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MODEL OF OPTIMAL CARGO TRANSPORT STRUCTURE BY FULL CONTAINER SHIP ON PREDEFINED SAILING ROUTE

ABSTRACT

This paper presents the mathematical model for solving the problem of defining optimal cargo transport structure, occurring when, on a predefined sailing route, adequate number of containers of various types, masses and sizes, possibly including RO/RO cargo, is to be selected, i.e., a "container lot" is to be established in loading ports with the aim of gaining maximum ship profit and, at the same time, of exploiting useful load and transport capacity of container ship as much as possible. The implementation of the proposed model enables considerable increase in the efficiency of container ship operations. The model was tested using a numerical example with real data. The applied post-optimal analysis examines the influence of change in some values of the mathematical model on the resulting optimal program.

KEY WORDS

container transport, structure of container transport by ship, linear programming, post-optimal analysis

1. INTRODUCTION

The containerised maritime transport represents higher level of transport services as compared to the classic maritime transport technology. In order to achieve the maximum possible technological exploitability rate of full container ship, before building such a ship the owner should do the following:

- define all geographical areas where the ship will operate (winds, temperatures, ice and freezing, usage of inland waterways and channels, passages under bridges, minimum UKC, maximum draft);
- analyze container flow rate on particular sailing route and, using appropriate method, determine average TEU mass, which will equal the "design TEU mass" in ship design
- monitor, in terms of statistics, and define, using descriptive statistical methods, significant number of 20' and 40' containers in order to make correct decision on installation of sufficient number of container cell guides for 20' containers and define adequate number of bays.

The design TEU mass and average number of TEUs in defined area of operation will determine constructional, technical, technological and exploitation characteristics of the ship.

As to the ships already in operation, higher coefficient of technological exploitability is achieved through "pyramidal stacking" providing more favourable ratio between quantities of cargo and ballast, and therefore better stability of the ship during navigation.

The rate of technological exploitability of container ships is the ratio between the number of actually loaded containers (expressed as TEU) and the total number of positions accommodating containers, expressed as TEU, where useful load must not be exceeded. The maximum rate of technological exploitability is achieved when the average mass of loaded containers equals the "design TEU mass" for each respective ship.

The transport process of maritime container technology can be considered from various points of view: technical, technological, economic, legal, etc. High technological exploitability rate of container ships influences their efficiency of operation measured in respect to the profit resulting from transport of cargo accommodated in containers on the particular sailing route. A ship will not yield profit if empty containers owned by the operator are transported, i. e., humanitarian aid, which is transported free of charge.

Since the operation of a ship is based on profit, it is important that cargo transported on the container ship is arranged so that costs are minimal or that it yields maximal possible profit, i. e., to determine such a structure of container ship transport, according to ISO container types, which would yield maximal profit, considering useful load and transport capacity of the container ship.

This paper considers the so-called full container ships[1] addressing exclusively ISO containers[2] and possible restrictions in respect to RO/RO cargo on trailers loaded on deck, unlike other ship types where the structure of cargo can be bulk cargo, general cargo, mixed cargo, liquid or gas cargo, which are more difficult to quantify unambiguously.

Containers loaded on full container ships are uniformly internationally standardized, which enables setting of universal conditions to be met in drawing up adequate mathematical model.

The objective of this paper is to show that adequate operation research methods, specifically linear programming, can be used to determine optimal cargo structure transported on container ships.

2. PROBLEM DEFINITION

The profit yielded from transport of containerised cargo by fleet of full container ships on particular sailing route is directly related to the structure of cargo stowed in containers loaded in ports of loading and transported to destination ports.

In ports of loading, agents, forwarders and ship-owners' representatives find cargo offered for transport in ISO containers. On the basis of the total cargo supply and number of containers available for loading, "container lots" are formed for each particular container ship of a fleet maintaining regular service on particular sailing route.

The ship management does not define the structure of cargo, i. e., container lot to be transported, but rather organises the loading, stowing, transport and unloading of containers, based on the pre-set container lot in port of loading.

Arrangement of containers planned for loading in ship cells is made taking into consideration the following factors: voyage rotation, stability and ship construction load, allowed support surface load, quantity, size and type of containers, hazardous cargo in containers and ship stay in port (loading/unloading standard rates).

The problem of defining optimal cargo structure for transport by full container ship occurs when it is necessary to select the adequate number of containers of various types and masses and possibly RO/RO cargo on trailers, out of the number of containers available in port of loading so that maximal profit is gained and, at the same time, to maximally exploit the payload and TEU capacity of a ship.

There are several solutions to the problem. One solution uses a series of simulations or attempts until the solution providing the maximum profit is found. This method is relatively time-consuming and slow and it is not considered for commercial use. In real life, such problems are most often solved intuitively by shipmasters, on the basis of their many years of experience in the trade.

Maximal profit as an optimization criterion is chosen because full container ships have very high daily fix expenses (over 5000 USD) and from the economic point of view the most important issue is to yield maximum profit per voyage.

3. FORMULATION OF THE MATHEMATICAL MODEL

In order to solve the previously defined problem using operations research, it is necessary to set adequate mathematical model as follow:

Criteria function

$$\max Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

with constraints

$$\sum_{j=1}^{n} a_j x_j \leq N_k \, , \tag{2}$$

$$\sum_{j=1}^{n} x_j \leq d_i \, , \quad i = 1, 2, 3, \ldots, m \tag{3}$$

$$\sum_{j=1}^{n} x_j \leq g_i \, , \quad i = 1, 2, 3, \ldots, m \tag{4}$$

$$x_j \geq 0 \, , \quad j = 1,2,3, \ldots, n„ \tag{5}$$

where:

Z – criteria function,

c_j – criteria coefficient of j-th variable, i. e., profit per one transported container or trailer,

x_j – quantity (amount) of j-th variable, i. e., number of containers or trailers for transport according to optimal solution found,

N_k – ship payload for respective voyage in tons,

d_i – ship capacity in TEU, i. e., number of free spaces or positions for loading containers or trailers,

g_i – number of containers and trailers of various types, sizes and masses available for loading,

a_j – quantity of restriction required for one unit of j-th variable, i. e. unit mass of container or trailer,

m – number of restrictions, i. e. ship payload, TEU transport capacity of ship and available number of containers and trailers,

n – number of variables, i. e. number of container and trailer types of various sizes and masses.

To solve the optimal full container ship transport structure according to the set mathematical model means to define the structure variable values x_j

($j = 1,2,3, \ldots, n$) that yield maximal value of the function of the criterion Z, and at the same time meet all the basic and additional conditions, i. e. set restrictions. The values of g_i, c_j, a_j, m and n can be any positive integer.

Some new restrictions can occur in maritime practice, for example, if certain quantity of some container or trailer type, size or mass are mandatory in the structure of optimal program regardless of the financial effects of such a decision. In that case, the mathematical model would be:

Criteria function

$$\max Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \tag{6}$$

with constraints

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n \le N_k \tag{7}$$

$$\begin{aligned} x_1 \quad & \le g_1 \\ x_2 \quad & \le g_2 \\ \ldots \quad & \\ x_n \quad & \le g_m \end{aligned} \tag{8}$$

$$\begin{aligned} x_1 + x_2 + \ldots + x_k \quad & \le d_1 \\ x_{k+1} + \ldots + x_f \quad & \le d_2 \\ \ldots \quad & \\ x_{f+1} + \ldots + x_n \quad & \le d_m \end{aligned} \tag{9}$$

$$x_i \le g_i \le d_i \,,\, i = 1,2,3, \ldots, m \tag{10}$$

$$x_j \ge 0 \,,\, j = 1,2,3, \ldots, n \tag{11}$$

As can be seen from the presented model, restrictions stated in the system of non-equations are set primarily because, in maritime practice, it is not possible to form a structure of container transport that would simultaneously fully exploit payload and transport TEU capacity of a container ship. Also, non-equations are more flexible than equations because they allow a solution to be selected within a certain interval. Specifically, if restrictions under (7)-(10) had "=" instead of "≤", the number of possible solutions in the set mathematical model would be considerably reduced.

The problem of finding optimal cargo transport structure for a container ship using the proposed mathematical model is solved by the method of linear programming, particularly, using simplex method.

For the sake of simplification, the application of personal computer software, e. g. QSB [4], is recommended, rather than manual calculation.

Also, it is recommended to perform post-optimal analysis after finding the optimal solution in order to establish the influence of changes in some elements of the mathematical model on the optimal solution found and to compare alternative solutions of the given problem, on the basis of which adequate business decision could be made.

Detailed explanations regarding implementation of particular solution methods and modes are available in respective literature on operations research [1], [2] and [9].

4. NUMERICAL EXAMPLE

An example using real data was chosen to illustrate the set mathematical model.

In ports of loading, there is a variety of containerised cargo available for maritime transport.

Mass and profit per container types are shown in Table 1.

Table 1 - Daily profit per one transported container ready for loading

Type and size of container	20'DB	20'OT	20'RF	20'OS
Unit mass in tons	11	15	13	12
Profit per 1 container in USD	48	63	60	59
Type and size of container	20'TC	40'PL	40'RF	40'DB
Unit mass in tons	16	23	25	21
Profit per 1 container in USD	69	70	72	67

The following quantity is ready for loading, presented by type, size and allowed container mass[3]:

160 x 20' DB à 11t,	400 x 20' TC à 16t,
120 x 20' OT à 15t,	160 x 40' PL à 23t,
200 x 20' RF à 13t,	82 x 40' RF à 25t,
300 x 20' OS à 12t,	80 x 40' DB à 21t.

Transport is to be carried out by means of full container ship of 29,434 tons total cargo capacity and 1762 TEU[4] transport capacity. The planned duration of the voyage is nine days. The average consumption of heavy fuel is 45t/day and water consumption is 15 t/day. Ship navigates in "winter zone". Ship tanks contain 300 tons of light fuel, 150 tons of lube oil, 1100 tons of heavy fuel oil, while the crew and other provisions weigh 200 tons. Ship maximal load is 77 x 40' RF containers, i. e., a total of 1154 x 20' and 304 x 40' containers can be stowed. Also, 1600 tons of ballast is necessary for ballasting (adequate stability, trimming).

The goal is to define optimal transport structure of container ship, by ISO container types, obtaining the maximal total profit.

Note: Ship payload[5] = 29434 – 3530 = 25904 tons.

The respective mathematical model for the given problem is as follows:

Criteria function

$$\begin{aligned} \max Z = 48x_1 + 63x_2 + 60x_3 + 59x_4 + \\ + 69x_5 + 70x_6 + 72x_7 + 67x_8 \end{aligned} \tag{12}$$

Constraints

$$11x_1 + 15x_2 + 13x_3 + 12x_4 + 16x_5 + 23x_6 + 25x_7 + 21x_8 \le 25904$$

$$x_1 \le 160$$
$$x_2 \le 120$$
$$x_3 \le 200$$
$$x_4 \le 300 \quad (13)$$
$$x_5 \le 400$$
$$x_6 \le 160$$
$$x_7 \le 77$$
$$x_8 \le 80$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \le 1154 \quad (14)$$

$$x_6 + x_7 + x_8 \le 304 \quad (15)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \ge 0 . \quad (16)$$

Using the simplex algorithm through eight iterations the following optimal structure program for container ship transport is obtained:

structural variables	additional variables
$x_1 = 134$	$u_1 = 3018$
$x_2 = 120$	$u_2 = 26$
$x_3 = 200$	$u_3 = 0$
$x_4 = 300$	$u_4 = 0$
$x_5 = 400$	$u_5 = 0$
$x_6 = 160$	$u_6 = 0$
$x_7 = 77$	$u_7 = 0$
$x_8 = 67$	$u_8 = 0$
	$u_9 = 13$
	$u_{10} = 0$
	$u_{11} = 0$.

All types of ISO containers are represented in the optimal solution. Total profit calculation is shown in Table 2.

Table 2 - Structure of optimal transport program

Type and size of container	Unit mass in tons	Number of containers	Profit per container (USD/day)	Total profit (USD/day)
-	a_j	x_j	c_j	$c_j x_j$
20' DB	11	134	48	6432
20' OT	15	120	63	7560
20' RF	13	200	60	12000
20' OS	12	300	59	17700
20' TC	16	400	69	27600
40' PL	23	160	70	11200
40' RF	25	77	72	5544
40' DB	21	67	67	4489

Value of optimal program:

$$\max Z = \sum_{j=1}^{8} c_j x_j = 92\,525 \text{ USD/day.}$$

Exploitability of ship payload:

$$N_k = \sum_{j=1}^{8} a_j x_j = 22\,886 \text{ tons,}$$

meaning that total ship payload is not fully exploited since there is $u_1 = 3018$ t, i. e. 11.65 percent of unexploited ship payload.

Exploitability of ship transport capacity:

$$P_k = \sum_{j=1}^{5} x_j + \sum_{j=6}^{8} x_j = 1154 + 304 = 1458 \text{ containers.}$$

Transport capacity P_k is completely used according to the obtained optimal program because the quantity of 304 x 40' containers corresponds to 608 x 20' containers, i. e. transport capacity of 1762 TEU, which is also confirmed by the optimal solution ($u_{10} = u_{11} = 0$).

The values of additional variables $u_2 = 26$ and $u_9 = 13$ show cargo, i. e. containers that should be "rejected", since all locations intended for storage of containers on board are occupied, it is obvious that 26 x 20' DB à 11 t containers and 13 x 40' DB à 21 t containers cannot be loaded on board.

A post-optimal analysis established that change of either coefficient in criteria function or amount of particular restriction influences the value and structure of variables included in the optimal solution.

The structure of the optimal program will not change if daily profit per one transported container for structural variables included in the optimal solution is in the following interval ranges:

$x_1 \rightarrow$ 20'DB à 11t : $0 \le c1 \le 59$ USD
$x_2 \rightarrow$ 20'OT à 15 t : $48 \le c2 \le \infty$ USD
$x_3 \rightarrow$ 20'RF à 13 t : $48 \le c3 \le \infty$ USD
$x_4 \rightarrow$ 20'OS à 12 t : $48 \le c4 \le \infty$ USD
$x_5 \rightarrow$ 20'TC à 16 t : $48 \le c5 \le \infty$ USD
$x_6 \rightarrow$ 40'PL à 23 t : $67 \le c6 \le \infty$ USD
$x_7 \rightarrow$ 40'RF à 25 t : $67 \le c7 \le \infty$ USD
$x_8 \rightarrow$ 40'DB à 21 t : $0 \le c8 \le 70$ USD .

The conclusion follows that the structure of optimal solution would change only if transport market offered 20'DB containers à 11 t and 40'DB containers à 21 t, where daily profit per one container would amount to 59 USD or more and 70 USD or more, respectively.

Intervals within which restriction amounts can be changed: payload, available quantity of containers for loading and maximal number of 20' and 40' containers for the given problems are as follows:

$$22886 \le N_k \le \infty$$
$$134 \le g_1 \le \infty$$
$$94 \le g_2 \le 254$$
$$174 \le g_3 \le 334$$
$$274 \le g_4 \le 434$$
$$374 \le g_5 \le 534$$

$$147 \le g_6 \le 227$$
$$64 \le g_7 \le 144$$
$$67 \le g_8 \le \infty$$
$$1020 \le d_1 \le 1180$$
$$237 \le d_2 \le 317 .$$

If the number of containers available for loading per particular container types were considerably greater than the transport capacity of container ship, then it would not be necessary to include restrictions in the mathematical model (13). In such case, the optimal solution would have the following structure:

$x_1 = 0 , x_2 = 0 , x_3 = 0 , x_4 = 0 ,$
$x_5 = 1154$ x 20' TC à 16 t ,
$x_6 = 80$ x 40' PL à 23 t ,
$x_7 = 224$ x 40' RF à 25 t ,
$x_8 = 0$
$u_1 = 0 , u_2 = 0 , u_3 = 0 .$

The ship payload and transport capacity would be fully exploited and the value of optimal program would amount to $Z = 101354$ USD/day.

Such a solution is probably not so often in real life, and a great number of a single container type (20'TC à 16 t) in optimal solution results from the favourable ratio between profit and container mass.

5. CONCLUSION

The process of maritime container transport technology is very complex and it requires a thorough study of all the elements included in the process, from various points of view. In order to justify the technical and technological advance in container technology, it is necessary to optimize the transport of cargo, i. e. to organize transport by container ships either with minimal expenses or with maximal profit.

The rate of success in operating a container ship depends directly on the forming of "container lots", which are defined for each particular ship. Therefore, utmost importance lies on the adequate selection and arrangement of various container types offered for transport on a particular line provided by the respective ship-owner. The selection of containers available for transport by ship is performed "ashore", while on board, containers are arranged depending on the actual situation during loading.

Since each full container ship operation is based on profit, the success in business can be considerably improved by implementation of the model of optimal container transport structure on some sailing route, as shown in this paper. Personal computers and adequate software relatively quickly and easily provide "optimal container lot" to be formed out of a great number of containers available on maritime container transport market.

The following preconditions must be met when defining the structure of transport for any container ship:

- maritime market must offer enough containers for transport (specifically, more than ship transport capacity) of various types, sizes and masses (20', 40'), thus enabling profit resulting from transport
- it must be a full container ship with a capacity greater than 700 TEU, because ships of smaller capacity are supposed to operate as "feeder service" ships, which are loaded regardless of profit since their basic function is to deliver all containers from a home port to all destinations, and vice versa
- transport must be "long distance voyage" i. e. minimum 1000 NM.

After having found the optimal solution using adequate procedure, it is necessary to perform post-optimal analysis in order to find out if there is any possibility of improving the optimum transport structure for a particular ship in terms of increasing profit or meeting technological and exploitation characteristics of such a ship.

The presented mathematical model is very useful in operation planning for making adequate business decisions with respect to the container ship transport structure.

The validity of the model is especially important considering continuous changes of conditions that owners experience on the market; the post-optimal analysis provides various alternatives for the solution of the problem when the number or profit of a particular ISO container type change.

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MODEL OPTIMALNE STRUKTURE PRIJEVOZA TERETA POTPUNO KONTEJNERSKIM BRODOM NA ODREĐENOJ MORSKOJ LINIJI

SAŽETAK

U ovom je radu prikazan matematički model za problem određivanja optimalne strukture prijevoza tereta potpuno kontejnerskim brodom, koji se pojavljuje kada na određenom unaprijed definiranom morskom prometnom pravcu od kontejnera koji su na raspo-laganju u lukama ukrcaja treba odabrati odgovarajući broj kontejnera raznih vrsta, masa, veličina uz eventualni RO/RO teret, tj. formirati "kontejnersku pošiljku" s ciljem da se ostvari maksimalna dobit broda i uz uvjet da se, što je više moguće, iskoristi nosivost i prijevozni kapacitet broda. Korištenjem predloženog modela moguće je bitno povećati uspješnost poslovanja kontejnerskog broda, odnosno flote kontejnerskih brodova. Model je testiran na numeričkom primjeru s realnim podacima. Primijenjena je postoptimalna analiza kojom se ispituje utjecaj promjene vrijednosti pojedinih

elemenata matematičkog modela na dobiveni optimalni program.

KLJUČNE RIJEČI

kontejnerski promet, struktura prijevoza tereta kontejnerskim brodom, linearno programiranje, postoptimalna analiza

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1. Full container ship is a ship intended exclusively for transport of 20' and 40' ISO containers; bays are equipped with cells to accommodate containers and it has no equipment for handling (loading/unloading) containers.
2. ISO containers are uniformly standardized by length, width, height and load.
3. Designations 20' and 40' refer to 20 and 40 feet container, respectively, thus defining container size. Additional designations refer to special types of containers: DB - dry box container, OS - open side container, OT - open top container, PL - container platform, RF - refrigerated container and TC - tank container.
4. The capacity is calculated for 20' containers because, considering size, 1x 40' container equals 2x20' containers.
5. The quantity of 3,530 tons refers to heavy and light fuel and potable water supplies as well as other supplies; however, considering that navigation in "winter zone" requires one-day supply for every three days of navigation, i. e. for a nine-day journey, supplies are calculated for 12 days.

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