# CALCULATION OF CRITICAL VEHICLE SPEEDS NEGOTIATING CURVES 

## SUMMARY

A great number of very severe traffic accidents occurs as the direct result of destabilisation of the vehicle negotiating a curve, most often caused by excessive speed of the vehicle for the given curve configuration, and for the given surface condition. Therefore, the issue of determing the speed in negotiating curve is not just a problem met by experts involved in the analysis of the actual traffic accidents, but also a question without which the preventive safety measures in road traffic cannot be thought of, both regarding the construction and design of the roads, and regarding speed limitations at certain points. The aim of this papers is to define a exactly analyticall method for case of actual vehicle negotiating a curve situation.

## 1. INTRODUCTION

A great number of very severe traffic accidents occurs as the direct result of destabilisation of the vehicle negotiating a curve, most often caused by excessive speed of the vehicle for the given curve configuration, and for the given surface condition. Therefore, the issue of determining the speed in negotiating curves is not just a problem met by experts involved in the analysis of the actual traffic accidents, but also a question without which the preventive safety measures in road traffic cannot be thought of, both regarding the construction and design of the roads, and regarding speed limitations at certain points. However, in previous practice commonly used method of analytically determining the speed of passing through a certain curve, based on the idealised consideration of vehicle as a material point, is due to its very gross approximation completely inadequate, so that the results, obtained in this way significantly deviate from the actual situation. The stability of a vehicle passing through a curve, does not depend only on the curve configuration and the current ground condition, but it also depend significantly on the vehicle design parameters, its load condition, and on the method of controlling the vehicle itself.

It is a physically generally known and proven fact that, with a certain surface condition the optimal vehicle deceleration, in its rectilinear movement, can be achieved when the relation between the braking forces of the front and rear wheels is proportional to their dynamic loading. Such distribution of the braking forces is in practice denoted as "Ideal distribution of braking forces". However, in actual traffic situations, the motorist is often forced to apply the brakes in a curve. This is then a situation of curvilinear movement of the vehicle, when, along with the negative longitudinal acceleration, or rather deceleration, caused by the braking forces, there comes also to a centripetal or socalled lateral acceleration, due to the curvilinear movement. Of course, the lateral acceleration of the vehicle will be related to the occurrence of lateral guiding forces at the wheels, which will most certainly affect the braking distribution, since the maximum force value, which can occur between the wheels and the surface at a given moment depends on the current value of the vertical load on the wheels and the current maximum value of the coefficient of adhesion between the treading area of the pneumatic protector and the surface. Namely, the resultant of the longitudinal and lateral force, which act simultaneously on the wheel of the vehicle, has to equal the contact force of the wheel towards the surface, at the limit of balance. Therefore, generally speaking, for any wheel the following relation will hold:
$F_{i, \text { rez }}=G_{i} \cdot \mu_{i}=\sqrt{O_{i}^{2}+B_{i}^{2}} ; \quad i=1,2,3,4, \ldots$
where:
$F_{i, r e z}$ - is the resulting force on the wheel,
$O_{i}$ - is the flange force on the wheel,
$B_{i}-$ is the lateral force on the wheel,
$G_{i}-$ is the vertical load on the wheel,
$\mu_{i}-$ is the coefficient of adhesion between the wheel and the surface,
whereas index " i " denotes single wheels on the vehicle.

The purpose of this paper is, therefore, to analyse the influence of lateral acceleration on the braking stability of the modern road vehicles, or more precisely, to analyse the influence of the lateral acceleration values on the distribution of braking forces which is characteristic of instaled braking systems, as to try to define such a method of analytical determination of speed in a curve, which will be generaly true, at last for a certain vehicle category, and which will by sufficienty simple, but naturally, sufficienty accurate, in order to be acceptable for direct application in practise

## 2. BRAKING OF THE VEHICLE NEGOTIATING A CURVE

Braking of the vehicle negotiating a curve presents a so-called case of non-stationary vehicle movement along a curvilinear path, which, due to the acting of inertia forces in the vehicle centre of gravity, results in the re-distribution of loading on the wheels. Strictly physically considered, such a case should be analysed
using the model presented in Figure 1. However, in conventional braking systems, installed in the modern road vehicles, both wheels of the same vehicle axle brake with the same intensity. In such a situation, the issue related to the problem of influence of the lateral acceleration on the distribution of braking forces of the installed braking systems can be analysed by means of a simplified model of the vehicle, presented in Figure 2, where the difference in the value of the vertical load on the outer and inner wheel of the same vehicle axle is neglected. Using, therefore, the presented simplified model of the vehicle, and by means of balance conditions, we can obtain the data on the values of dynamic vertical loads.

From the balance conditions, in relation to the supports of the front i.e. rear wheels, the following relation can be obtained as well:
$G_{P} \cdot l=m \cdot g \cdot l_{2}+m \cdot a \cdot h \quad G_{S} \cdot l=m \cdot g \cdot l_{1}-m \cdot a \cdot h$
$G_{P}=m \cdot g \cdot\left(\frac{l_{2}}{l}+\frac{a}{g} \cdot \frac{h}{l}\right) \quad G_{S}=m \cdot g \cdot\left(\frac{l_{1}}{l}+\frac{a}{g} \cdot \frac{h}{l}\right)$


Figure 1 - Model of a vehicle for the general case of non-stationary movement of the vehicle through a curve

Introducing into the substitution analysis for the position of the vehicle centre of gravity, in relation to the wheel supports, the relative values
$\chi=\frac{h}{l} ; \quad \Psi=\frac{l_{1}}{l} ; \quad(1-\Psi)=\frac{l_{2}}{l}$
the expressions for dynamic vertical loads of the front i.e. rear vehicle axle are transformed into the following form
$G_{P}=m \cdot g \cdot\left(1-\Psi+\chi \cdot \frac{a}{g}\right) \quad G_{S}=m \cdot g \cdot\left(\Psi-\chi \cdot \frac{a}{g}\right)$
Furthermore, and as shown in Figure 2, on the axles of the considered and simplified vehicle model, there will occur, e.g. in the right bend, lateral reaction forces "Bp" and "BS", whose values can be also determined by means of the balance conditions, i.e.:
$B_{P} \cdot l=m \cdot b \cdot l_{2}$

$$
\begin{equation*}
B_{S} \cdot l=m \cdot b \cdot l_{1} \tag{6}
\end{equation*}
$$

Using the already mentioned relative values, the analytically simple expressions for the lateral reaction forces are obtained:

$$
\begin{equation*}
B_{P}=m \cdot b \cdot(1-\psi) \quad B_{S}=m \cdot b \cdot \psi \tag{7}
\end{equation*}
$$

If we neglect the rolling resistance, air drag, and engine braking, as flange i.e. longitudinal forces on the wheels of our simplified vehicle model presented in Figure 2, only braking forces will occur. Since in the installed braking systems, the relation of the braking forces of the front and rear wheels, as a design value defined by dimensions of operating braking cylinders of the wheel braking mechanisms, and by the pressure of the working medium, is expressed by means of the so-called braking forces distribution coefficient
$\Phi=\frac{F_{k S}}{F_{k}}=\frac{F_{k S}}{m \cdot a}$


Figure 2 - Simplified model of a vehicle for approximate qualitative analysis of lateral acceleration influence on the distribution of braking forces
i.e.
$1-\Phi=\frac{F_{k P}}{F_{k}}=\frac{F_{k P}}{m \cdot a}$
we obtain the analytical expressions for the values of braking forces of the front and rear wheels
$F_{k P}=m \cdot a \cdot(1-\Phi) \quad F_{k S}=m \cdot a \cdot \Phi$
In the introduction it has already been mentioned that the resulting forces of the longitudinal and lateral forces on the wheels have to be equal to the forces of adhesion of the wheels to the surface. This means that for the front wheels the following condition has to be fulfilled
$F_{k P}^{2}+B_{P}^{2}=\mu^{2} \cdot G_{P}^{2}$
whereas the following condition has to be fulfilled for the rear wheels
$F_{k S}^{2}+B_{S}^{2}=\mu^{2} \cdot G_{S}^{2}$

### 2.1. Conventional braking systems

The conventional braking systems in vehicles have either so-called linear or so-called bi-linear characteristics. This means that in conventional braking systems the relation between the braking forces of the front and rear wheels is constant and defined by means of the already mentioned distribution coefficient of the braking forces. In order to determine the limitations of adhesion capabilities of the front and rear wheels, the braking problem will be analysed separately for the front and for the rear wheels.

## A) Limit of adhesion of the front wheels

$F_{k P}^{2}+B_{P}^{2}=\mu^{2} \cdot G_{P}^{2}$
By introducing the already mentioned expressions for the braking force, lateral force and dynamic load into this equation, it follows:
$m^{2} a^{2}(1-\Phi)^{2}+m^{2} b^{2}(1-\Psi)^{2}-$
$-\mu^{2} m^{2} g^{2}\left(1-\Psi+\chi \frac{a}{g}\right)^{2}=0$
$b^{2}(1-\Psi)^{2}+a^{2}(1-\Phi)^{2}-\mu^{2} g^{2}(1-\Psi)^{2}-$
$-2 g \mu^{2} \chi(1-\Psi) a-\mu^{2} \chi^{2} a^{2}=0$
$b^{2}(1-\Psi)^{2}+a^{2}\left[(1-\Phi)^{2}-\mu^{2} \chi^{2}\right]-$
$-2 \mu^{2} g \chi(1-\Psi) a-g^{2} \mu^{2}(1-\Psi)^{2}=0$
$\frac{(1-\Psi)^{2} b^{2}}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}+a^{2}-\frac{2 g \mu^{2} \chi(1-\Psi) a}{(1-\Phi)^{2}-\mu^{2} \chi}-$
$-\frac{g^{2} \mu^{2}(1-\Psi)^{2}}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}=0$
$\frac{(1-\Psi)^{2} b^{2}}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}+a^{2}-2 \frac{\mu^{2} \chi(1-\Psi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}} \cdot a+$
$+\left[\frac{g \mu^{2} \chi(1-\Psi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}\right]^{2}-\left[\frac{g \mu^{2} \chi(1-\Psi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}\right]^{2}-$
$-\frac{g^{2} \mu^{2}(1-\Psi)^{2}}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}=0$
$\frac{(1-\Psi)^{2} b^{2}}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}+\left[a^{2}-\frac{g \mu^{2} \chi(1-\Psi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}\right]^{2}-$
$-\frac{g^{2} \mu^{2}(1-\Psi)^{2}(1-\Phi)^{2}}{\left[(1-\Phi)^{2}-\mu^{2} \chi^{2}\right]^{2}}=0$
$\frac{b^{2}}{g^{2} \mu^{2}(1-\Phi)^{2}}+\frac{\left[a-\frac{g \mu^{2} \chi(1-\Psi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}\right]}{g^{2} \mu^{2}(1-\Psi)^{2}(1-\Phi)^{2}}=1$ $(1-\Phi)^{2}-\mu^{2} \chi^{2} \quad\left[(1-\Phi)^{2}-\mu^{2} \chi^{2}\right]^{2}$

If we substitute:
$a_{1}=\frac{g \mu^{2} \chi(1-\Psi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}$
$r_{a}=\frac{g \mu(1-\Psi)(1-\Phi)}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}$
$r_{b}=g \mu(1-\Phi) \sqrt{\frac{1}{(1-\Phi)^{2}-\mu^{2} \chi^{2}}}$
the final expression for the relation between the lateral acceleration and deceleration of the vehicle at the limit of adhesion of the front wheels is obtained:
$\frac{b^{2}}{r_{b}^{2}}+\frac{\left(a-a_{1}\right)^{2}}{r_{a}^{2}}=1$
It is obvious that this expression presents an equation of ellipse with the centre shifted towards the axis (a). Therefore, the limit of adhesion of the front wheels, in case of simultaneous longitudinal deceleration and lateral acceleration of the vehicle, in a rectangular co-ordinate system with co-ordinates (a) and (b) is represented as an ellipse with semi-axes $\left(\mathrm{r}_{\mathrm{a}}\right)$ and $\left(\mathrm{r}_{\mathrm{b}}\right)$ and the shifted centre towards the axis (a).

## B) Limit of adhesion of the rear wheels

$F_{k S}^{2}+B_{S}^{2}=\mu^{2} \cdot G_{S}^{2}$
Similar to the analysis of the front wheels braking, by introducing the already mentioned expressions for the braking force, lateral force and dynamic axle load, it follows:
$\frac{b^{2}}{r_{b}^{2}}+\frac{\left(a-a_{1}\right)^{2}}{r_{a}^{2}}=1$
$m^{2} a^{2} \Phi^{2}+m^{2} b^{2} \Psi^{2}-\mu^{2} m^{2} g^{2}\left(\Psi-\chi \frac{a}{g}\right)^{2}=0$
$\Psi^{2} b^{2}+\Phi^{2} a^{2}-\mu^{2} g^{2} \Psi^{2}+$
$+2 \mu^{2} g \Psi \chi a-\mu^{2} \chi^{2} a^{2}=0$
$\Psi^{2} b^{2}+\left(\Phi^{2}-\mu^{2} \chi^{2}\right) a^{2}+$
$+2 \mu^{2} g \chi \Psi a-\mu^{2} g^{2} \Psi^{2}=0$
$\frac{\Psi^{2} b^{2}}{\Phi^{2}-\mu^{2} \chi^{2}}+a^{2}+\frac{2 \mu^{2} g \chi \Psi a}{\Phi^{2}-\mu^{2} \chi^{2}}-\frac{\mu^{2} g^{2} \Psi^{2}}{\Phi^{2}-\mu^{2} \chi^{2}}=0$
$\frac{\Psi^{2} b^{2}}{\Phi^{2}-\mu^{2} \chi^{2}}+a^{2}+\frac{2 \mu^{2} g \chi \Psi a}{\Phi^{2}-\mu^{2} \chi^{2}}+\left[\frac{\mu^{2} g \chi^{2}}{\Phi^{2}-\mu^{2} \chi^{2}}\right]^{2}-$
$-\left[\frac{\mu^{2} g \chi^{\Psi}}{\Phi^{2}-\mu^{2} \chi^{2}}\right]^{2}-\frac{\mu^{2} g^{2} \Psi^{2}}{\Phi^{2}-\mu^{2} \chi^{2}}=0$
$\frac{\Psi^{2} b^{2}}{\Phi^{2}-\mu^{2} \chi^{2}}+\left[a^{2}+\frac{\mu^{2} g \chi^{\Psi}}{\Phi^{2}-\mu^{2} \chi^{2}}\right]^{2}-$
$-\frac{\mu^{2} g^{2} \Psi^{2} \Phi^{2}}{\left(\Phi^{2}-\mu^{2} \chi^{2}\right)^{2}}=0$
$\frac{b^{2}}{\mu^{2} g^{2} \Phi^{2}}+\frac{\left[a^{2}+\frac{\mu^{2} g \chi^{\Psi}}{\Phi^{2}-\mu^{2} \chi^{2}}\right]^{2}}{\mu^{2} g^{2} \Psi^{2} \Phi^{2}}=1$
$\Phi^{2}-\mu^{2} \chi^{2} \quad\left(\Phi^{2}-\mu^{2} \chi^{2}\right)^{2}$
If we substitute
$a_{1}=-\frac{\mu^{2} g^{2} \chi^{\Psi}}{\Phi^{2}-\mu^{2} \chi^{2}} \quad r_{2}=\frac{\mu g \Psi \Phi}{\Phi^{2}-\mu^{2} \chi^{2}}$
$r_{b}=\mu g \Phi \sqrt{\frac{1}{\Phi^{2}-\mu^{2} \chi^{2}}}$
the final expression for the relation of the longitudinal deceleration and lateral acceleration of the vehicle at the limit of adhesion for the rear wheels is obtained:
$\frac{b^{2}}{r_{b}^{2}}+\frac{\left(a-a_{1}\right)^{2}}{r_{a}^{2}}=1$
It is obvious that, as with front wheels, in the case of adhesion limit of the rear wheels, the relation of acceleration and deceleration of the vehicle, in the rectangular co-ordinate system with co-ordinate axes (a) and (b), is represented as an ellipse with semi-axes ( $\mathrm{r}_{\mathrm{a}}$ ) and ( $\mathrm{r}_{\mathrm{b}}$ ) and a shifted centre towards the axis (a). Although the relation of the lateral acceleration and deceleration of the vehicle is presented as an ellipse, the mentioned ellipses, i.e. the ellipse referring to the wheels of the front axle and the ellipse referring to the wheels of the rear axle, differ because of the differences in the above mentioned substitution values.


Figure 3 - Limits of adhesion features of the front and rear wheels in ready-for-movement condition with various road surface conditions for the installed braking system with linear characteristic

### 2.2. Braking systems with the ideal distribution of braking forces

The modern road vehicles use sometimes also the braking systems with electronic regulation of the working medium pressure, which includes also the braking forces of the wheels. These types of braking systems provide the so-called ideal distribution of braking forces, where the braking forces of the wheels of the front and rear vehicle axles are proportional to the dynamic axle loads. A braking system that would achieve the so defined ideal distribution of braking forces, would insure the maximum possible vehicle deceleration at a certain coefficient of adhesion value i.e. in certain conditions of the vehicle pneumatics and the road surface. In order to study the influence of the defined ideal distribution of braking forces on the limits of adhesion of the front and rear wheels in case of lateral acceleration i.e. braking during cornering, the following analysis will be performed:

From the mentioned expressions for the roadholding limits of the front and rear wheels, the braking forces distribution coefficient will be eliminated. According to the definition, namely, the braking forces distribution coefficient equals the ratio of the braking forces of the rear wheels and the overall braking force of the vehicle, i.e.
$\Phi=\frac{F_{k S}}{F_{k P}+F_{k S}}$
Since in the ideal distribution of braking forces the condition must be fulfilled according to which the braking forces of the wheels are proportional to their dynamic loads, i.e.


Figure 4 - Limits of adhesion of the front and rear wheels of a fully loaded vehicle in different road surface conditions for the installed braking system with linear characteristic.


Figure 5 - Limits of adhesion of the front and rear wheels of a ready-for-movement vehicle at various conditions of the road surface for the installed braking system with bi-linear characteristics
$\frac{F_{k P}}{F_{k S}}=\frac{G_{P}}{G_{S}}$
it follows that
$F_{k S}=\frac{G_{S}}{G_{P}} F_{k P}$
If this expression for the braking force of the rear wheels is introduced into the expression for the braking forces distribution coefficient, the following is obtained:


Figure 6 - Limits of adhesion of the front and rear wheels of a ready-for-movement vehicle at various road surface conditions for the braking system with the ideal distribution of braking forces per vehicle axles.
$\Phi=\frac{\frac{G_{S}}{G_{P}} \cdot F_{k P}}{F_{k P}+\frac{G_{S}}{G_{P}} \cdot F_{k P}}=\frac{G_{S}}{G_{P}+G_{S}}$
Dynamic loads of the front and rear vehicle axle, as mentioned before, are:
$G_{P}=G \cdot\left[(1-\Psi)+\frac{a}{g} \chi\right]$
$G_{S}=G \cdot\left[\Psi-\frac{a}{g} \chi\right]$
If we introduce these expressions for dynamic axle loads in the above expression for the barking forces distribution coefficient, it follows:
$\Phi=\frac{\Psi-\frac{a}{g}}{1-\Psi+\frac{a}{g} \chi+\Psi-\frac{a}{g} \chi}=\Psi-\frac{a}{g}$
By introducing this value of the braking forces distribution coefficient into the above mentioned expressions for limits of adhesion of the front and rear wheels of the considered simplified vehicle model, the expressions are transformed into the following forms:

## A) LIMIT OF ADHESION OF THE FRONT WHEELS OF THE VEHICLE

$(1-\Psi)^{2} b^{2}+(1-\Phi)^{2} a^{2}-\mu^{2} g^{2}\left(1-\Psi+\frac{a}{g} \chi\right)^{2}=0$

$$
\begin{align*}
& (1-\Psi)^{2} b^{2}+\left(1-\Psi+\frac{a}{g}\right)^{2} a^{2}- \\
& -\mu^{2} g^{2}\left(1-\Psi+\frac{a}{g} \chi\right)^{2}=0  \tag{38}\\
& (1-\Psi)^{2} b^{2}+\left(a^{2}-\mu^{2} g^{2}\right)\left(1-\Psi+\frac{a}{g}\right)^{2}=0  \tag{39}\\
& (1-\Psi)^{2} b^{2}+(1-\Psi)^{2} a^{2}+2(1-\Psi) \frac{a^{3}}{g} \chi+\frac{a^{4}}{g^{2}} \chi^{2}- \\
& -\mu^{2} g^{2}(1-\Psi)^{2}-2 \mu^{2} g(1-\Psi) a \chi-\mu^{2} a^{2} \chi^{2}=0 \tag{40}
\end{align*}
$$

By dividing the above equation by $\left(g^{2}\right)$ we get the expression for the relation between the lateral acceleration and the deceleration of the vehicle at the road-holding limit of the front wheels
$(1-\Psi)^{2} \frac{b^{2}}{g^{2}}+\chi^{2} \frac{a^{4}}{g^{4}}+2(1-\Psi) \chi \frac{a^{3}}{g^{3}}+$
$+\left[(1-\Psi)^{2}-\mu^{2} \chi^{2}\right] \frac{a^{2}}{g^{2}}-2 \mu^{2} \chi(1-\Psi) \frac{a}{g}-$
$-\mu^{2}(1-\Psi)^{2}=0$

## B) LIMIT OF ADHESION OF THE REAR WHEELS OF THE VEHICLE

$\Psi^{2} b^{2}+\Phi^{2} a^{2}-\mu^{2} g^{2}\left(\Psi-\frac{a}{g} \chi\right)^{2}=0$
$\Psi^{2} b^{2}+\left(\Psi-\frac{a}{g} \chi\right)^{2} a^{2}-\mu^{2} g^{2}\left(\Psi-\frac{a}{g} \chi\right)^{2}=0$
$\Psi^{2} b^{2}+\left(a^{2}-\mu^{2} g^{2}\right)\left(\Psi-\frac{a}{g} \chi\right)^{2}=0$
$\Psi^{2} b^{2}+\Psi^{2} a^{2}-2 \Psi \frac{a^{3}}{g} \chi+\frac{a^{4}}{g^{2}} \chi^{2}-$
$-\mu^{2} g^{2} \Psi^{2}+2 \mu^{2} g \Psi a \chi-\mu^{2} a^{2} \chi^{2}=0$
By dividing the above expression, as with the front wheels, by $\left(g^{2}\right)$, we obtain the expression for the relation between the lateral acceleration and deceleration of the vehicle at the limit of adhesion of the rear wheels of the vehicle.
$\Psi^{2} \frac{b^{2}}{g^{2}}+\chi^{2} \frac{a^{4}}{g^{4}}-2 \Psi \chi \frac{a^{3}}{g^{3}}+\left(\Psi^{2}-\mu^{2} \chi^{2}\right) \frac{a^{2}}{g^{2}}$
$+2 \mu^{2} \chi \Psi \frac{a}{g}-\mu^{2} \Psi^{2}=0$

## 3. TRANSITORY VEHICLE SPEED TROUGH A CURVE

In today's procedure, and in several science papers, school and faculty books, vehicles are due to analyses of vehicle movement through a curve, very
often regarded as material spot, while such idealistic analysis of vehicle formula is given in order to calculate the limit speed, i.e., the vehicle speed negotiating a curve:
$v_{\text {gran }_{0}}=\sqrt{\mu \cdot g \cdot R_{0}}$
where:
$\alpha$-Coulomb friction coefficient
$g$ - gravity
$R_{0}$ - radiance of vehicle weight centre
But investigations of vehicle speeding negotiating a curve show that in realistic traffic situations, lateral skidding of the vehicle wheels happen considerably lower values of speeds of those calculated by analitical expression (47).

Figures (7) and (8) show, according to DESOYER and SLIBAR the flows of road-holding coefficients of the wheels for two characteristic vehicle design concepts, i.e. for the rear-drive and for the front-drive vehicles, depending on the speed of movement in the curve with a constant radius of curvature and for the dry asphalt surface condition. Based on the presented diagrams the following can be concluded:

1. that the flows of road-contact coefficients of the same wheels of the rear-drive and front-drive vehicles differ slightly, meaning that the position of the driving wheels has no major significance on the analysis of the critical conditions of the movement through a curve. This is proved also by the studies of ROMPE, according to which the location value of vehicle skidding when the wheel road-holding limit is exceeded. of the driving wheels has major effect only on the value of vehicle skidding when the wheel adhesion limit is exceeded
2. that with the certain value of lateral vehicle acceleration, due to reduction in load caused by centrifugal force, the realised values of adhesion


Figure 7
coefficients of the inner wheels, meaning the wheels closer to the rotation centre, are substantially greater, not only than the realised values of the adhesion coefficients of the outer wheels, but also than the coefficient values of the lateral acceleration of the vehicle centre of gravity, whose acceleration otherwise matches the idealised observation of the vehicle as a material point, which proves without doubt, that the described and in the previous practice most frequently used idealised view of vehicle as a material point, with the


Figure 8


Figure 9
application of Coulomb's friction, presents a very rough and not allowed approximation of the actual condition.
3. that in case of exceeding the speed limit of vehicles negotiating a curve, first skidding of the inner wheel of the driving axle will occur, and, since the loss of stability in one of the wheels on the axle, due to reduction in load caused by the centrifugal force, results immediately in loss of stability of the whole axle, i.e. lateral skidding occurs, it means that the driving axle is of relevance for the analysis of the critical movement conditions of the vehicle in a curve.
4. since the flows of adhesion coefficients of the inner wheels differ slightly, and the same case is also with the outer wheels, this means that in case of the lateral skidding of one axle, soon lateral skidding of the other axle will occur, which was confirmed by the analyses performed by SORGATZ and AMMESDÖRFER.
These statements, proved in practice, hold of course regardless of the road surface condition, and as presented in Figure (9) and (10), where according to SLIBAR, DESOYER, and LUGNER presented flows of adhesion coefficients of the single wheels and average turning angles of the controllable wheels of the rear-drive vehicles in the function of lateral acceleration, i.e. speed of vehicle negotiating a curve with the constant radius of curvature, in case of the dry and wet asphalt surface condition.

## SAŽETAK

## ANALIZA VELIČINA KRITIČNIH BRZINA KRETANJA VOZILA KROZ ZAVOJ

Velik broj vrlo teških prometnih nesreća nastaje kao izravna posljedica destabilizacije vozila u zavoju, najčěšće uzrokovane prevelikom brzinom kretanja vozila za odredenu konfiguraciju zavoja i za određeno stanje podloge.S time u vezi, pitanje odredivanja veličine prolazne brzine kretanja vozila kroz zavoj nije samo problem s kojim se susreću eksperti koji se bave problematikom analize realnih prometnih nesreća, nego je to i pitanje bez kojega se praktički ne može zamisliti provodenje preventivnih mjera sigurnosti u cestovnom prometu, $i$ to kako u pogledu konstrukcije i izvodenja prometnica, tako i glede propisivanja veličina dopuštenih brzina kretanja vozila na odredenom mjestu. Cilj rada je pokušati definirati točnu analitičku metodu za realni sluc̆aj prolaza vozila kroz zavoj.

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