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A RESTITUTION MODEL OF TWO-CAR COLLINEAR COLLISIONS

ABSTRACT

In the paper two-car collinear collisions are discussed using Newton's law of mechanics, conservation of energy and linear constitutive law connecting impact force and crush. Two ways of calculating the mutual restitution coefficient are given: one based on car masses and one based on car stiffness. A numerical example of an actual test is provided.

KEY WORDS

car accidents, car stiffness, impact mechanics, restitution

1. INTRODUCTION

For the modelling of the collinear car collision two methods are usually used. The first is the so-called impulse-momentum method based on classical Poisson impact theory, which replaces the forces with the impulses ([3], [11]). The second method treats a car as a deformable body; so the constitutive law connecting contact force with crush is necessary. For the compression phase of impact the linear model of force is usually adopted and the models differ in the way the restitution phase of collision is treated ([7], [13], [14], [17]).

The purpose of this paper is to extend the linear force model discussed in [1] to the collinear impact of two cars. In the quoted article it is proposed that a car is characterized by its mass, stiffness and limit velocity for permanent crush. The latter properties can be established by a fixed barrier crush test. Also, the proposed restitution model is simple: rebound velocity is constant. The question arises as to how these characteristics can be incorporated into the two-car collision model since it is well known that the mutual coefficient of restitution is the characteristic of impact; i.e. it is a two-car system and not the property of an individual car ([2], [17]).

To answer the above question, first the well-known theory of central impact is specialized for collinear car collisions. The kinetic energy losses are then discussed and the restitution coefficient is related to them. The third section of the paper discusses two models for calculating the mutual restitution coefficient based on individual car characteristics. The last section is devoted to a description of the use of the present theory in accident reconstruction practice. The section ends with a numerical example.

2. TWO-CAR COLLINEAR COLLISION

Consider a collinear impact between two cars where collinear impact refers to rear-end and head-on collisions. Before impact the cars have velocities v_1 and v_2 respectively and after impact they have velocities u_1 and u_2 (Figure 1).

In the collision phase the movement of cars is governed by Newton's 2nd and 3rd laws (Figure 2). On the basis of these laws equations of motion of the cars can be written as follows

$$m_1 \frac{dv_1}{dt} = -F \quad \text{and} \quad m_2 \frac{dv_2}{dt} = F \tag{1}$$

where m_1 and m_2 are the masses of the cars and F is contact force.

Following Poisson's hypothesis ([16]) the impact is divided into two phases: compression and restitution. In the compression phase the contact force F rises and the cars are deformed. The compression phase terminates when the relative velocity of cars vanishes; i.e., when cars have equal velocity (Figure 1). The compression phase (1) thus integrates the changes from initial velocities to common velocity u. This leads to the following system of equations

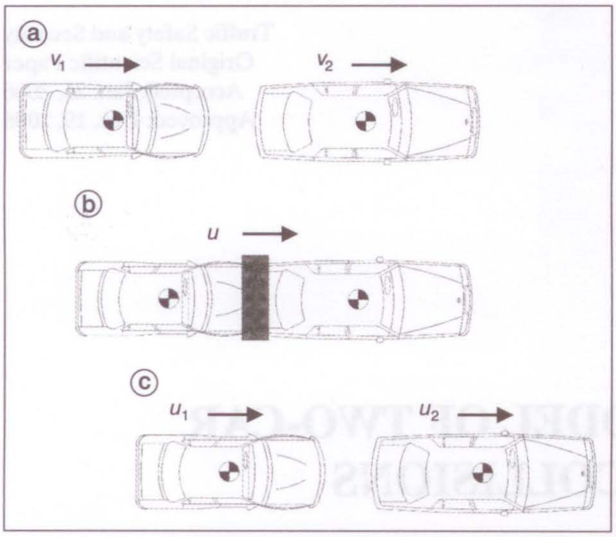

Figure 1 - The two-car impact: (a) pre-impact velocities, (b) end of compression velocity, (c) post-impact velocities

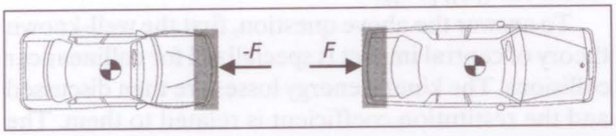

Figure 2 - Newton's 3rd law applied to collinear impact of two cars

$$m_1(u - v_1) = -P_c \quad m_2(u - v_2) = P_c \tag{2}$$

where $P_c \equiv \int_0^{\tau_c} F dt$ is compression impulse and τ_c compression time. From (2) one obtains the velocity after compression

$$u = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{3}$$

and the compression impulse

$$P_c = \frac{m_1 m_2}{m_1 + m_2}(v_1 - v_2) \tag{4}$$

In the restitution phase the elastic part of internal energy is released. Equations (1) are integrated from u to the end velocities, which gives two equations for three unknowns

$$m_1(u_1 - u) = -P_r \quad m_2(u_2 - u) = P_r \tag{5}$$

where $P_r \equiv \int_0^{\tau_c} F dt$ is restitution impulse and τ_r is restitution time. In order to solve system (5) for an unknown's post-impact velocity and restitution impulse the constitutive equation is needed. According tothe Poisson hypothesis the restitution impulse is proportional to compression impulse

$$P_r = e P_c \tag{6}$$

where e is the restitution coefficient. Because contact force is non-negative, so are the compression and restitution impulse. From (6) this implies that $e \geq 0$.

Note: Instead of one can use Newton's kinematical definition of restitution coefficient

$$e = \frac{u_2 - u_1}{v_1 - v_2}$$

which is in the case of centric impact without friction equivalent to Poisson's definition. However, in the case of non-centric impact with friction Newton's model could lead to overall energy increase ([12]).

The total impulse is $P = P_c + P_r$ so by using (4) and (6)

$$P = (1+e)\frac{m_1 m_2}{m_1 + m_2}\Delta v$$

Solving (5) and (6) and taking into account (4) gives the well known formulae (see for example [3], [11]) for the cars post-impact velocities

$$u_1 = u - e\frac{m_2}{m_1 + m_2}\Delta v = v_1 - \frac{(1+e)m_2}{m_1 + m_2}\Delta v$$
$$u_2 = u + e\frac{m_1}{m_1 + m_2}\Delta v = v_2 - \frac{(1+e)m_1}{m_1 + m_2}\Delta v \tag{8}$$

where $\Delta v = v_1 - v_2$. The above equations can be used for calculation of post-impact velocities if pre-impact velocities are known, masses of cars are known and, in addition, the restitution coefficient is known.

3. ENERGY CONSIDERATION

At car impact the kinetic energy is dissipated. Applying the principle of conservation of energy one obtains, after compression,

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{(m_1 + m_2)u^2}{2} + \Delta E_m \tag{9}$$

where ΔE_m is maximal kinetic energy lost (or maximal energy absorbed by crush). By using (3) one gets

$$\Delta E_m = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\Delta v^2 \tag{10}$$

Similarly, by applying the principle of conservation of energy to the overall impact process

$$\frac{m_1 v_1^2}{2} + \frac{m_1 v_1^2}{2} = \frac{m_1 u_1^2}{2} + \frac{m_1 u_1^2}{2} + \Delta E \tag{11}$$

one finds the well known formula for total kinetic energy lost (see for example [11])

$$\Delta E = \frac{1}{2}(1 - e^2)\frac{m_1 m_2}{m_1 + m_2}\Delta v^2 \tag{12}$$

Since, by the law of thermodynamics, $\Delta E \geq 0$, it follows from (12) that $e \leq 1$. Now, from (10) and (12) one has $\Delta E = (1 - e^2)\Delta E_m$, so the mutual restitution coefficient is given by ([11])

$$e=\sqrt{1-\frac{\Delta E}{\Delta E_a}}=\sqrt{\frac{\Delta E_0}{\Delta E_m}} \quad (13)$$

where $\Delta E_0 \equiv \Delta E_m - \Delta E$ is the rebound energy. The formula obtained is the basis for relating the mutual coefficient of restitution e with the restitution coefficients obtained for individual cars in the fixed barrier test.

4. THE MUTUAL COEFFICIENT OF RESTITUTION

Let v_{T1} be a barrier test velocity of the first car and v_{T2} a barrier test velocity of the second car. Let these velocities be such that the maximal kinetic energy lost can be written as

$$\Delta E_m = \frac{m_1 v_{T1}^2}{2}+\frac{m_2 v_{T2}^2}{2} \quad (14)$$

and in addition the rebound energy can be written as (see [9])

$$\Delta E_0 = \frac{m_1 e_1^2 v_{T1}^2}{2}+\frac{m_2 e_2^2 v_{T2}^2}{2} \quad (15)$$

The mutual restitution coefficient is therefore from (13), (14) and (15), by using (10),

$$e=\sqrt{\frac{m_1 e_1^2 v_{T1}^2+m_2 e_2^2 v_{T2}^2}{m_1 v_{T1}^2+m_2 v_{T2}^2}} \quad (16)$$

For the model of the barrier test proposed in [1] the restitution coefficients of cars are

$$e_1=\min\left(1,\frac{v_{01}}{v_{T1}}\right) \quad \text{and} \quad e_2=\min\left(1,\frac{v_{02}}{v_{T2}}\right) \quad (17)$$

where v_{01} and v_{02} are limited impact velocities where all the crush is recoverable ([1]). The task is now to determine appropriate test velocities of cars which satisfy (14).

4.1 Model A – stiffness-based mutual restitution coefficient.

Let v_{T1} be the barrier test velocity (or barrier equivalent velocity [8]) of the first car for the same crush as in a two-car impact and v_{T2} the barrier test velocity for the same crush for the second car. Then the test velocities for the same crush must satisfy relations ([1], [8])

$$\frac{m_1 v_{T1}^2}{2}=\frac{k_1\delta_{m1}^2}{2} \quad \text{and} \quad \frac{m_2 v_{T2}^2}{2}=\frac{k_2\delta_{m2}^2}{2} \quad (18)$$

where k_1 and k_2 are stiffness of the cars and δ_{m1} and δ_{m2} are actual maximal dynamics crush of the cars. From (18) one has

$$v_{T1}=\sqrt{\frac{k_1}{m_1}}\delta_{m1} \quad \text{and} \quad v_{T2}=\sqrt{\frac{k_2}{m_2}}\delta_{m2} \quad (19)$$

On the other hand, from (10), (14) and (18) it follows that

$$\Delta E_m=\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\Delta v^2=\frac{k_1\delta_{m1}^2}{2}+\frac{k_2\delta_{m2}^2}{2} \quad (20)$$

Defining overall maximal crush $\delta_m \equiv \delta_{m1}+\delta_{m2}$ and taking into account the law of action and reaction $k_1\delta_{m1}=k_2\delta_{m2}$ one obtains

$$\delta_{m1}=\frac{k_2}{k_1+k_2}\delta_m \qquad \delta_{m2}=\frac{k_1}{k_1+k_2}\delta_m \quad (21)$$

Substituting (21) into (20) yields

$$\Delta E_m=\frac{m\Delta v^2}{2}=\frac{k\delta_m^2}{2} \quad (22)$$

where m is system mass and k is system stiffness, given by

$$m\equiv\frac{m_1 m_2}{m_1+m_2} \qquad k\equiv\frac{k_1 k_2}{k_1+k_2} \quad (23)$$

From (22) one has $\delta_m=\sqrt{\frac{m}{k}}|\Delta v|$ and therefore from (19) the required test velocities are (see also [8])

$$v_{T1}=\sqrt{\frac{k}{k_1}\frac{m}{m_1}}|\Delta v| \quad \text{and} \quad v_{T2}=\sqrt{\frac{k}{k_2}\frac{m}{m_2}}|\Delta v| \quad (24)$$

Substituting (24) into (14) leads to identity $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$ and substituting it into (16) provides the required mutual restitution coefficient

$$e=\sqrt{\frac{k_2 e_1^2+k_1 e_2^2}{k_1+k_2}} \quad (25)$$

This equation for the calculation of e was published by various authors ([4], [5], [15]). Knowing the mass and stiffness of the cars and Δv one can calculate test velocities from (24), restitution of individual cars from (17), the mutual restitution coefficient from (25) and post-impact velocities from (8).

4.2 Model B – mass-based mutual restitution coefficient.

This model does not include cars' stiffness and it is based on (10) and (14) only. Equating (10) and (14) results in the equation

$$m\Delta v^2 = m_1 v_{T1}^2+m_2 v_{T2}^2 \quad (26)$$

for two unknowns. To solve it one could set

$$v_{T1}=v_1-v_0 \qquad v_{T2}=v_2-v_0 \quad (27)$$

where v_0 is a new unknown velocity. Substituting (27) into (14) one obtains after simplification

$$[m_1(v_1-v_0)+m_2(v_2-v_0)]^2=0,$$

thus

$$v_0 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{28}$$

This is in fact the velocity of the centre of the mass of colliding cars. Substituting (28) into (27) yields unknown test velocities

$$v_{T1} = \frac{m_2(v_1 - v_2)}{m_1 + m_2} \qquad v_{T2} = -\frac{m_1(v_1 - v_2)}{m_1 + m_2} \tag{29}$$

Note that in calculation of restitution coefficients (17) the absolute values of test velocities should be used. Substituting (29) into (16) gives the mutual restitution coefficient

$$e = \sqrt{\frac{m_2 e_1^2 + m_1 e_2^2}{m_1 + m_2}} \tag{30}$$

This formula was derived by different arguments of Howard et al. ([9]) and was also qoted by Watts et al. ([18]).

4.3 Comparison of the models

Comparing (24) and (25) one finds that test velocities of both models are the same if stiffness is proportional to the mass; i.e., $k_1 = k_0 m_1$ and $k_2 = k_0 m_2$ where k_0 is a constant.

While the test velocities of the models differ, the mutual restitution coefficient differs only in the case when just one car is crushed permanently, since

- when $v_{T1} \le v_{01}$ and $v_{T2} \le v_{02}$ then both $e_1 = e_2 = 1$ so by (25) or (30) it follows $e = 1$ and
- when $v_{T1} > v_{01}$ and $v_{T2} \le v_{02}$ then substituting (17) and appropriate test velocities into (25) or (30), and taking (10) into account, yields

$$e = \sqrt{\frac{m_1 v_{01}^2 + m_2 v_{02}^2}{m \Delta v^2}} \tag{31}$$

Note that (31) cannot be used directly for calculating the mutual restitution coefficient in advance since the clasfication of impact-fully elastic, fully plastic or mixed -depends on test velocities.

Finally, the question arises as to which model is more physically justified. While Model A has a sound physical base connecting test velocities with crushes, Model B requires some additional analysis. It turns out that it can be interpreted as follows. The compression impulse (4) can be written by using $(23)_1$ as $P_c = m\Delta v$. Using (2) one could define test velocities of individual cars as velocities resulting at the end of the compression phase in a fixed barrier test as the same impulse as in an actual two-car collision; i.e.,

$$P_c = m|\Delta v| = m_1 v_{T1} = m_2 v_{T2} \tag{32}$$

The result of this equation are the test velocities given already by (29). Now, by (6) restitution impulse is $P_r = eP_c = em|\Delta v|$, therefore, by (5) and (32) one must have $em|\Delta v| = e_1 m_1 v_{T1} = e_2 m_2 v_{T2}$. But this can be fulfilled only in the special case when $e_1 = e_2$, and consequently, by (30), when $e = e_1$. This consequence raises a doubt about Model B's adequacy for general use.

4.4 Examples

The above formulae were implemented into the spreadsheet program (Table 1). As an example, a full scale test (Test no. 7) reported by Cipriani et al. ([6]) was executed. In this test the bullet car made impact with the rear of the target car at a velocity of 5 m/s or 18 km/h. The mass of the cars and their stiffness were taken from the report; however, the limit speed was taken to be 4 km/h for both cars ([1]). The result of the calculation is shown in Table 2. The calculated velocity difference for the target car is 14.8 km/h, which differs from that measured (3.9 m/s or 14.0 km/h) by about 5%. The calculated velocity change for the bullet car is 11.3 km/h and the measured one was 2.9 m/s or 10.4 km/h. The discrepancy is thus about 7%. If one takes the limit speed to be 3 km/h, then the calculated value of velocity change for the bullet car is 13.6 km/h, differing from that measured by about 2%, and the calculated value of velocity change for the target car is 10.4, which actually matches the measured value.

Table 1 - Spreadsheet program for calculation of post-impact velocities
Full scale test 7 of Cipriani et al. ([6])

		Vehicle 1		Vehicle 2
mass	kg	1146		1495
stiffness	kN/m	886.07		1564.687
limit velocity	km/h	4		4
impact velocity	km/h	18		0
Delta V	km/h		18.00	
velocity after compression	km/h		7.81	
system mass	kg		648.72	
system stiffness	kN/m		565.71	
test velocity	km/h	10.82		7.13
test restitution		0.37		0.56
restitution			0.45	
post impact velocity	km/h	3.24		11.31
Delta V	km/h	14.76		-11.31
Maximal crush	m	0.11		0.06
Residual crush	m	0.07		0.03

5. ACCIDENT RECONSTRUCTION

In a real car accident the problem is not to determine the post-impact velocities but usually the opposite; i. e., to calculate the pre-impact velocities. For determining pre-impact velocities, however, the post-impact velocities determined from skid-marks should be known. If only the permanent crushes of cars are known then only the velocity changes for individual cars in an accident can be calculated. If the characteristics of cars are known - i. e. mass, stiffness and limit velocity - then the problem is solved as follows. Let δ_{r1} be residual crush of the first vehicle. The maximal crush, then, is ([1])

$$\delta_{m1} = \delta_{r1} + \delta_{01} \tag{33}$$

where the recoverable part of crush is calculated as $\delta_{01} = v_{01}\sqrt{\frac{m_1}{k_1}}$. The maximal crush of the second car can be calculated in the same way or from Newton's 3rd law as

$$\delta_{m2} = \frac{k_1}{k_2}\delta_{m1} \tag{34}$$

The maximal energy lost at impact is then calculated from

$$\Delta E_m = \Delta E_{m1} + \Delta E_{m2} \tag{35}$$

where $\Delta E_{m1} = \frac{k_1 \delta_{m1}^2}{2}$ and $\Delta E_{m2} = \frac{k_2 \delta_{m2}^2}{2}$. The pre-impact velocity difference is thus, from (22),

$$\Delta v = \sqrt{\frac{2\Delta E_m}{m}} \tag{36}$$

To calculate velocity changes of individual vehicles the first test velocities are calculated by (18)

$$v_{T1} = \sqrt{\frac{2\Delta E_{m1}}{m_1}} \qquad v_{T2} = \sqrt{\frac{2\Delta E_{m2}}{m_2}} \tag{37}$$

From (17) the restitution coefficient for individual cars are calculated and from (25) the mutual coefficient of restitution. From (8) the velocity differences of individual cars at impact are

$$\begin{aligned} \Delta v_1 &= v_1 - u_1 = \frac{(1+e)m_2}{m_1+m_2}\Delta v \\ \Delta v_2 &= v_2 - u_2 = \frac{(1+e)m_1}{m_1+m_2}\Delta v \end{aligned} \tag{38}$$

The above formulae were programmed into a spreadsheet program (Table 2). As an example, the car-to-car test described by Kerkhoff et al. ([10]) is considered. In this test the test car (bullet) struck the rear of the stationary car (target) at a speed of 40.6 mph or 65 km/h. The actual measured Δv was 22.6 mph or 36.2 km/h. As can be seen from Table 2, the calculated value Δv_1 for th e bullet car is 36.1 km/h; i.e. the discrepancy between actual and calculated value is 0.2% and the calculated impact velocity 64.14 km/h differs from the actual by 1.3 %. Note that the deformation of the stationary car was not reported, so (34) is used for calculation of its maximal dynamic crush. The limit speed for both cars was taken to be 4 km/h ([1]). The discrepancy of calculated values in the previous case is so minimal because the actual low impact velocity tests were used for determination of stiffness. If one used for the calculation the default values of CRASH stiffness and appropriate calculated limit velocity for class 1 cars the discrepancy would increase. Thus, in this case the calculated velocity change of the bullt car is 38.5 km/h, which differs from the actual change by about 6% and the calculated Δv is 52.2 km/h, differing by about 20%.

Table 2 - Spreadsheet program for calculation of velocity differences at impact.
Car-to-car test no 1 by Kerkhoff et al. ([10])

			Vehicle 1		Vehicle 2
Data	mass	kg	1100.44		1101.11
	stiffness	kN/m	1681.91		872.89
	limit speed	km/h	4.00		4.00
	crush	m	0.16		?
	recoverable crush	m	0.03		0.04
	maximal crush	m	0.19		0.36
	system mass	kg		550.39	
	system stiffness	kN/m		574.65	
	max energy lost	kJ	29.86		57.53
	test velocity	km/h	26.52		36.80
	test restitution		0.15		0.11
	restitution			0.12	
	Delta V	km/h	36.09	64.15	-36.06

CONCLUSION

The main results of the article can be summarized as follows:

1) The restitution model originally proposed in [1] for modelling the impact of car with fixed barrier can be extended to two-car collinear collision where two models are possible: stiffness-based model and mass-based model;
2) Interpretations of the models show that the mass-based restitution model is not adequate for general use;
3) It is found that the discrepancy of measured speeds and the calculated speeds after collinear collision can be very small – let us say less than 5% - if one provides accurate data about car stiffness;

4) The proposed model can be a valuable tool in actual car accident reconstruction.

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POVZETEK

Članek obravnava čelni centrični trk vozi z uporabo Newtonovih zakonov mehanike, zakona o ohranitvi energije in linearnega konstitutivnega zakona, ki povezuje trčno silo in deformacijo. Podana sta dva načina izračuna restitucijskega koeficienta: prvi, ki temelji na togost vozil in drugi, ki temelji na masah vozil. Podan je tudi numerični primer dejanskega testa.

KLJUČNE BESEDE

prometne nesreče, togost vozil, mehanika trka, restitucija

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