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AIRPORT SYSTEM CONTROL IN CONDITIONS OF CONTINUOUS RANDOM PROCESSES OF TRAFFIC FLOW

SUMMARY

The article presents a systemic approach to air traffic operation and control in which the airport represents a basic technical-technological and organisational system. Determining optimum control of the airport system therefore represents the most important component in the process of controlling air traffic system.

1. Introduction

Air traffic can be defined as an extremely complex dynamic system. When referring to air traffic as a system, we understand its inner structure, i. e. the set of subsystems, elements and their interrelationships as well as their relationships with the environment. In /6/ an air traffic system is divided and interrelated in vertical and horizontal directions. In vertical analysis four strata are found. They behave as technical, technological, organisational and economic subsystems. In the horizontal direction, the system of air traffic is a synthesis of three subsystems of activities: transport, disembarkation/embarkation and flight control.

Traffic systems are generally quite complex. For this reason, in creating their theoretical mathematical model, we would have to take into account an extremely large number of variables and their interrelationships. However, with methods of logical and methodological decomposition, a traffic system may be divided into a finite set of simpler subsystems which are then studied and analysed separately [5]. Each of the subsystems represents an independent system while other subsystems represent its more or less relevant environment. Such a scientific approach is based on the axiomatic principle of describing a traffic system with the set of elementary statements which are true on the basis of evidence of the traffic science theory. In this way consistency, completeness and minimality of the traffic system structure are guaranteed while scientific precision and generality are maintained.

In this part we are interested in the airport subsystem within the framework of the air traffic system. We are going to deal with it as a system of aircraft, passenger, cargo, luggage and postal operations. All the necessary and relevant activities are carried out by airports, which are organised as business companies [6], [7].

Rapid development of air traffic has emphasised the importance of airports, which have in many places become the bottleneck in the process of air traffic dynamics. In most cases this bottleneck is felt as lack of capacity. Airports have become complex technological and organisational structures which follow the laws of dynamic systems. Therefore technical and technological modernisation alone cannot provide satisfying results in optimal exploitation of existing capacities and planning the new ones. For this reason we have to adopt a scientific approach to managing airports as complex dynamic systems.

2. Properties of an airport system

Traffic systems (and all their subsystems) are stochastic, as there are always random variables in their operation. The stochastic feature of technological subsystems of a traffic system is conditioned by the very nature of traffic. Traffic system controllers' interest is to study these random processes intensely, and to make them - to a certain degree of accuracy - statistically predictable. In an air traffic system the airport as its subsystem has a known infrastructure and terminal capacity, but traffic flow in this system is stochastic. Stochasticity of passenger and aircraft flows causes periodical overloading of facilities, and here a basic question arises of how to solve this problem as on average the capacities are not surpassed by demand. Optimal control has to depend on a clearly defined function of the goal which will be defined in structuring the mathematical model of control.

All real life problems change in time and are therefore dynamic, and the same is with traffic systems and all their subsystems including the airport system. According to the type of input and output signals, dynamic systems can be continuous or discrete. Artificial systems, such as traffic systems with all their subsystems are discrete by nature as the events taken as input functions do not take place continually in the strict mathematical sense. But if the rule determining the function defined over a given time interval [0,T] is such that it can be applied at almost any point of this interval (definition area), we can replace it with continuous function. In this article we shall make up and solve the mathematical model of system control for continuous functions.

Dynamic system is linear when the transformation of a linear combination of input functions is the same as linear transformation of these input functions. On these grounds traffic systems can be viewed as linear systems. In the case of an airport system the requirement for linearity means solving the problem of passenger flow by optimising the capacity for passengers and aircraft handling. The linearity of the system enables us to determine the optimum number of check-in counters, so that the passenger flow is co-ordinated with the timetable and with the operations on the platform.

Linear dynamic systems, including traffic systems with all their subsystems, are time-independent (invariant) when the structure and duration of an output function (signal) do not depend on the chosen start of observation. Weather dimension can move uniformly in one direction depending on the input which in the system produces an equally formed output signal in real time. Linear dynamic time-independent systems are called stationary systems. As everything that happens in traffic systems (and their subsystems) takes place in real time, these systems are stationary.

Within the framework of an air traffic system airports perform two key functions: the first one is the infrastructure which allows aircraft to take off and land, and the second one is the traffic function which allows passengers to board and disembark the planes. In this sense the airport is divided into two basic functional parts: infrastructure or air part and terminal or ground part. For this reason airports as systems are studied from the point of view of traffic infrastructure and from the point of view of traffic processes.

Organisation of technological processes is the basic function of an airport. It must take care that all the aircraft land and take off safely and on time, and that they are provided with timely and quality service. It should provide adequate loading and unloading of passengers, luggage and cargo, and all the necessary procedures to ensure the operation of all the complementary activities. If an airport does not fulfill these conditions, negative consequences occur, such as late arrivals and delays in air traffic, inadequately used airport capacities, poor airport services and financial loss. However, unavoidable problems often crop up in the operation of an airport. These are late arrivals and departures due to bad weather or technical malfunctions, overloading during peak hours and security related measures in exceptional circumstances.

During the operation of a dynamic system, processes can take place on several levels. As a rule, each level has its partial objective and with this its local criterial function of control. Different partial objectives in the same system can be either consistent or conflicting. The efficiency of such systems depends on co-ordinating and synchronising the operation of different parts of the system. An airport system operates on three levels which comprise its functional activities [5]: strategic, administrative-organisational or coordinative and operational levels. Characteristics of each level differ according to the relationships with the system environment, time dimension, requirements and expectations criteria, and decision making technique. Airports as dynamic systems are therefore studied, analysed and optimised according to their basic functions from the point of view of these hierarchical levels.

The operational level of an airport system is the same as the operational level of most production systems. It consists of three interactive subsystems: the subsystem of demand, expressed as a flow of passengers, luggage, cargo and aircraft; the subsystem of production, expressed as a technological process; the subsystem of facilities, expressed as infrastructure, terminals and other vital airport facilities.

In an airport system the users are airlines, passengers and luggage and cargo owners. Based on these traffic flows, airport systems organise and carry out all the necessary technological operations. These operations result in airport services (production) which, in the dynamics of the traffic flow, occur in phases. Offering airport services involves the use of proper infrastructure and technical facilities. In traffic technological potential acts as stock as these usable resources are made to meet the demand at all times, including peak times.

The co-ordination level of airport systems is represented by the sales and administrative subsystems. Each of them can be studied and analysed separately. The sales subsystem consists of the following elements: the market, users, services, income and calculations. The administrative subsystem consists of the following elements: the operations, services, income, costs, net income.

The strategic level represents the subsystem of planning which consists of the following elements: the operations, business effects, investments, capacities, decision making.

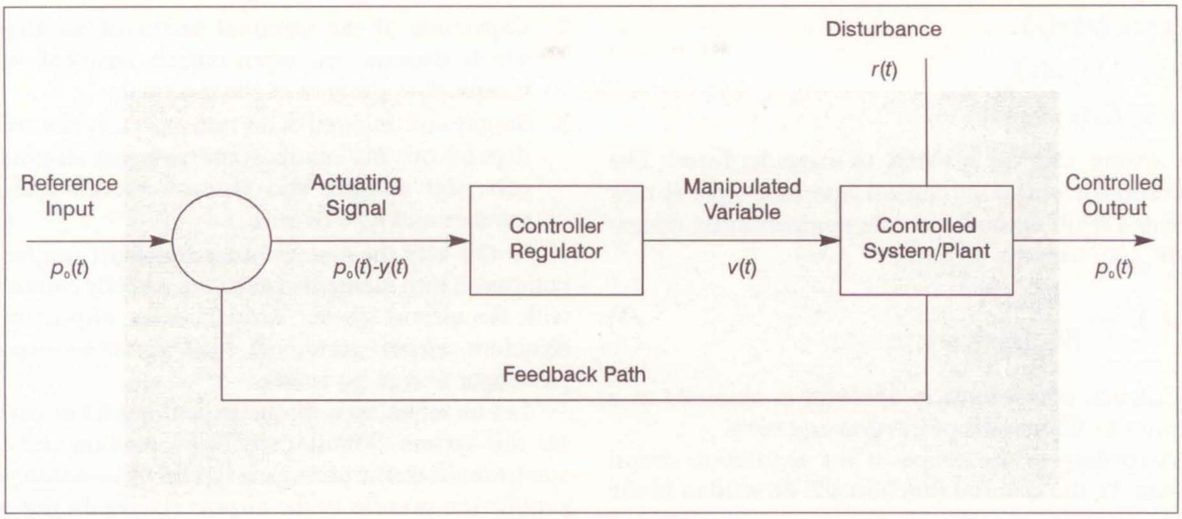

Figure 1

3. Linear theoretical model of regulation

The optimisation model of dynamic system regulation is determined by the system and by the optimality criterion. The system as regulation circuit generally consists of a regulator, the object of regulation, feedback, input and output information [3], [4]. (See Fig. 1).

We will restrict ourselves to dynamic linear continuous system where the input is a random process with known statistical properties. We want to determine a linear system in which the real output signal $y(s)$ will be as similar to the desired (planned) response signal $p_0(s)$ as possible. Comparison of planned and factual system responses with input signals is made by means of optimality criterion. It allows us to assess the quality of control [10]. Good control therefore requires permanent co-ordination of planned output functions with the factual ones.

With Laplace transform [8] functions are translated into the complex area and the regulation circuit is described with a block diagram. We want to define a linear system or a regulation operator which will make sure that the factual output signal will be as similar to the desired ideal output as possible. Because of relative computing simplicity, we will be using Wiener's filter, i. e. we will be looking for the smallest mean square error [9]. The criterion of regulation therefore is the minimum of mathematical hope of square error $e(s)$. With a parallel shift we can always achieve $p_0(s)=0$, therefore $e(s)=-y(s)$ means that the output itself denotes the error which must be minimised (Figure 2).

The equations of the system are:

$$y(s)=G_p(s)[v(s)-r(s)] \tag{1}$$

$$v(s)=G_f(s)\,u(s) \tag{2}$$

$$u(s)=-G(s)\,y(s) \tag{3}$$

Operators $G_p(s)$ and $\widetilde{G}_f(s)$ in concrete systems are known, while regulation operator $G(s)$ must be defined so that the average value of mean square error will be minimal.

With the introduction of functions

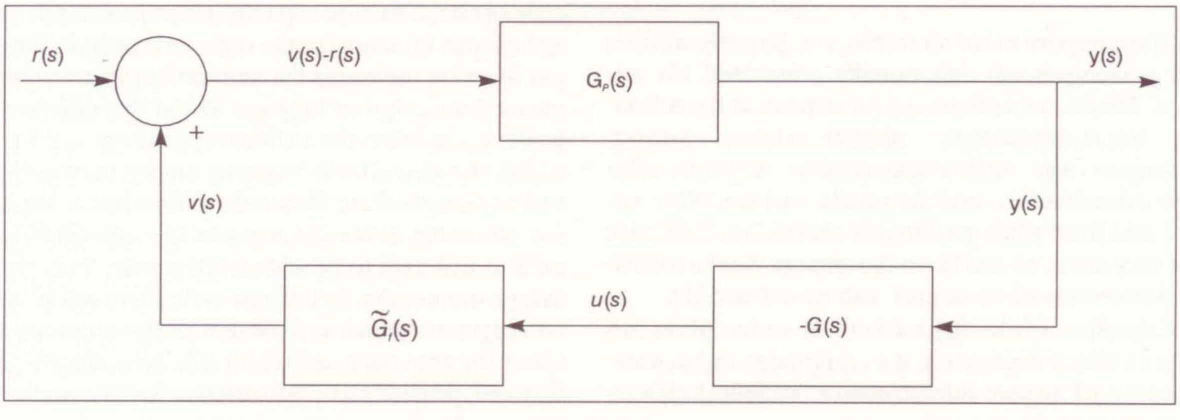

Figure 2

$$R(s)=G_p(s)\,r(s),$$
$$V(s)=G_f(s)\,u(s)$$
$$G_f(s)=\widetilde{G}_f(s)\,G_p(s)$$

the system can be created in cascade form. The searched-for optimum control operator $G(s)$ is now obtained from optimal cascade compensation operator $W_{\text{opt}}(s)$ with the formula:

$$G_{\text{opt}}(s)=\frac{W_{\text{opt}}(s)}{1-W_{\text{opt}}(s)\,G_f(s)} \tag{4}$$

Cascade compensation operator is obtained as a solution to Wiener-Hopf integral equation.

According to the shape of the regulation circuit (Figure 2), the criterial function will be written in the form

$$Q=K_y\,E\left(y^2(t)\right)+K_u\,E\left(u^2(t)\right) \tag{5}$$

In (5) K_y and K_u are empirically defined positive real numbers, while E denotes the mathematical hope of a random variable.

As $E\left(y^2(t)\right)=R_{yy}(0)$ and $E\left(u^2(t)\right)=R_{uu}(0)$ where $R_{yy}(0)$ and $R_{uu}(0)$ are autocorrelations of stationary random processes $\{y(t)\}$ and $\{u(t)\}$, we get the optimum of criterial function with the operator of the cascade compensation [10]:

$$W_{\min}(s)=\frac{\left[\dfrac{G_f(-s)\,\Phi^{+}_{RR}(s)}{\left(G_f(s)\,G_f(-s)+\rho^2\right)^{-}}\right]_{+}}{\left(G_f(s)\,G_f(-s)+\rho^2\right)^{+}\,\Phi^{+}_{RR}(s)},$$

$$\rho^2=\frac{K_u}{K_y}\geq 0 \tag{6}$$

From (6) we get the optimum control operator using formula (4), and the responses of the system using formulae (1), (2) and (3).

4. A theoretical model of the airport system control

The components of air traffic, i. e. airports, airlines and passengers are functionally connected via airports. These connections appear in pairs as the following interrelationships: airport-airlines, airport-passengers and airlines-passengers. Airports offer their infrastructure and terminals, airlines offer aircraft and lines while passengers themselves take care that they arrive at and leave the airport. So the following parameters of an airport can be defined [1]:

1. Capacities of the infrastructural sector of an airport, which depend on: the equipment and maintenance of airport infrastructure, technical characteristics of the ground operative.
2. Capacities of the terminal sector of an airport, which depend on: input-output terminal units, technical equipment of the terminal.
3. Supply and demand of air transport services, which depend on: the number and category of passengers, the number and type of aircraft and the number and type of lines.

In this way the system of air transport can be decomposed into elements which are directly connected with the airport system: aircraft, lines, airport infrastructure, airport terminals, road access to airports, passenger and cargo traffic.

Let us make up a mathematical model of control for this system. Simultaneity of production and consumption of traffic services is typical of technological-production process in the airport system. In this process there is no stock in the classical sense, as traffic services cannot be produced in advance for a known customer or stock built up for unknown customers. The demand of traffic services is neither uniform in time nor known in advance. It varies, has its ups and downs and it can only be met by installing and activating proper technological capacities. Because of this, the function of stock in traffic belongs to the whole technological potential which is large enough to meet periods of extra demand. The demand of traffic services is not given and known explicitly in advance. With market research we can only learn about the probability of our expectation of a certain intensity of demand. The demand is usually not given with explicitly expressed mathematical function, we only know the shape and type of the whole family of functions. The demand is a random process for which all the statistical indicators are known. The system input represents the demand for products/services which a given subject offers. They are airport system services, which eventually mean air transport of passengers, goods and/or cargo. Let demand be a stationary random process with known statistical characteristics - mathematical hope and autocorrelation. Any given demand should be met with current and standard services according to transport timetables and order. The difference between current capacity of services and demand is the input function for the object of control. The output function measures the amount of (un)transported passengers, cargo or luggage. When this difference is positive, i. e. when the airlines capacity exceeds the demand, the aircraft will be partly empty, certain flights will be cancelled etc. When the difference is negative, i. e. when the demand surpasses the capacities, extra aircraft will have to be added. Otherwise, there will be delays, queues etc. In the new cycle there will be a system regulator which will contain all the necessary data about the true state and which will, according to given demand, provide basic information for the production process. In this way the regulation circuit is closed.

Based on given demand, our task is to determine such (optimal) airport system control that total costs will be minimal. With optimal control we will understand the situation where all the passengers, luggage and cargo are transported with the minimum involvement of additional facilities.

Let us denote continuous stationary random processes:

$Z(t)$ – activated facilities/resources at moment t,

$u(t)$ – the amount of services performed (production) at moment t,

$d(t)$ – the demand for services at moment t.

Let the system be modelled with equations [10]:

$$\dot{Z}(t)=\psi\left[v(t)-r(t)\right], \quad \psi\in\Re^{+} \tag{7}$$

$$v(t)=u(t-\lambda) \tag{8}$$

$$u(t)=-\int_{0}^{\infty} G(\tau)\,Z(t-\tau)\,\mathrm{d}t \tag{9}$$

$$Q(t)=K_{z}\,E\left\{\left(Z^{2}(t)\right)\right\}+K_{u}\,E\left\{\left(u^{2}(t)\right)\right\} \quad \min \tag{10}$$

$G(t)$ is the regulation function, λ is time elapsed between the moment the data are received and the carrying out of a service. Let us take a real life situation where each service is checked via airport information system database, which represents delay of λ time units. We are looking for a system control with minimum operation costs. In this model, let us take into account the total cost of involved capacities and the total cost of services.

Equations (7) - (10) form a stationary stochastic linear model of control where the minimum of criterial function is calculated on the basis of Wiener's filter. This means that we will obtain the searched-for solution using Wiener-Hopf ´s equation (6).

Let us take a situation in which the function of demand complies with distribution with the autocorrelation

$$R_{dd}(\tau)=\frac{\zeta^{2}}{2\alpha}\,\mathrm{e}^{-\alpha|\tau|}=\sigma^{2}\,\mathrm{e}^{-\alpha|\tau|}, \quad \sigma>0 \tag{11}$$

In this case we calculate

$$W_{\mathrm{opt}}(s)=\frac{s\,(Cs+1)}{\rho s+\psi}, \quad C=\frac{\psi+\alpha\rho-\psi\,\mathrm{e}^{-\alpha\lambda}}{\alpha(\psi+\alpha\rho)} \tag{12}$$

and from equations of model in (s)-plane we calculate $u(s)$ and $Z(s)$.

With inverse Laplace transform we obtain the following functions in the time zone:

a) services that were performed:

$$u(t)=\mathcal{L}^{-1}\left\{u(s)\right\}=$$

$$=\frac{\psi C}{\rho}\,d(t)+\frac{\psi(\rho-\psi C)}{\rho^{2}}\int_{0}^{t}\mathrm{e}^{-\frac{\psi}{\rho}t}\,d(t-\tau)\,\mathrm{d}\tau \tag{13}$$

b) facilities that were operating:

$$Z(t)=\mathcal{L}^{-1}\left\{Z(s)\right\}=\frac{\psi}{\rho}D(t-\lambda)-D(t)+$$

$$+\frac{\psi(\rho-\psi C)}{\rho(\rho+\psi)}\int_{0}^{t}\mathrm{e}^{-\frac{\psi}{\rho}\tau}\,D(t-\lambda-\tau)\,\mathrm{d}\tau \tag{14}$$

In the demand-services-result relationship during optimum control process, with known demand, all the searched-for optimums are the sums of the exponential functions, and with increasing t they approach boundary, mutually co-ordinated values. The system is regulated in such a way that it responds permanently and synchronously to outside disturbances.

5. Conclusion

For the study of structure, interrelationships and operation of a phenomenon with system characteristics, the best method is the general systems theory, and within the latter, the systems regulation theory. When we refer to traffic technology as a synthesis of organisation, information technology and operations, we have to consider, in creating a mathematical model, its dynamic dimension. As each such complex phenomenon makes up a system, the traffic technology in this article is again dealt with as a dynamic system. Elements of the technological system make up an ordered entity of interrelationships and thus allow the system to perform production functions. Because of condition of linearity, response functions of the system are, with reference to the type of traffic, either continuous or discrete. Generally speaking, airports, airlines and passengers are the component parts of air traffic. Their functional relationships via airport appear in pairs: airport-airlines, airport-passengers and airlines-passengers. For air traffic operation many conditions have to be fulfilled, such as highly developed infrastructure, the use of up-to-date transport technologies, market operations, reliable operation of integral information system etc. During the control process a great deal of information must be processed, which can only be done if transparent and properly developed information system is available. During the operation of the airport an enormous amount of data is used which can only be processed into information for control if high quality software and powerful hardware are available. Communications also play a major role, as it is necessary to contact and use a number of international databases interactively.

Models of optimum control can also be used in the airport system. The mathematical model describing the system can be analytically more or less complex, but generally the procedure always follows the same rule. With appropriate mathematical tools the algorithm is numerically manageable even in cases when the functions of the system are vector functions with n $(n \in \mathcal{N},\quad n < \infty)$ components.

POVZETEK

UPRAVLJANJE SISTEMA ZRAČNE LUKE V POGOJIH ZVEZNIH SLUČAJNIH PROCESOV PROMETNEGA TOKA

V članku je predstavljen sistemski pristop k delovanju in upravljanju zračnega prometa, kjer predstavlja letališče osnovni tehnično-tehnološki in organizacijski sistem. Določitev optimalnega upravljanja sistema zračne luke zato predstavlja najpomembnejšo komponento v procesu upravljanja sistema zračnega prometa.

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