI^X / G^{3,3} / 1$ ,  $GI^X / G^{4,4} / 1$ ,  $GI^X / G^{1,1} / 2$ ,

$GI^X / G^{2,2} / 2$ ,  $GI^X / G^{3,3} / 2$  and  $GI^X / G^{4,4} / 2$  queuing systems are presented by applying the mentioned assumptions, when the number of ships in convoys uniformly distributed of the arriving batches/convoys with the following probabilities:

$$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = 0.25$$

The coefficients of variation for inter-arrival times of arriving batches of ships ( $C_{AX}$ ) and of batch service time  $C_B$  are adopted as usual values in the pairs: (1.2, 1.2), (1, 1), (1.2, 0), (1.1, 0.7), (0.7, 1.1), (0, 1.2), (0.7, 0.7), and (0.5, 0.5), and the lock occupancy  $\rho = 0.2, 0.5$ , and  $0.7$ .

If the number of ships in batches/convoys is uniformly distributed, then the variance of random variable of convoy size ( $X$ ),  $\sigma_X^2 = 0$  has been adopted,

**Table 2 - Mean queue length of ships,  $L_q$ , for a  $GI^X / G^{b,b} / 2$  ship lock queuing system with uniform distributed batch sizes of the arriving ships with  $P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = 0.25$**

Lock occupancy $\rho$	Coefficients of variations		Approximations for ship lock queuing system			
	$C_{AX}$	$C_B$	$GI^X / G^{1,1} / 2$	$GI^X / G^{2,2} / 2$	$GI^X / G^{3,3} / 2$	$GI^X / G^{4,4} / 2$
			KLB	KLB	AC	AC
0.2	1.2	1.2	0.0336	0.5496	1.0660	1.5779
0.2	1.0	1.0	0.0274	0.5361	1.0458	1.5541
0.2	1.2	0	0.0168	0.5210	1.0300	1.5300
0.2	1.1	0.7	0.0211	0.5514	1.0374	1.5415
0.2	0.7	1.1	0.0197	0.5244	1.0404	1.5505
0.2	0	1.2	0.00005	0.5038	1.0360	1.5479
0.2	0.7	0.7	0.0135	0.5128	1.0224	1.5265
0.2	0.5	0.5	0.0020	0.5010	1.0114	1.5135
0.5	1.2	1.2	0.7310	0.5243	2.3198	3.0598
0.5	1.0	1.0	0.5197	1.2323	1.9165	2.5832
0.5	1.2	0	0.4181	0.9804	1.5998	2.0999
0.5	1.1	0.7	0.4777	1.1199	1.7491	2.3307
0.5	0.7	1.1	0.3983	1.0750	1.8091	2.5107
0.5	0	1.2	0.1198	0.5753	1.7191	2.4593
0.5	0.7	0.7	0.2759	0.8145	1.4491	2.0308
0.5	0.5	0.5	0.1242	0.5717	1.2291	1.7708
0.7	1.2	1.2	3.1187	4.7222	6.2366	7.7953
0.7	1.0	1.0	2.1966	3.4837	4.6808	5.8717
0.7	1.2	0	1.9497	2.6191	3.4211	3.9213
0.7	1.1	0.7	2.0930	3.0273	4.0232	4.8528
0.7	0.7	1.1	1.6196	2.8945	4.2653	5.5792
0.7	0	1.2	0.7612	2.0884	3.9055	5.3740
0.7	0.7	0.7	1.1293	1.9624	2.8126	3.6421
0.7	0.5	0.5	0.5566	1.0952	1.9248	2.5929

AC – Allen/Cunneen approximation formula, KLB – Kraemer/Langenbach-Belz approximation formula

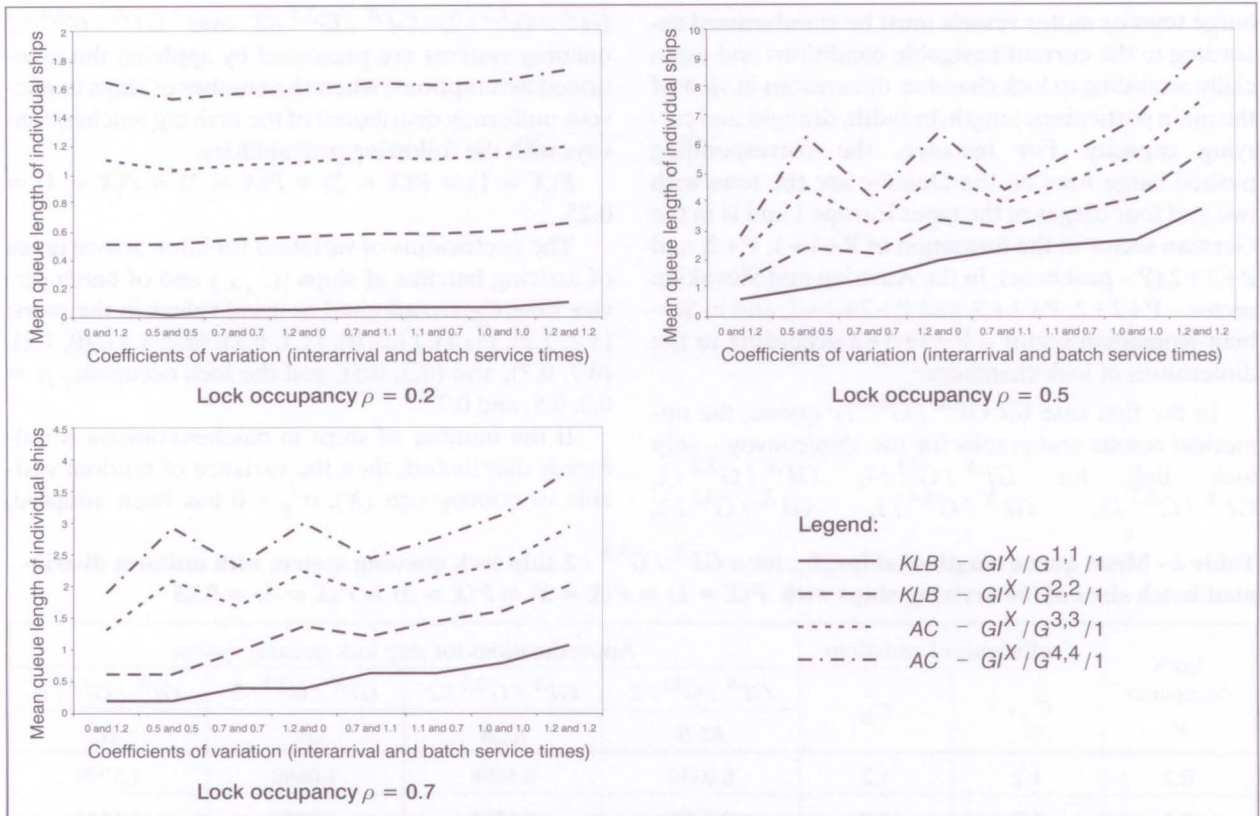


Figure 1 - Queuing analytical solutions for mean queue length of individual ships of  $G1^X/G^{b,b}/1$  ship lock queuing system

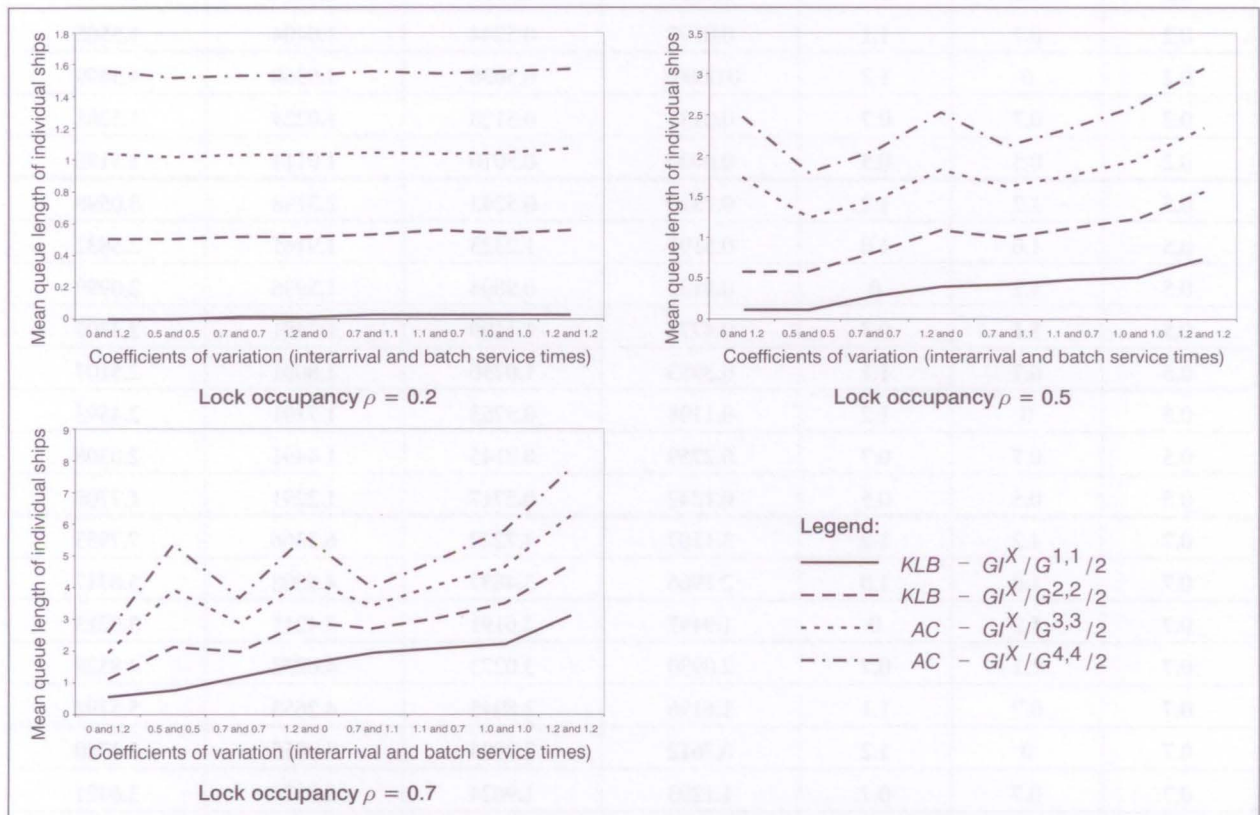


Figure 2 - Queuing analytical solutions for mean queue length of individual ships of  $G1^X/G^{b,b}/2$  ship lock queuing system

and then the coefficient of variation of the size of the arriving convoys of ships according to Eq. (1) is  $C_X^2 = 0$ .

By using Allen/Cunneen approximation formula (*AC* – Eq. (8)) and Kraemer/Langenbach-Belz approximation formula (*KLB* – Eq. (9)) some of our results are given in Tables 1 and 2. *AC* formula and *KLB* formula have been applied for  $\bar{X} \leq b$  and  $\bar{X} > b$ , respectively, when the number of parallel ship locks or chambers is equal to  $c = 1$  and  $c = 2$ .

The Figs. 1 and 2 show the variation of the mean queue length ( $L_q$ ) of the individual ships with  $b = 1, 2, 3$ , and 4, depending on the coefficients of variation of inter-arrival and batch service times in a full batch service policy in the ship lock.

In the second case for  $GI^X / G^{b,b} / c$  queue, the mean queue length of the individual ships can be presented with numerical results for the ship/convoy – ship lock link in  $GI^X / G^{1,4} / 1$  and  $GI^X / G^{1,4} / 2$  queuing systems, when the number of ships in convoys uniformly distributed of the arriving batches/convoys with the following probabilities:

$$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = 0.25$$

and

$$P(X = 1) = 0.1, P(X = 2) = 0.135, \\ P(X = 3) = 0.135, P(X = 4) = 0.63,$$

according to the methods of navigation used on the Danube. Push-towing has become predominant on the Danube with a share of 63%, while the share of

**Table 3 - Mean queue length of ships,  $L_q$ , for a  $GI^X / G^{a,b} / 2$  ship lock queuing system with uniform distributed batch sizes of the arriving ships**

Lock occupancy $\rho$	Coefficients of variations		Approximations for ship lock queuing system - <i>KLB</i>			
	$C_{A^X}$	$C_B$	$GI^X / G^{1,4} / 1$	$GI^X / G^{1,4} / 2$	$GI^X / G^{1,4} / 1$	$GI^X / G^{1,4} / 2$
			P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = 0.25		P(X = 1) = 0.1, P(X = 2) = 0.135, P(X = 3) = 0.135, P(X = 4) = 0.63	
0.2	1.2	1.2	0.5313	0.1753	0.5570	0.1857
0.2	1.0	1.0	0.4290	0.1271	0.4742	0.1530
0.2	1.2	0	0.3874	0.1274	0.4046	0.1386
0.2	1.1	0.7	0.4097	0.1283	0.4478	0.1495
0.2	0.7	1.1	0.3650	0.1040	0.3896	0.1092
0.2	0	1.2	0.3226	0.1002	0.3226	0.1002
0.2	0.7	0.7	0.3159	0.1002	0.3309	0.1002
0.2	0.5	0.5	0.3004	0.1000	0.3011	0.1000
0.5	1.2	1.2	3.0835	2.0531	3.3413	3.0913
0.5	1.0	1.0	2.2839	1.8667	2.5532	1.6952
0.5	1.2	0	1.6434	1.0928	1.8416	1.2311
0.5	1.1	0.7	1.9572	1.2921	2.2367	1.4910
0.5	0.7	1.1	1.9771	1.2358	2.3401	1.0082
0.5	0	1.2	1.6564	0.9793	1.6564	0.9793
0.5	0.7	0.7	1.2822	0.7900	1.3227	0.5000
0.5	0.5	0.5	0.8763	0.5469	1.1704	0.1762
0.7	1.2	1.2	8.6848	7.1601	9.5600	7.8227
0.7	1.0	1.0	6.2287	5.1071	6.9785	5.7419
0.7	1.2	0	3.9805	3.2568	4.7466	3.9089
0.7	1.1	0.7	5.0660	4.1629	5.9064	4.8640
0.7	0.7	1.1	5.5735	4.5186	5.9994	4.8857
0.7	0	1.2	4.9075	3.9094	4.9077	3.9091
0.7	0.7	0.7	3.2385	2.6017	3.6552	2.9584
0.7	0.5	0.5	3.0171	1.4598	2.0531	1.6237

*KLB* – Kraemer/Langenbach-Belz approximation formula

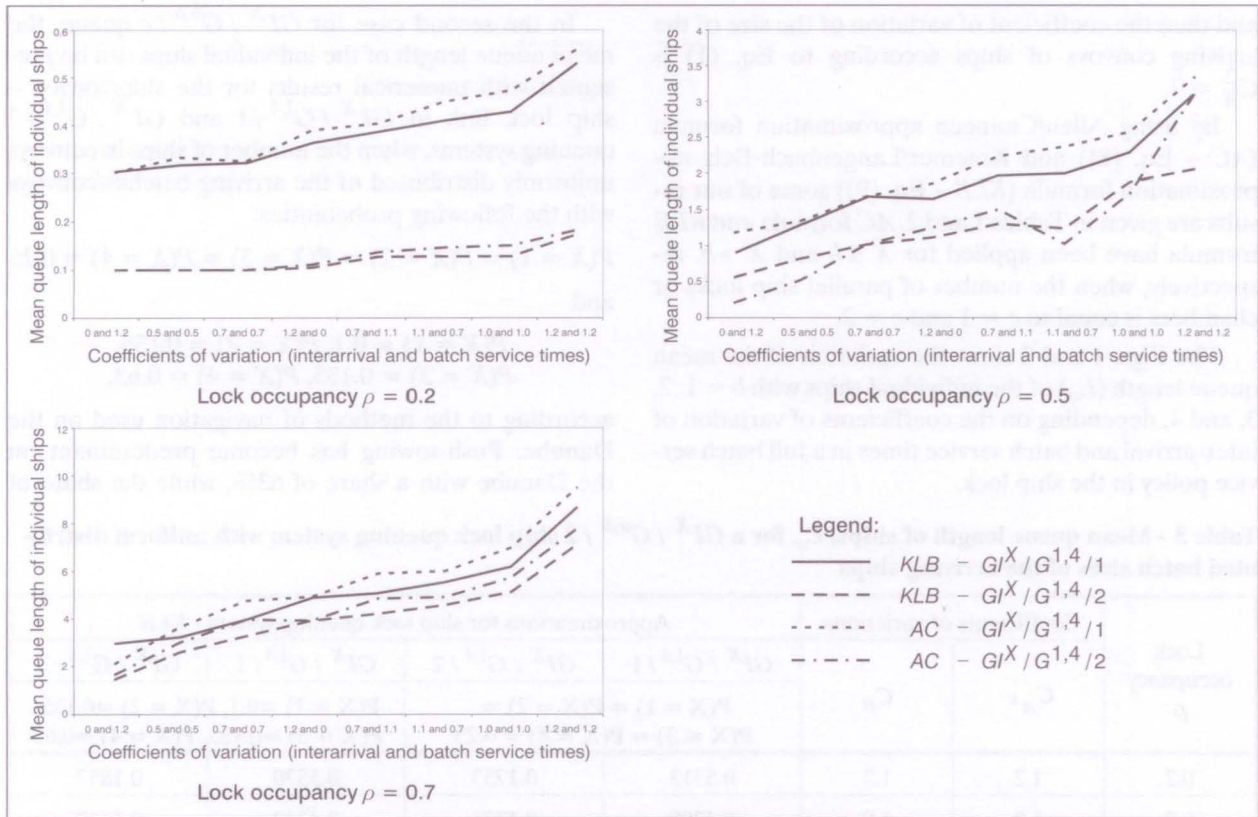


Figure 3 - Queuing analytical solutions for mean queue length of individual ships of  $GI^X/G^{a,b}/1$  and  $GI^X/G^{a,b}/2$  ship lock queuing system

self-propelled ships remains at about 10%, combined tows-motor cargo ships plus one barge – 13.5% and pull-towing system – 13.5% of the total fleet tonnage [13]. The coefficients of variation for inter-arrival times of arriving batches of ships  $C_{A^X}$  and the batch service time  $C_B$  have values as in the previous section. The lock occupancies are equal to  $\rho = 0.2, 0.5,$  and  $0.7$ . These numerical results are presented in Table 3. Figure 3 shows the variation of the mean queue length ( $L_q$ ) of an individual ships with  $a = 1$  and  $b = 4$ , when number of lock chamber equals  $c = 1$  and  $c = 2$ .

#### 4. CONCLUSIONS

The major conclusions obtained in this paper can be divided into general and particular conclusions in terms of application of general queuing systems with batch arrivals and batch service in the ships/convoys – ship lock link on waterways.

The general conclusions are as follows:

- (1) The curves in Figures 1, 2 and 3 and computation results in Tables 1, 2 and 3 may be used for the analysis of ship lock operations or the mean queue length of ships/convoys at lock anchorage depending on the probabilities by uniformly distributed batch sizes of the arriving ships, number

of locks or lock chambers, lock occupancy and coefficient of variation of the size of arriving batches of ships, coefficient of variation of inter-arrival time of arriving batches of ships, and coefficient of variation of batch service time.

- (2) The quality level of the lock is determined by the mean queue length or the average waiting time of ships/convoys at the ship lock. It means that the lock capacity that corresponds to this quality level can be determined by taking into account the diversity of vessel types and an imbalance in traffic intensity per direction.
- (3) There are few examples of ship lock, which offer quantifications of the relation between the intensity of traffic and congestion at the lock [14]. Our results offer different possibilities of determining the congestion costs for inland waterway transport. In the theoretical framework, general mean queue length of ships/convoys can be valued and analysed for the different type of vessels and different types of cargo according to the provided lock occupancy.
- (4) The analysis may be also carried out in the following cases: the changes of existing conditions (increase of lock capacity, changes in the number of lock chambers, changes of convoy sizes, and changes of lock occupancy) and forecasts for the lock development.



In particular, for the general queuing systems with batch arrivals and batch service when there are full and minimum batch service policies the following laws can be derived:

- (1) At all degrees of lock occupancy  $\rho$  and all coefficients of variation of inter-arrival time and all coefficients of variations of service time, more values have the mean queue length of ships/convoys when the batch size ( $b$ ) increases in case of full batch service policy. This value decreases with increasing the number of lock chambers, as shown in Tables 1 and 2 and in Figures 1 and 2.
- (2) The mean queue length of ships/convoys mainly depends on the coefficients of variation of inter-arrival time for batch arrivals then on the coefficient of variation of service time. If the coefficient of variation of inter-arrival time increases then the values for the mean queue length also increase for all degrees of lock occupancy and full and minimum batch service policies.
- (3) The mean queue length of ships/convoys has fewer values when the probabilities of occurrence by one ship and two ships, three ships and four ships in tows are equal as the arrivals at ship lock for minimum batch service policy, as shown in Table 3 and Fig. 3.

Finally, the results obtained in this paper are restricted because of the approximations about inter-arrival and service time distributions, as well as the convoy size uniform probability distribution. In other words, the variables that affect these results are numerous and it is not possible to take all of them into account.

Nevertheless, the conveniences of the methodology are the obvious applications in the estimates of existing conditions and the planning of lock requirements, lock management, and better utilization of inland waterway transport. All the numerical results can be very easily extended for other requirements.

Dr. sc. **ZORAN RADMILOVIĆ**

E-mail: z.radmilovic@sf.bg.ac.yu

Mr. sc. **VLADISLAV MARAŠ**

E-mail: marasv@inffo.net

Univerzitet u Beogradu, Saobraćajni fakultet

Vojvode Stepe 305, 11000 Beograd, Republika Srbija

**SASA JOVANOVIĆ**, dipl.ing.

E-mail: sjovanovic73@yahoo.co.uk

Universitat Politècnica de Catalunya

Departament de Ciència i Enginyeria Nàutiques

Pla del Palau 18, 08003 Barcelona, Reino de España

## SAŽETAK

### BRODSKA PREVODNICA KAO OPŠTI SISTEM OPSLUŽIVANJA SA GRUPNIM DOLASCIMA I GRUPNIM OPSLUŽIVANJEM

*Realne operacije prevodenja brodova i sastava plovila razmatraju se zavisno od primenjene transportne tehnologije, ili, tačnije, vrste brodova/sastava koji zahtevaju prevodenje. Flota može biti podeljena na sledeći način: (1) grupe pojedinačnih brodova, (2) sastavi potisnica i tegljenica i (3) razne kombinacije prethodnih sistema (1) i (2). Grupe brodova i sastava koji prolaze kroz prevodnicu imaju ekstremno stohastičke osobenosti koje uzrokuju razni dolasci i vreme opsluživanja. To znači da ravnomerno kretanje ili strogi red plovidbe između prevodnica i operacija prevodenja nije moguće čak iako se primenjuju najsavremenija sredstva upravljanja. Prema tome, u ovom radu se razvija analitički metod koji koristi grupne sisteme opsluživanja za analizu i planiranje zahteva za prevodenjem koji je podržan sa numeričkim primerom. Razvijena metodologija može biti primenjena za određivanje srednje dužine reda brodova-sastava na sidrištu prevodnice bez međusobnog uticaja uzvodne i nizvodne plovidbe ili za jednosmerni saobraćaj.*

## KLJUČNE REČI

*monitoring saobraćajne infrastrukture, upravljanje brodom prevodnicom, grupni sistemi opsluživanja, unutrašnja plovidba*

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