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Traffic Technology
Review
U.D.C. 656.11
Accepted: Oct. 29, 1997
Approved: Jan. 30, 1998

OPTIMISATION OF THE MOTOR FLEET WORK IN ROAD HAULAGE

SUMMARY

The fleet of cars as one of the factors in the technological processes in transport has to be dimensioned in the optimal way. Market requirements are continually subject to changes, hence the demand for transportation services has to be modelled by means of quantitative methods. A mathematical model for determining the optimum size of the motor pool finds in the paper its hypothetical presentation.

1. INTRODUCTION

The size and structure of the motor fleet in road haulage depends mainly on the demand for transportation services. The correct selection and degree of its homogenisation is the basis of rational exploitation and cost-efficient business operations. Any business decision in connection with the fleet of cars has to be made as the result of studious research regardless of whether it is a question of renewing the existing motor fleet, or of purchasing new transportation means. The range of scientific and specialist methodology offered in this context can be found within the framework of quantitative methods with adequate programme backing:

(a) Methods of operational research and standard parametric statistics.
(b) Methods of multidimensional statistic analysis, nonparametric statistics, and the so-called fuzzy groups.

The basic postulates which have to be kept in mind when applying the said methods are without doubt: clearness and simplicity, qualitative character of data (not their numerical sense only), and the possibility of their adequate presentation and relevant practical application. In actual practice the size of the motor fleet is as a rule expressed with the number of vehicles (or their static and dynamic capacity), and the following methods for the capacity analysis and evaluation are used:[1]

(a) The method of linear programming, by means of which the optimalvalue of the criterion function is established (maximum and minimum), with a certain number of variables from x_1 ..., x_n interlinked with linearequations or non-equations
(b) Simulation, which presupposes formation of an abstract model of a realsystem and simulating this model on an electronic computer
(c) Theory of queuing lines, solving the request of the traffic service userin conformity with the capacity possibilities of the traffic supply
(d) Method of disposition or the Hungarian method, which is used to dispose "n" activities (jobs, tasks, vehicles) to "m" performers or places, at maximum efficiency
(e) Empirical method, which uses adequate empirical formulae and graphs to determine connections between transport parameters and to compute the extent of the required capacity.

The successful application of quantitative methods in planning the motor pool is dependent on the clear definition of the problem, good acquaintance with the technical-technological and economic-organisational elements of the transport process, existence of adequate methods and programme backing, as well as on the correct choice of mathematical instruments regarding the correct application of the method to the selected problem and correct interpretation of the results obtained.

2. THE PROCEDURE OF MATHEMATICAL MODELLING

As a specimen copy in the optimisation of the motor pool, a transport subject has to be presumed who has decided on purchasing a certain number of trucks type 1,2, ..., n. The investment limitation for the undertaking has been determined by the quantity b_1 of currency units, and depending on the type of vehicle and supplier, the purchase price is as follows:

for a truck type 1	a_{11} currency units
for a truck type 2	a_{12} currency units
for a truck type n	a_{1n} currency units

Trucks type 1, ..., n have a static capacity (possibility of one filling of the vehicle according to quantity and kind of goods) of a_{21} to a_{2n} tons. On the basis of relevant data about the directions of the flow of goods and quantities of goods on the market, the expected quantity of cargo is determined by applying the method of anticipation. The prognosis of the anticipated volume of the transport service contains data on the quantity, kind and structure of the cargo, as well as data on the types and load capacity of the transport units. The totality of the relevant parameters is seen in the sequence:

a_{1j} – purchase price of the j-type trucks, j = 1, ..., n
a_{2j} – load capacity for the j-type truck, j = 1, ..., n
a_{3j} – fuel consumption in the primary time unit for the j-type truck j = 1, ..., n
a_{4j} – number of driver for j-type truck, j = 1, ..., n
a_{5j} – costs of regular maintenance for the j-type truck, j = 1, ..., n
b_1 – amount of currency units for the purchase of the truck
b_2 – amount of cargo planned to be transported in the observed unit of time
b_3 – available quantity of fuel
b_4 – total number of truck drivers
b_5 – planned amount for regular maintenance in currency units
b_6 – number of trucks
c_j – coefficient in the function of criterion (income, expenditure, profit, purchase price, number of trips, effect realised, number of turnovers etc.)
x_j – number of trucks of the j-type, j = 1, ..., n

Income per one truck of the j-type, j = 1, ..., n for 1 hour of work amount to c_{pl}, ..., c_{pn} currency units.

Expenditure per one truck of the j-type, j=1, ..., j=1 ..., n for 1 hour of work amounts to c_{rl}, ..., c_{rn} currency units.

Profit per 1 truck of the j-type, j = 1, ..., n for 1 hour of work amounts to c_{dl}, ..., c_{dn} currency units.

The presumption is the average value of the employment of the vehicle 5 hours/shifts, resulting in the daily values of income $\sum_{i=1}^{n} 5c_{pi}$, expenditure $\sum_{r=1}^{n} 5c_{pr}$, and profit $\sum_{d=1}^{n} 5c_{pd}$ per one j-type truck.

In values marked b_1, ..., b_n are the ***limiting quantities*** of the model which are, according to the content of the table, minimal or maximal, and the values a_{ij} are *technological* **coefficients**, (normatives) or quantities of the i-th amount of limitation (i = 1, ..., m) required for one j-type truck.

The function of the criterion is function Z, for which the maximal or the minimal value of the criterion function can be looked for. The coefficient c_j, j = 1, ..., n are the days for each particular truck type, and can be expressed as:

- income in currency units per one j-type truck, j=1,..., n
- effect realised by one j-type truck, j=1,..., n
- number of turnovers per one j-type truck, j=1,..., n, etc.

The motor fleet optimisation procedure begins with the determination of extreme values of the aim function Z, hence:

$$\mathrm{Min}Z=c_1 x_1+c_2 x_2+\ldots+c_n x_n \Rightarrow \mathrm{Min}Z=\sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\mathrm{Max}Z=c_1 x_1+c_2 x_2+\ldots+c_n x_n \Rightarrow \mathrm{Max}Z=\sum_{j=1}^{n} c_j x_j$$

respecting the following limitations:

$$\begin{aligned} &a_{11} x_1+\ldots+a_{1n} x_n \leq b_1 \rightarrow \sum_{j=1}^{n} a_{1j} \leq b_1 \\ &a_{21} x_1+\ldots+a_{2n} x_n \leq b_2 \rightarrow \sum_{j=1}^{n} a_{2j} \leq b_2 \\ &a_{31} x_1+\ldots+a_{3n} x_n \leq b_3 \rightarrow \sum_{j=1}^{n} a_{3j} \leq b_3 \\ &a_{41} x_1+\ldots+a_{4n} x_n \leq b_4 \rightarrow \sum_{j=1}^{n} a_{4j} \leq b_4 \\ &a_{51} x_1+\ldots+a_{5n} x_n \leq b_5 \rightarrow \sum_{j=1}^{n} a_{5j} \leq b_5 \end{aligned} \tag{2}$$

$$\left.\begin{aligned} &x_1 \leq b_{61} \\ &x_2 \leq b_{62} \\ &\ldots\ldots \\ &x_n \leq b_{6n} \end{aligned}\right\} \sum_{i=1}^{n} x_j \leq b_6 \tag{3}$$

$$x_j \leq 0 \quad j=1,\ldots n \tag{4}$$

The shown model holds good for **n** trucks and **m** limitations, and the given list of limitations can also be extended as circumstances require, i.e. in order to obtain as complete and correct results as possible, new limitations can be added to the model. The symbol of non-equation can be changed according to certain circumstances, as for example if all drivers are required to be employed the symbol of equality will be used, and conversely the symbol $\leq$. The programme is solved by means of the method of linear programming, implying that absolute perfection is almost impossible, but that it is realistic to expect the so-called sub-optimal solution "oblique".

The solution of the model for determining the optimal fleet of cars is constituted by: the number of vehicles (trucks and the like) which have to be procured (or which exist), in order to realise the maximal or the minimal function value of the criterion (maximal income, profit, effect, or minimal costs etc.), taking into consideration the given limitations (amount for investment, quantity of cargo to be transported, number of drivers, number of available vehicles, amount for maintenance of vehicles, supply of fuel, and the like), i.e. the solution of the optimisation model reveals the number and structure of the vehicles, constituting the preparation of the instruments of labour. By the method of disposition the number and type of vehicles obtained is used for their disposition to certain loading and unloading stations, thus achieving the optimal plan of the vehicles movement, created as a result of the preparing stage of transport technology.

3. PROCESS OF POSTOPTIMISATION

The analysis of optimisation results can be structured in a number of stages:

3.1. Analysis of the primal

The results of the primal refer to the values of structural and additional variables, as well as to the value of the criterion function. The primal problems are articulated on the basis of the verbal formulation of the given task, whose analysis can be (hypothetically) shown in the following table:

Table: Analysis of the primal (hypothetically)

Type of truck	Number of trucks (x_i)	Profit per truck (c_i)	Amount of currency unit profit ($c_i x_i$)
K_1	10	100	1000
K_2	5	80	400
Value of programme Z =			1400

Testing the dual is based on the theorem on the duality of linear programming. The model for which the maximal value of the criterion function is required is analogous with form (1), and in matrix form it looks like this:

$$\mathrm{Max}Z=\underset{(1xn)}{\left[c_1\, c_2 \ldots c_n\right]} \cdot \underset{(nx1)}{\begin{bmatrix} x_1 \\ x_2 \\ \cdot\cdot \\ x_n \end{bmatrix}} \tag{5}$$

... with the limitation

$$(a)\quad \underset{(mxn)}{\begin{bmatrix} a_{11}\, a_{12} \cdots a_{1n} \\ a_{21}\, a_{22} \cdots a_{2n} \\ \cdots\cdots\cdots\cdots \\ a_{m1}\, a_{m2} \cdots a_{mn} \end{bmatrix}} \cdot \underset{(nx1)}{\begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}} \leq \underset{(mx1)}{\begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_m \end{bmatrix}} \tag{6}$$

$$(b)\quad \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix} \tag{7}$$

3.2. Analysis of the dual

There is an inversion between the primal and the dual as regards the requirements, i.e. if the maximum of the function was required in the primal, the minimum of the function will be required in the dual, i.e. $\mathrm{Max}Z \rightarrow \mathrm{Min}W$. Hence the formulation of the dual will be in the following matrix form:

$$\mathrm{Min}W=\underset{(1xm)}{\left[b_1\, b_2 \ldots b_m\right]} \cdot \underset{(mx1)}{\begin{bmatrix} y_1 \\ y_2 \\ \cdot\cdot \\ y_m \end{bmatrix}} \tag{8}$$

... with the limitation

$$(a)\quad \underset{(nxm)}{\begin{bmatrix} a_{11}\, a_{21} \ldots a_{m1} \\ a_{12}\, a_{22} \ldots a_{m2} \\ \cdots\cdots\cdots\cdots \\ a_{1n}\, a_{2n} \ldots a_{mn} \end{bmatrix}} \cdot \underset{(mx1)}{\begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_m \end{bmatrix}} \geq \underset{(nx1)}{\begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_n \end{bmatrix}} \tag{9}$$

$$(b)\quad \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_m \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix} \tag{10}$$

The primal and the dual problems have to be solved integrally, and their values can be seen in the simplex table. The dual variables are seen in the last iteration in the line (Z_j - c_j) under the vectors unifying the initial and the basic solution in the said table. The value of the primal criterion function and of the dual criterion function are the same, hence:

$$\mathrm{Max}\sum_{j=1}^{n} c_j\, x_j = \mathrm{Min}\sum_{i=1}^{m} b_1\, y_1$$

3.3. Analysis of the limitations

The analysis of the limitations consists of the analysis of the quantity of limitations mentioned in the model. If the values of the additional variables equal 0, it means that the limitations have been fully exploited, or that there is no surplus in excess of the quantity of limitation laid down in the model. The most frequent case in practice is that the limitations are not coordinated, leading to the occurrence of the so-called “bottlenecks”.

3.4. Postoptimal sensitive analysis

The optimal solution of the problem need not necessarily satisfy the interests of the user, or changes have occurred in the set model in the meantime.[2] That is why it is very interesting to call attention to the possibility of modifying the existing model by means of the post-optimal analysis examining the influence of changing the values of particular parameters and other alterations in the model on the obtained optimal programme.

The post-optimal analysis contains:

(a) Changes of the components of vector A_o, or changes in the limitations.

(b) Changes of the components of vector C, or changes of the coefficient c_j in the criterion function.

(c) Changes of the structural coefficients a_{ij}.

(d) Introduction of new variables.

4. CONCLUSION

The application of quantitative methods in planning the motor fleet is one of the possibilities which can be used when planning the motor fleet. In order to enable the application of the quantitative methods, a clear definition of the size and structure of the motor fleet is required, as well as adequate and reliable data on the means of transport, and the use of adequate methods and programme backing when planning the fleet of cars. The mathematical model of linear programming depending on the limitations, can be written in standard, canon or general form. It consists of criterion or aim functions, limitations and non-negativity conditions, which are expressed by the system of equations and non-equations. In order to make it possible to use the simplex algorithm and draw up a simplex table, it is necessary to translate the mathematical model of linear programming into the canon form, which is done by adding additional or artificial variables, depending on the non-equations from the limitations. What follows is the drawing up of the simplex table, from which after a certain number of iterations the optimal solution emerges, i.e. the values of the new vectors in the basis, and the values of the criterion function. With adequate modifications, the shown model can be applied to dimensioning, or to the optimisation of any motor fleet.

SAŽETAK

OPTIMIZACIJA RADA VOZNOG PARKA U CESTOVNOM PRIJEVOZU

Vozni park kao jedan od čimbenika tehnološkog procesa u transportu, mora biti optimalno dimenzioniran. Tržišne potrebe kontinuirano su podložne promjenama, stoga se prijevozna potražnja mora modelirati pomoću kvantitativnih metoda. U radu je hipotetski predstavljen matematički model za određivanje optimalne veličine voznog parka.

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