

DAMIR BUDIMIR, Ph.D.<sup>1</sup>

(Corresponding author)

E-mail: damir.budimir@fpz.hr

MARKO ŠOŠTARIĆ, Ph.D.<sup>1</sup>

E-mail: marko.sostaric@fpz.hr

KREŠIMIR VIDOVIĆ, Ph.D.<sup>2</sup>

E-mail: kresimir.vidovic@ericsson.com

<sup>1</sup> University of Zagreb

Faculty of Transport and Traffic Sciences

Vukelićeva 4, 10000 Zagreb, Croatia

<sup>2</sup> Ericsson Nikola Tesla

Krapinska 45, 10000 Zagreb, Croatia

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# DATA ENVELOPMENT ANALYSIS FOR DETERMINING THE EFFICIENCY OF VARIANT SOLUTIONS FOR TRAFFIC FLOW ORGANISATION

## ABSTRACT

*There is a small number of empirical modelling study cases available that are related to the calculation of variant solutions efficiency from the aspect of sustainable mobility in the urban areas. In practice, it is often necessary - especially when it comes to the urban transport network - to evaluate the solutions for traffic flow organisation and routing, in order to implement the one(s) with the maximum potential to reduce the possibility of congestion during peak travelling periods i.e. during transport network peak load. The paper presents an approach to the aforementioned problem by the application of the transport system efficiency analysis. The aspect of traffic flow organisation and routing efficiency in variant solutions is clarified through the analysis model development, built on the premises of Data Envelopment Analysis (DEA) method and the principles of unnecessary traffic flow intersections (TFI) theory. The proposed model defines the efficiency limit for data attributed to variant solutions, based on the calculation of the optimal TFI model and the possibilities of DEA method that include comparison and definition of relative routing efficiency for every optional traffic flow against the efficiency limit (optimal model) in order to calculate relative efficiency in relation to other solutions.*

## KEY WORDS

*data envelopment analysis; modelling methodology; traffic flow organisation; traffic flow intersection; efficiency analysis; transport network; sustainable mobility;*

## 1. INTRODUCTION

The accelerated social development is conditioned upon the fast development of the transport systems. The benefits from urban traffic flow enhancement are multifaceted: reduced number of traffic accidents, improved capacity of crossroads and road network, increased average vehicle speed, reduced pollution, savings generated by the decreased need for additional

infrastructural investment costs (since the existing infrastructure is better utilised), decreased individual and public transport vehicle operational costs [1, 2].

Every transport network can be reorganised i.e. routes of traffic flows can be changed. By using the mathematical calculation of intersection quantity, inflows and outflows, it is possible to identify the current number of flow conflicts in the network. The aim of the traffic flow re-routing is to achieve a lower intensity of flow intersections, which indicates a higher quality of traffic flow organisation. Reduction of the number of traffic flow intersections increases permeability i.e. these two parameters are correlated [3, 4]. As an example, the reorganisation also includes a modification of traffic lights operation regime and a general traffic flow regulation together with a modification of the traffic surfaces utilisation in terms of the number and direction of the traffic lanes [5].

Practical application of relative technical efficiency measurement in variant solutions for traffic flow regulation and routing in sustainable mobility models is reflected in the evaluation of the existing network or its parts, from the perspective of fulfilling the sustainable mobility requirements during peak load, with the objective of identifying realistic possibilities of network utilisation improvement in time dynamics. The increase of the number of participants in the transport network (the network load, which is especially expressed in the peak hours defines the entry of the model), results in an increase in the intersection of traffic flows (defining the output) in the observed time. The intensity of the output (intersection of traffic flows) is the relation of the process of movement of users of the transport network. In general, the efficiency measurement that is a relation of a process, can be described by input and output [6]. For example, the intersection of traffic flows

is the realization of the process of running traffic flows or the number of participants in a transport network whose routes intersect.

Most of the literature dedicated to the transport network efficiency focuses on the observation of traffic flows through statistical data and other parameters related to the traffic flow volume and density. There is a certain deviation of the results presented in papers that include transport network efficiency assessment, since these results are based on empirical research executed in a specific timeframe (e.g. morning, afternoon, work day, weekend) [7] and there are no unified criteria for the results quality.

Opposite to the aforementioned approaches, this research is focused on the vector analysis of traffic flow conflicts (using *OptaGIS* software tool), primarily at one-level crossroads, with an application of Data Envelopment Analysis (DEA). Within this approach, the key challenge is to define a decision-making unit (DMU). The efficiency results of the ranked solutions are given as relative values, in relation to inputs and outputs. The variables that are analysed because of their importance for the variant solutions efficiency are presented as decision-making units (e.g. the number of traffic flow conflicts on the observational network).

Taking the aforementioned conclusions into consideration, the following research hypotheses are proposed:

- H1: Efficiency and effectiveness of the traffic flows are an important indicator for the selection of the variant solutions.
- H2: Efficiency and effectiveness of the traffic flows are affected by numerous impacts.
- H3: Approaches to the development of the key criteria for variant solutions efficiency and effectiveness evaluation are insufficiently explored.
- H4: Variant solutions can be improved by the application of an appropriate efficiency and effectiveness evaluation model.

Statistical regression methods measure the productivity or efficiency of the decision-making units in relation to the average or the trend. The application of DEA model enables an identification of the best decision-making unit from the set of compared units but also allows the less efficient units to be compared to the best DMU, based on the model inputs and outputs analysis [8]. By using DMU, the research implies an alternative transport network solution through the coefficient of unnecessary traffic flow intersections. In the research, the decision-making units are defined through the variant solutions, while the coefficients of traffic flow intersections on the analysed routes are taken as outputs. Inputs are defined as the number of vehicles on the network causing traffic congestion during the peak hour. Variant solutions are defined through the variations of traffic flow routing and regulation and are relevant for the assessment of efficiency – quality of

the transport network. It is generally considered that the DMU is the subject responsible for the generation of outputs in relation to inputs and its effect needs to be evaluated [9, 10].

DEA is being extensively developed and applied in different fields. There is a significant number of DEA-based models, all having in common the techniques of linear mathematical programming of outputs vs. inputs, which are used for the efficiency analysis and comparison of the different entities. By choosing a suitable DEA model, data can be analysed from multiple perspectives, depending on the dependency between inputs or outputs [11]. An overview of models is available in the papers published for the occasion of DEA method 30<sup>th</sup> anniversary [12-14]. There are several thousand publications about DEA in the form of articles in scientific journals, books and studies; even the regularly updated bibliographies cannot guarantee being complete in terms of DEA. Publications can be divided into two groups: one being dedicated to DEA application on specific issues of efficiency measurement and the second one, encompassing more theoretical publications focused on the development of the original model. Both groups are at least indirectly facing data quality assessment issues [15]. Despite that, the literature dedicated to the aspects of data quality assessment in variant solutions that need to be analysed before the implementation, is scarce. From the concept of the DEA method, one can easily notice its significant potential for measuring technical relative efficiency of the transport network variant solutions.

The core of such analysis is identification of an optimal, “virtual” decision-making unit, positioned on the efficiency boundary for every realistically compared DMU which belongs to the set of alternative solutions. If the virtual DMU is better than the real one i.e. results in a higher value of unnecessary TFI coefficient for an equal or higher number of input flows, the observed DMU is efficient. In practice, it is very difficult to evaluate inputs or outputs and achieve a common set of weighting coefficients, mostly because of different impacts of individual units and a lack of understanding of unit effects on inputs and outputs [16].

In the applicative part, the efficiency calculations were performed using a non-commercially developed Excel Solver solution at the Faculty of Transport and Traffic Sciences. The data analysis was used to optimize a part of the traffic network of the city of Makarska.

## 2. TECHNICAL EFFICIENCY OF VARIANT SOLUTIONS

The creators of DEA method [9], that has been modified and extended during the past years, have confirmed that an objective process for defining the value of weighting coefficients within the evaluation of inputs or outputs efficiency (units, parameters etc.) is

not necessary. However, all variables whose efficiency is being assessed have to be agreed, i.e. inputs, outputs and minimum values of weighting coefficients have to be defined. The problem of scaling is uniformly solved by expressing the efficiency of an individual parameter as a number between 0 and 1. For every value (unit) the values of weighting coefficients can be determined in the most appropriate way, thus maximising its efficiency. Additional analysis can enable a distinction between the (in)efficiency of the units considered.

The DEA method evaluates if a specific DMU shows higher efficiency when compared to other units included in the analysis, i.e. if it is positioned on the efficiency boundary.

The efficiency boundary in terms of traffic is an empirically identified maximum, expressed through the output variables, where every variable (in our case – an indicator) can be connected to the predefined input and behaves as an envelope for the inefficient variable values. The method analyses every output value and tests if the related input parameters could be enveloped bottom-up (tests if the specific outputs could be achieved with less TFIs i.e. with more efficiency), having in mind the input values of the remaining variables, but also tests if the outputs can be enveloped top-down (i.e. if the predefined inputs could result in more satisfactory outputs), considering the output values of the remaining variables. If a DMU could be enveloped, it is relatively inefficient. However, in case that a DMU cannot be enveloped, it is used in defining the efficiency boundary which presents an equivalent to boundary function of TFI.

According to the DEA premises, the method of calculating the relative efficiency of the transport network variant solutions uses a mathematical linear programming technique, enabling identification of variant solution efficiency (depending on the input and output data), in comparison to other variant solutions included in the analysis. It is a non-parametric approach, since it does not require an a priori assumption regarding the analytical form of the efficiency function. While the parametric approaches focus on the averaging and performance assessment for a specific entity which is carried out in relation to the average value [17], DEA is a method based on the borderline values calculation, consisting of a series of optimisations (one for each variant solution included in the analysis). For each DMU a maximum performance measurement is calculated, in relation to other units (proposed variant solutions) which have to meet the requirement of being positioned on or below the extreme boundary called the efficiency boundary. The efficiency measure provided by DEA is relative, since it depends on the number of variant solutions (entities) included in the analysis, but also on the number and the structure of inputs and outputs used.

Therefore, the key feature of this model is that each DMU is being assessed as relatively efficient or relatively inefficient. This statement is in line with the one made by the DEA method authors [9] claiming that one DMU could be characterised as efficient only if the following conditions have not been fulfilled:

- 1) It is possible to increase any of the outputs without increasing any inputs and without decreasing any other outputs.
- 2) It is possible to decrease any of the inputs, without decreasing any outputs and without increasing any other inputs.

In line with this point of view, the definition of relative efficiency is as follows: A decision-making unit will be treated as absolutely efficient (100%) on the basis of available data only if the realisation of other DMUs does not indicate, by using the same inputs or outputs, the possibility of achieving improvement without negatively affecting any other input or output [18].

The theoretical cognitions related to the quality issues of variant solutions for the traffic flow routing and regulation are empirically tested on one part of the transport network of the City of Makarska. The implemented research shows a possibility to standardise the methodology for the assessment of technical efficiency of the aforementioned solutions by using DEA as a useful and applicable tool for decision-making and performance evaluation in relation to other alternative solutions.

## 2.1 Factors for calculating technical efficiency

The growing frequency of congestions in traffic flows strengthens the need for an applied research in the field of traffic flow routing and organisation optimisation. Unfortunately, in the aforementioned field, there is a very small number of empirical modelling available, especially for urban areas. In general, the purpose of optimisation is to select the optimal solution from the set of all possible alternatives, in line with the defined objectives. Formally, optimising means defining the extremes (minimum or maximum) of the criteria function by the application of different methods, depending on the relation types occurring within the mathematical model and the conditions set.

The objective is to develop a model for variant solutions optimisation and evaluation, starting from the key premises of DEA model and the principles of traffic flow organisation and regulation theory. The relations between the factors in the decision-making process are presented in *Figure 1*. The proposed model contributes to the exploration of the main factors supporting the decisions related to the transport network optimisation, especially when the traffic loads are causing congestion on the specific parts of the network.

Each variant solution in the process of traffic flow re-regulation and re-routing, has to be founded on a reliably identified baseline and an assessment of modification possibilities. It can be described as an optimisation of traffic flows through the DEA-based recursive process (Figure 1).

The selection of the variant solution for traffic flow routing and regulation is significant, as it can enable an improved permeability within the transport network and the avoidance of the critical points where traffic flows are conflicting, which causes frequent congestions at certain levels of traffic loads (e.g. frequent congestions are the key feature of traffic flows in Makarska during the tourist season). The concept of traffic flows organisation and regulation efficiency, described in this paper, is in direct correlation with the avoidance of traffic flow anomalies at the given traffic load levels. The primary objective in such circumstances is to improve the traffic flow organisation (e.g. unidirectional into bidirectional or vice versa, turn prohibition, etc.) for the purpose of mitigating the negative impacts of transport network peak loads.

The initial step within the process of defining the new traffic flow organisation (the variant solution) is the analysis of the existing traffic flows in order to identify the deficiencies. The problem of proposing a solution lies in the endeavour to achieve maximum efficiency at the specific traffic load levels (where congestion occurs). Therefore, the objective is to enable an increase of the traffic load levels without causing anomalies in the traffic flows.

The calculation of the relative efficiency for a set of variant solutions, where identical criteria and loads are used, is a valid indicator of the relative relations between various solutions. As shown in Figure 3, a simple case with four input and two output parameters is considered in the case study (Makarska). The following parameters are used: traffic loads  $p$  and  $q$  which have, in the existing traffic flow organisation, led to significant congestions and increased in-vehicle time spent within the transport network (taken with the premise that the flow of traffic towards the defined points is unhindered, adding the requirement to achieve the most time-efficient routes when designing the solution).

As shown in Figure 1, the recursive process with the relative efficiency calculation for the set of variant solutions leads to the identification of the new traffic flow organisation solutions, on which new DMUs are based, as a part of input and output variables in the DEA model (*OptaGIS* application was used to identify the values of inputs and outputs for each variant solution). By introducing every new DMU into the DEA model, new relative relations are created between the DMUs, where the effects of the traffic flow modifications can be analysed through the analysis of the input and output variables (further through the premises of the DEA method). Therefore, the result of the processing is sorting the set of DMUs according to the relations between their effectiveness, expressed as the individual efficiency of the outputs in relation to the inputs for each DMU. The results provide guidelines towards the potential new solutions (as visible from the calculation explained in more detail further in the text).

The key determinants for proposing and/or improving the variant (alternative) solution are achieved through data quality, i.e. by enhancing the values of the chosen comparable units within the inputs and/or outputs of the set model, guided by the following principles important for the decision makers:

- At the beginning, a model based on the existing transport network is used and the variant solutions are defined only after a detailed exploration of the problems.
- The model is developed in order to consider different routes and relations between traffic flows, which can be easily achieved using GIS (Geographic Information System) interface based on vector graphics. General models have not shown as practical because of the significant number of requirements related to the parameters defined as vectors in space.
- Before proposing the variant solutions, whose efficiency will be compared, it should be determined whether the solution in question is technically and economically feasible and whether it will be, in the organisational terms, well accepted by the local community (it would be best if the potential users participated in the design process).

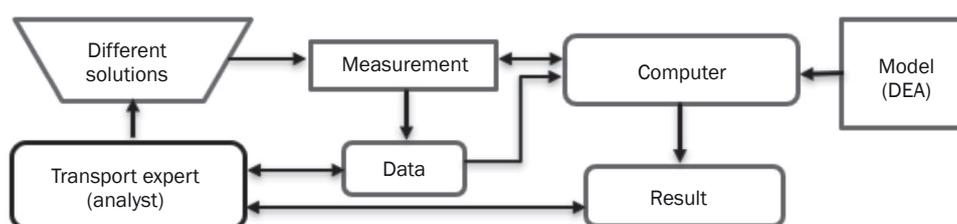


Figure 1 – The relations between factors in the decision-making process

- d) Data generation by the individual variant solution is carried out in parallel with the model development (Figure 1).
- e) The process of perfecting the model is carried out on the basis of recognised deficiencies, potential new requirements and variables that might appear in the development model, primarily for the purpose of an improved optimisation in practice (most often there is an increased number of vehicles in the analysed transport network access points).

### 2.2 Comparable decision-making units for efficiency

Transport network efficiency is of fundamental significance for the consideration of traffic flows in urban areas. In this context, the technical efficiency indicates a successful implementation of transport processes i.e. the efficiency of transport infrastructure in terms of its response to the transport demand.

Defining the set of alternatives (DMUs) to the existing solution of traffic flow routing and regulation is usually the initial step in calculating the relative efficiency of the transport network and it is directly connected to the selection of model inputs and outputs. Therefore, the choice of DMUs is the first and basic problem of data quality and, as previously mentioned in the decision-making process (Figure 1), can be recursively upgraded, achieving better results. The DEA is, due to its non-parametric approach towards the objective, especially vulnerable to the data selection and quality [19]. This implies a significant impact of DMUs efficiency boundary on other variables, but also other DMUs which determine the final relative efficiency of DMUs (variant solutions).

Because there are no common borderline values for a comprehensive analysis of the DMU efficiency boundaries, the empirical analysis was applied, using the assumed boundaries chosen according to the ideal model of TFIs (OptaGIS application). In line with the key premises of the DEA model, the selected units can

be evaluated regardless of whether they have been calculated or evaluated, whereby every unit value is equal to the rank of suitable modality [20].

Figure 2b shows an isolated part of the transport network. The TFI model, presented in Figure 2a, is an example of a minimum quantity of TFIs model within the isolated segment (between points A, C, D, F) while Figure 2c presents a model of traffic flow intersections on 4-leg crossroads. The assumed efficiency boundary is determined on the basis of minimum quantity model and defines the parameters of an ideal number of TFIs [3]. The parameters obtained are used for efficiency boundary testing and represent one of the main criteria of DEA results dependency on individual DMUs from variant solutions.

The intensity of intersections in conflicting points is determined by mathematical methods for calculating TFIs quantity at crossroads, according to the mathematical expressions 1-3 [3]. For the purpose of variant solutions efficiency assessment, a coefficient of TFI  $K_{INT}$  is introduced and defined. In order to define the coefficient, three measurement methods are set, presenting the intensity of traffic flow conflicts within the network:

- A method of minimum flow in the point of conflict:

$$K_{INT}(t) = \sum_{k=1}^n \min(p_k, q_k) \text{ [veh/h]} \tag{1}$$

- A method of summary traffic flows in the point of conflict; the deficiency is in the fact that the conflict exists even when one of the flows equals zero, which requires an additional condition:

$$K_{INT}(t) = \sum_{k=1}^n (p_k + q_k), \forall p, q > 0 \text{ [veh/h]} \tag{2}$$

- A method of square root of the surface area, for the area of conflict between traffic flows, is a product of traffic intensity in conflict points:

$$K_{INT}(t) = \sum_{k=1}^n \sqrt{(p_k \cdot q_k)} \text{ [veh/h]} \tag{3}$$

where  $p_k$  and  $q_k$  represent the conflicting traffic flows.

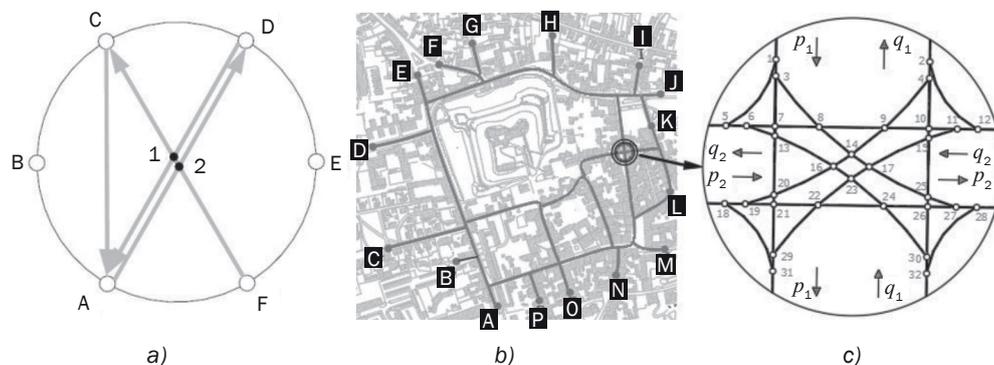


Figure 2 – Models of traffic flow intersections, source [3]

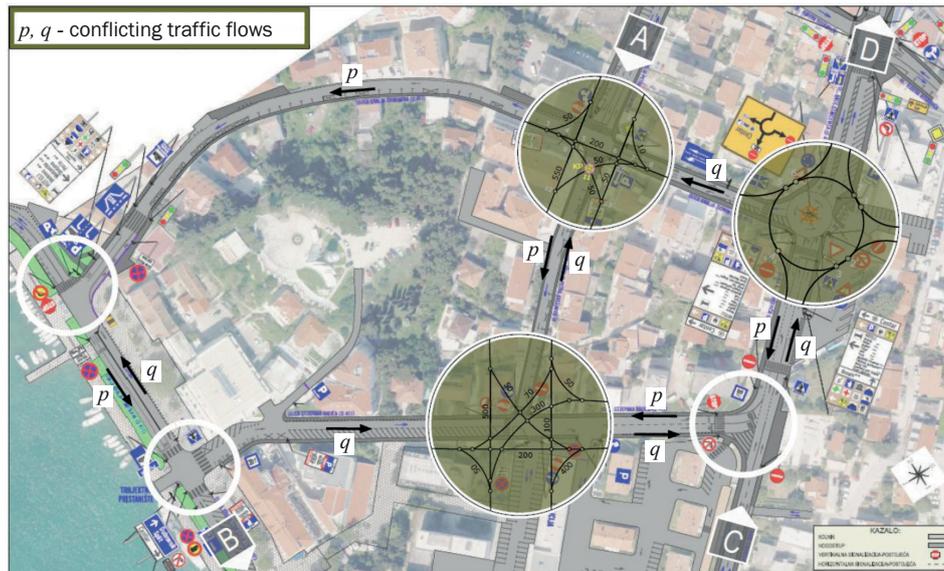


Figure 3 – Traffic solution on the segment of the transport network in the City of Makarska (OptaGIS)

The regulation and the relations between traffic flows are illustrated on the segment of transport network of the City of Makarska, Figure 3. The relations between the traffic flows at the intersections are one of the causes of the reduced capacity [3]. Avoidance of the unnecessary conflicts and the reduction of traffic flows breaking are some of the solutions used to increase the capacity of the intersection. In order to establish the optimal flows of traffic, the relations between them have to be identified. When it comes to the transport network efficiency, especially in the urban areas, the analyses of the interaction intensity between traffic flows have to be carried out. The interaction can be classified as passing, intersecting, interweaving, inflowing and outflowing [21].

Starting from the fact that the intensity of all types of traffic flow interaction is affected by the routing within the road network. The variant solutions for efficiency improvement of the traffic flow organisation and regulation should aim to achieve the flow of traffic with as little interaction intensity indicators as possible. Therefore, the road network is regarded as a set of vector elements, elements being the sections of the road network, while the interaction intensity is calculated on the basis of intersection intensity of the vector model.

The intersection, inflow and outflow, as the most important relations between traffic flows, occur at the intersections and are in direct functional relationship with the transport network routing, affecting the choice of vehicle routes. If the traffic routing is not appropriate, needless intersecting occurs and the network load is not equally distributed, which usually results in bottlenecks within certain elements [21].

By observing the relations between traffic flows, it is possible to realize the existing deficiencies in traffic flow management [22]. Such observations are a basis

for further strategic orientation i.e. the starting point for traffic flow organisation in variant solutions and an application of methods which can minimise excessive intersection of flows. Reorganising the traffic flows (alternative solutions) can lead to significant improvements [23].

The evaluation of traffic flows can be perceived through the outlier parameters, which define the transport network from the efficiency perspective and encompass the following: traffic routing in the transport network (mathematical methods for the calculation of the number of possible routes on the intersection access points), the number of intersection points at road intersections and the identified quantity of TFI at road intersections.

### 2.3 Defining the mathematical model

The development of the mathematical model for the efficiency and effectiveness evaluation is based on the general principles of traffic flow organisation and routing but also on the DEA method. The DEA can be input- or output-oriented. In output-oriented models, a DMU is inefficient if any output can be increased without increasing the inputs and decreasing any of the remaining outputs.

In input-oriented models, an analysed DMU is inefficient if any input can be decreased without decreasing the outputs and increasing any of the remaining inputs. The efficiency boundary is taken as an ideal number of TFIs and represents a key point of orientation in the analysis. In both cases there is an outer, efficient boundary for the unnecessary TFI coefficient  $K_{bp}$ , determined according to the ideal model. In each model, for the purpose of the DMU efficiency quantification, a distance between  $K_{bp}$  and the efficiency boundary is measured by the remoteness function

[24, 25]. Without a more detailed familiarisation with DEA production and theoretical assertions, the set of all variables used in the decision-making process can be formally defined (4):

$$SR = \{z \in R^{r+s} | z \text{ is feasible (possible)}\} \tag{4}$$

where  $r$  represents the number of inputs,  $s$  the number of outputs and  $z$  the correctly valorised vector of  $r+s$  dimension, which forms all the observed DMU results  $z_k := (x_k, y_k)$ ,  $k=1, \dots, n$  from SR. Expression  $x_k := (x_{k,1}, \dots, x_{k,r})$  describes an input vector (entity input), while  $y_k := (y_{k,1}, \dots, y_{k,s})$  describes an output vector.

It is assumed that all SR elements can be described by the same metrical, non-scaled and non-negative inputs and outputs (programmed in a linear way) and that the realisations of this set of results are generally of an unknown process [26]. The possibility of generating the outputs without using the inputs is excluded. All other features can be derived from the following formalisation (5):

$$SR_\xi = \left\{ z \in R^{r+s} \mid z = \sum_{k=1}^n \lambda_k \cdot z_k; \lambda \in \Lambda_\xi \subseteq R^+ \right\} \tag{5}$$

The specific linear combinations of observation represent the possible variations. A specific nature of a set of solutions (especially: the coefficient of unnecessary TFI) is achieved by the suitable additional condition  $\xi$  for the weights related to DMU (linear factors)  $\lambda = (\lambda_1, \dots, \lambda_n)$ . A specification of a set of solutions (SR) determines the referent quantity, thus having a significant impact on the expressed inefficiency of identified DMUs [16].

According to Charnes / Cooper / Rhodes (1978) [9], the DEA model assumes a constant return to the scale of comparison i.e.  $\lambda_\xi \in R^n$ . Basically, the DEA model determines the efficiency of a specific DMU, and it is possible to identify the inefficiency on the side of the inputs or outputs, depending upon the input- or output- orientation of the model [15]. If an efficiency of an entity 0 is to be analysed,  $X_0 = (x_1, \dots, x_n)$  will be the appropriate input vector and  $Y_0 = (y_1, \dots, y_n)$  the appropriate output vector. In case that the mode is input-oriented, the DEA determines the efficiency of a specific DMU by solving the following *Linear programme 6*:

$$\begin{aligned} &\text{Min } \theta_0 \text{ subject to the constraints} \\ &\theta_0 \cdot x_{0,i} \geq \sum_{k=1}^n \lambda_{0,k} \cdot x_{k,i} \quad \forall i = 1, \dots, r \\ &y_{0,j} \leq \sum_{k=1}^n \lambda_{0,k} \cdot y_{k,j} \quad \forall j = 1, \dots, s \\ &\lambda_{0,k} \geq 0, k = 1, \dots, n \end{aligned} \tag{6}$$

where:  $\theta_0$  – relative efficiency of entity 0,  $\lambda_i$  – weight coefficients.

*Linear programme 6* demands a linear combination of the compared units, which provide an output, good at least as  $DMU_0$ , but requires only a fraction

( $\theta_0$ -product) of inputs. Because of its direct relation to the concept of the results set (SR), which envelopes every observation, this model pattern is called DEA.

Pattern 6 is (as shown in the model of relations between factors in the process of decision-making, *Figure 1*) usually taken a step further by allowing an identification of inefficiencies through the inputs and outputs analysis. According to the duality theory, each solution from 6 can be connected to a new limitation. The variables of one *Linear programme 6* match the limitations in another on and vice versa. If *Formulation 6* could be solved, then its target values (in this case: the efficiency values) are equal. In many cases, both DEA model formulations are used for the purpose of expanding the results interpretation [27]. These results are available within all usual DEA software (e.g. *Excel – Solver*). The following represents a *Linear programme 6* in a dual variant:

$$\begin{aligned} &\text{Max } \sum_{j=1}^s W_{\text{output}_{0,j}} \cdot y_{0,j} \leq 1 \\ &\text{subject to the constraints} \\ &\sum_{i=1}^r W_{\text{input}_{0,i}} \cdot x_{0,i} = 1 \\ &\sum_{j=1}^s W_{\text{output}_{0,j}} \cdot y_{k,j} \leq \sum_{i=1}^r W_{\text{input}_{0,i}} \cdot x_{k,i} = 1 \quad \forall k = 1, \dots, n \\ &W_{\text{output}_{0,j}} \geq 0 \quad \forall j = 1, \dots, s \\ &W_{\text{input}_{0,i}} \geq 0 \quad \forall i = 1, \dots, r \end{aligned} \tag{7}$$

where:  $W_{\text{output}_{0,j}}$  – outputs, weight coefficients;  $W_{\text{input}_{0,i}}$  – inputs, weight coefficients.

In terms of content, *Linear programme 7* allows for every DMU to choose the optimal ponderation of input and output factors (efficiency criteria). However, if this ponderation is used in order to valorise all other DMUs, it should be accomplished that its maximum efficiency equals 1 or 100% (the normalisation condition). Within the DEA, the performance as a quotient of outputs and inputs is used as an efficiency criterion: the act of dividing the left side of the second limitation, with the right side, results in a quotient of weighted sum of results and weighted sum of inputs. In terms of the decision-making theory, the weighting of different efficiency criteria matches the so-called compromise solution of the vector decisions model [27].

*Figure 4* shows the model of the variables' classification. The objective of the classification is to substitute the complexity of impacts present in the practical traffic flows with the most significant measurable variables (e.g. by using *OptaGIS* tool), also expressing inputs and outputs whose interdependency is analysed by the application of the DEA principles. The Makarska case study has addressed the combination of issues: the global traffic flow organisation and the variants of traffic flow routing in the urban transport network, i.e. the micro-organisation of traffic through seven variant solutions.

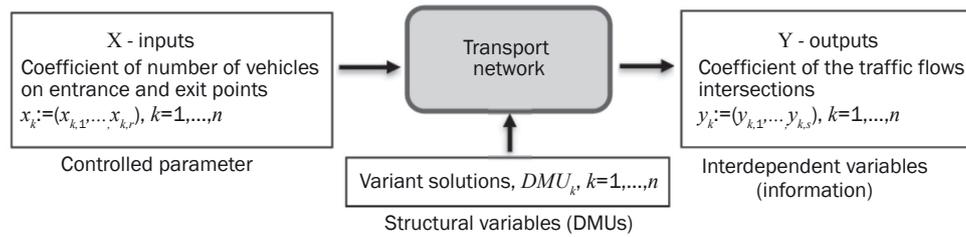


Figure 4 – Classification of the variables for the operation of DEA evaluation of the variant solutions

Figure 3 presents an existing organisation of traffic flows in Makarska. By *OptaGIS* application, the variables were identified for this specific DMU, e.g.  $DMU_1$  ( $x_{11}, \dots, x_{14}$ ) represents the coefficients of traffic flow intersections on 4 crossroads included in the calculation, Figure 5. The output variable  $y_{11}$  is a theoretical value of traffic flow intersection coefficient at the traffic load levels with occurring congestions, where A, B, C and D are the traffic flow directions. Output variable  $y_{12}$  is determined according to the time required to complete the routes from the points observed in the network, in the conditions of the free flow of traffic. The same procedure was carried out for each of the variant solutions, thus providing all variables of the DEA model.

The DEA model implementation can be divided into four key phases, where the selection of inputs data and outputs data is a key phase of model definition. The choice of relevant data is crucial to the effective interpretation, utilisation and acceptance of DEA results. During the identification of relevant inputs and outputs, special attention was given to the rules which are part of DEA premises (described below). Input and output data have to be controlled through the verification process in order to prevent easy manipulation without a possibility to detect an error. Faulty data or a lack of information can significantly impact the results and their interpretation. However, the distance between the DMUs and the efficiency boundary, i.e. its (in)efficiency, is no longer a deterministic parameter, but is defined in accordance with the expected efficiency boundary [28].

Because of the nature of the variant solution selection problem, this paper presents only the output-oriented model. However, the set-up and the interpretation of the input-oriented model could be achieved by applying the same analogy. Output-oriented models, unlike the input-oriented ones, maximise the criteria, which opens the possibility of enhancing the analysed variant solutions i.e. the new ideas.

### 3. WEIGHT COEFFICIENTS OF THE BASIC MODEL

Defining the criteria weight is not always simple, and every decision-maker defines the coefficients in a subjective manner. In some methods, the weighting

coefficients have a decisive impact on the result. As a consequence, the introduced values do not necessarily guarantee “the right solution” and an analysis is required to understand the behaviour of the result in dependence of the possible real variants of criteria weight. The simplification of the problem can be achieved by applying the weighting coefficients with the same ratio in all variant data, as envisaged in the DEA method, thus ensuring an absolute priority for each of the criteria.

The criteria final result is a function in a compromise programming frequently applied in the cases when a DMU can be given or modified during the assignment resolution. Within this paper, it is regarded as a relation between the factors in the decision-making process, Figure 1.

Therefore, the weight coefficients are subjective measures of the individual criteria relevance, defined by the decision-maker on the basis of their structural perception (the desired effects).

According to the model, the objective of weighting is the maximisation of the ratio representing efficiency:

$$\frac{\text{virtual output}}{\text{virtual input}} \tag{8}$$

Relative efficiency has to be determined for every individual DMU from the total of  $n$  DMUs observed,  $DMU_k, k=1, \dots, n$ . Every DMU uses  $r$  inputs and achieves  $s$  outputs. The optimal weights can differ between DMUs and are derived from the data. It is very important to ensure that the optimal values do not depend upon the measurement unit. The input and output weights are determined by associating each DMU with a set of optimal weights. “Optimal” is the weight for which a summary ratio of input and output for each DMU is maximised in comparison to other DMUs, when these weights are associated to the specific inputs and outputs for every DMU. As the efficiency takes values from 0 to 1, the condition puts limitations on the maximisation, Formula 7. For this reason, *Solver* - a software add-on for *MS Excel*, was used in the research, which is illustrated further in the paper. DEA consists of  $n$  linear optimizations when observing  $n$  variant solutions; where for each  $DMU_k$  the input  $m$   $x_{ki}$  ( $i= 1,2,3, \dots, r$ ) is used to make the output  $y_{kj}$  ( $j = 1,2,3, \dots, s$ ). The application part referred to the case where relative efficiency is developed on the

basis of seven decision units defined with one input and four outputs. In the analysis, the calculation of the input and output coefficients was determined by applying the software tool *OptaGIS*, which enabled the measurement of the interruption of traffic flows at a particular network load (default inputs). The obtained values are classified up to the value of 20. In this section, it is left to the decision-maker to classify further each of the points (location) according to various circumstances that may be of a subjective nature (using *OptaGIS* are standard inputs and outputs on a scale of up to 20). As shown in *Figure 5*, the weights needed to be determined are positioned in cells B10 to G10 (initially, all weights are set to 1). The weighted output for every DMU can be seen in column H and the weighted input in column I. The efficiency for every DMU (considering the current weights) is calculated in column J. Column K is the difference between the weighted output and the weighted input. This is a working column, necessary to operate the *Solver* model in *Excel*. It should be highlighted that the operative research includes a limitation of the difference between the weighted outputs and the weighted inputs – it should be  $\leq 0$ . Therefore, the current weights set to 1 are not feasible (row 10).

The target variable is presented in cell H3, weighted output for DMU 2 (variant2), that is to be maximised. The expected values are in cells B\$10 to G\$ and will

be calculated by the application of the *Solver Excel*. The applied limitations are that the I3 cell has to be equal to 1 and cells \$K2 to \$K8 have to be  $\leq 0$ . These cells contain the difference between the weighted output and the weighted input. The weighted coefficients are calculated under the specified conditions in accordance with *Formula 6*. After resolving the *Solver* for each DMU, the standard Weight values are obtained. *Figure 6* shows these values in a grouped bar graph, while the dotted black line shows the median value of Weight used for the calculation of virtual inputs and outputs, and then the efficiency according to the pattern (8). In the spreadsheet (*Figure 5*), weight efficiency is calculated in cells J\$2 to J\$8.

Determining weight in relation to inputs and maximizing the pattern (7) to each DMU is given the same importance. The disadvantage of the shown model for efficiency measurement is that it gives the efficient efficiencies to efficient DMUs. This implies that all efficient DMUs work equally well. However, this is not realistic because between the efficient DMUs there is a certain difference in the achieved efficiency. This reality cannot be considered with the model described above because it assigns to all efficient DMUs the efficiency rating equal to 1, so it is not possible to make a sequence of efficient DMUs. For this purpose, several analytical approaches have been developed to rank

	A	B	C	D	E	F	G	H	I	J	K
1	Structural variables	Output1	Output2	Output3	Output4	Input1	Input2	Weighted output	Weighted input	Efficiency	Working
2	DMU 1	$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	$x_{1,4}$	$y_{1,1}$	$y_{1,2}$	$=B2*\$B\$10+\$C2*\$C\$10+\$D2*\$D\$10+\$E2*\$E\$10$	$=F2*\$F\$10+\$G2*\$G\$10$	$=H2 / I2$	$=H2 - I2$
3	DMU 2	$x_{2,1}$	$x_{2,2}$	$x_{2,3}$	$x_{2,4}$	$y_{2,1}$	$y_{2,2}$	$=B3*\$B\$10+\$C3*\$C\$10+\$D3*\$D\$10+\$E3*\$E\$10$	$=F3*\$F\$10+\$G3*\$G\$10$	$=H3 / I3$	$=H3 - I3$
4	DMU 3	$x_{3,1}$	$x_{3,2}$	$x_{3,3}$	$x_{3,4}$	$y_{3,1}$	$y_{3,2}$	$=B4*\$B\$10+\$C4*\$C\$10+\$D4*\$D\$10+\$E4*\$E\$10$	$=F4*\$F\$10+\$G4*\$G\$10$	$=H4 / I4$	$=H4 - I4$
5	DMU 4	$x_{4,1}$	$x_{4,2}$	$x_{4,3}$	$x_{4,4}$	$y_{4,1}$	$y_{4,2}$	$=B5*\$B\$10+\$C5*\$C\$10+\$D5*\$D\$10+\$E5*\$E\$10$	$=F5*\$F\$10+\$G5*\$G\$10$	$=H5 / I5$	$=H5 - I5$
6	DMU 5	$x_{5,1}$	$x_{5,2}$	$x_{5,3}$	$x_{5,4}$	$y_{5,1}$	$y_{5,2}$	$=B6*\$B\$10+\$C6*\$C\$10+\$D6*\$D\$10+\$E6*\$E\$10$	$=F6*\$F\$10+\$G6*\$G\$10$	$=H6 / I6$	$=H6 - I6$
7	DMU 6	$x_{6,1}$	$x_{6,2}$	$x_{6,3}$	$x_{6,4}$	$y_{6,1}$	$y_{6,2}$	$=B7*\$B\$10+\$C7*\$C\$10+\$D7*\$D\$10+\$E7*\$E\$10$	$=F7*\$F\$10+\$G7*\$G\$10$	$=H7 / I7$	$=H7 - I7$
8	DMU 7	$x_{7,1}$	$x_{7,2}$	$x_{7,3}$	$x_{7,4}$	$y_{7,1}$	$y_{7,2}$	$=B8*\$B\$10+\$C8*\$C\$10+\$D8*\$D\$10+\$E8*\$E\$10$	$=F8*\$F\$10+\$G8*\$G\$10$	$=H8 / I8$	$=H8 - I8$
9											
10	Weight	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000				

Figure 5 - Structure of the variables for calculating the efficiency of DMU in Excel

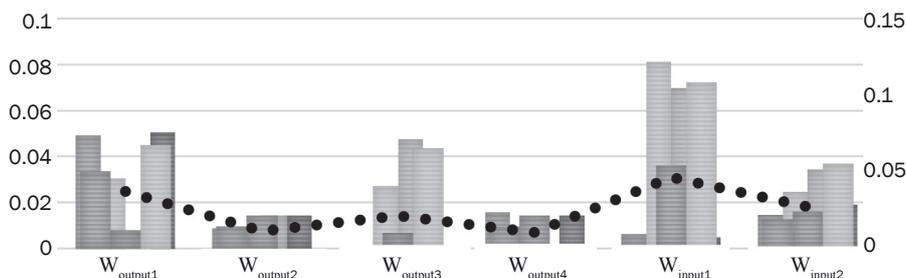


Figure 6 - Graph of weight coefficients (columns) and median values (dotted line) for the final calculation of the relative efficiency of DMUs

the efficient DMUs. An overview of analytical approaches for ranking such DMUs is shown in [29, 30]. Some authors propose a ranking model that allows effective decision-making units to assign a rating of greater than 1 (input orientation) or less than 1 (exit orientation) to make the difference between them. In this way, the rating is known as Super-efficiency Score. In fact, these authors have modified the basic model by making  $DMU_x$  absent from the list of efficient DMUs. This is the estimate of the  $DMU_x$  distance from the efficiency limit developed without its participation. It should be noted that in the work of Banker and Chang [31], it has been proven that super-efficiency measurement models can be used to detect non-standard DMUs. The practice is to exclude DMUs that have a super-efficiency rating of more than three or three times the efficiency of any other DMU (input orientation) and less than 0.3 or three times smaller than any other DMU (output orientation). In the paper, the problem of ranking efficient DMUs is solved by an extended thinking about the nature of the problem that is caused by traffic regulation and infrastructure constraints in the way of setting additional conditions or value judgments.

#### 4. ANALYSIS BASED ON JUDGMENTS

Most often, the DEA is defined as a methodology encompassing several different approaches and models which are interconnected and are used for the evaluation of relative efficiency of the units to be decided upon. One of the important model features is that the overview of weights leads to further insight. For example, in the initial Solver solution, weight  $W_{output2}$  for the intersection of the traffic flows on the arterial road (more lanes in one direction) equalled 0.01147. Value  $W_{output3}$  for the intersection of traffic flows related to a lower ranked road equalled 0.01204. It implies an equal importance of the intersections i.e. a contribution of the lower ranked in the final solution  $0.01204 / 0.01147 = 10.5$ . After considering the ratio of 10.5, a decision-maker might conclude that the traffic flow intersections on lower ranked roads, according to the expert opinion, reflect a much bigger congestion. For example, the conclusion might be that they are more valuable than the ones of a higher rank and should, as such and according to the weight parameter  $W_{output2}$  and  $W_{output3}$ , satisfy the following limitation:  $W_{output3}/W_{output2} \geq 10$ . It shows that one additional traffic flow intersection on a lower ranked road has a higher contribution to the congestion (of at least 12 intersections on a higher ranked road). This limitation is a judgement that better reflects the reality. In order to satisfy the  $W_{output3}/W_{output2} \geq 10.5$  ratio, it is necessary to transform it into a linear form:  $W_{output3} - 10.5 \cdot W_{output2} > 0$ , by entering formula  $W_{output3} - W_{output2}$  in cell X, but also to add the cell  $X \geq 0$  condition in Solver. The solution obtained a score

indicating that the efficiency of DMU 7 when added to this limit (value judgment) falls to 98%. The restriction addition includes value judgments. Just as a judgment was used in selecting inputs and outputs, our judgment of what are the (appropriate) constraints to add to the basic model to solve the problem of more efficient DMUs, Figure 7.

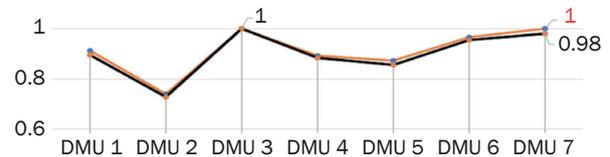


Figure 7 – Relative efficiency of seven variant solutions before (orange) and later (black) value judgments

#### 5. DISCUSSION

The traffic organisation and regulation in urban areas is extremely complex. Some of the requirements that need to be met are: sufficient capacity, desired vehicle speed, pleasant and comfortable transportation, maximal traffic safety, minimum construction and operational costs, minimum environmental impacts, etc. Therefore, the organisation of traffic flows is a key challenge in the significantly increased passenger and freight transport demand. Since the existing research is mainly focused only on some of these requirements, it is not possible to identify the total effects of the solutions proposed, which is the primary objective of this paper. In order to accomplish this goal, it is necessary to encompass a more significant number of input and output variables which, in terms of value, correlate to the aforementioned requirements or interact, thus becoming DMUs, on the basis of which objective discrete relative efficiency values could be identified for every DMU using the same set of criteria. The research is carried out within a simple case study in order to present the DEA-based methodology in a simple manner. The key determinant of reaching the objective is present in the validated hypothesis [21]; The increase of the network (nodes) permeability is achieved by the reduction of TFI. The intersections correlate to the traffic loads. The identification of the critical points (TFI), where certain levels of traffic load lead to congestions, points to an insufficient road capacity or, more often, to an insufficient intersection (crossroads) permeability. In both cases, the solution is sought in a different traffic flow organisation, with the aim of preventing (i.e. bypassing) the problems by reducing TFI (with the optimal duration of presence in the network observed). In reality, it implies that the traffic flow organisation is founded on a theory of TFI reduction (the amount of intersections is calculated according to Formula 2.

The aforementioned characterisation that simultaneously encompasses O-D (origin-destination) orientation of traffic flows, can be considered as an extension of the traffic flow unnecessary intersection concept from the intersections to the transport network. It also defines the technical efficiency of the network as the key indicator during the evaluation of variant solutions for urban traffic flow routing and regulation.

Unlike the parametric statistical models, the efficiency of every individual variant solution with the highest achieved efficiency level from the set of variant solutions has been compared. The DEA-based analysis model is a non-parametric approach, since it does not require an *a priori* assumption about the analytical form of the observational inputs and outputs. The following important advantage of such analysis is the fact that a different number of heterogeneous inputs and outputs can be used and presented by different metric types. The efficiency boundary represents an empirically identified maximum DMU value, which every variant solution is capable of achieving with the given input data and behaves like an envelope for the inefficient DMUs. The key aspects of data quality, related to the variant solutions' efficiency calculation had been only marginally considered until now [32]. The research uses the TFI variables which are a result of a multiannual work on the identification of variant solutions based on the sustainable mobility principles. Thereby, it is very important to ensure the causal relations between outputs and inputs [19, 33, 34].

The following features of DEA-based method confirm the hypotheses set during the research and contribute to the pre-defined objectives of the technical efficiency analysis for the transport network variant solutions:

- The focus is on the analysis of the individual traffic flows on every intersection (achieved by using the *OptaGIS* application) through the vector analysis of TFI;
- An individual summary measure is determined for every DMU on the basis of input factors value, before the desired outputs are obtained;
- It is possible to include exogenous variables (variables dependent upon external conditions, not upon the model itself) in order to present input and output factors which are under the control of the surroundings;
- It is possible to include categorical variables in order to present input and output factors that encompass only discreet values from the admissible set of values (e.g. the number of parking places within the variant traffic solution);
- A priori values of the results or the weight of input and output factors are not required (within the calculation they are defined by using *Solver* add-on for *MS Excel* for every weighted output);

- The functional form in terms of input-output relation is not required;
- The analysis which points to the needed modifications of the inputs and/or outputs is enabled, in order to design the DMU below the efficiency boundary (inefficient DMU) in a way to position it on the boundary;
- equal criteria are applied on the obtained efficiency values during the evaluation of every DMU from the set of variant solutions.

In principle, the aim is to reduce the conflicts between traffic flows (outputs) and to increase the number of vehicles in the transport network inputs), and the efficiency index should reflect this approach. Through the application of judgements, the existing model can be improved by the new concepts and approaches. However, it has to be mentioned that the solutions within this research are exclusively based on the quantitative variables, however, the research could also be extended to the qualitative argumentation.

## 6. CONCLUSION

Due to the impossibility of new roads construction in urban areas, it is often necessary to build new models of traffic flow organisation (new models of traffic flow routing and regulation), which will result in a more efficient utilisation of infrastructural resources, thus reducing resistance i.e. increasing capacity of nodes and the existing roads. Moreover, such approach will increase the capacity of the transport networks, improve traffic safety and reduce environmental pollution, which are the basic requirements for achieving sustainable urban mobility.

The development of the efficiency calculation model was accomplished in *Excel*. While measuring the variation of traffic flow variables (output variables in the model) in relation to the network load (input variables most commonly used in the peak load time) realized using the software tool *OptaGIS* developed at the Faculty of Transport and Traffic Sciences in Zagreb. A variant solution comparison model was developed on the DEA idea that allows comparison of comparable units' efficiency, in this case - variant solutions. A model with a larger number of output variables was used, which signifies the interruption of traffic flows that are in relation to the inputs i.e. the number of participants in the observed traffic network.

The key features of the developed model are as follows:

- the DEA-based model results in relative efficiency only i.e. the efficiency in relation to the data considered. It cannot provide an estimation of the absolute efficiency.
- *OptaGIS* application was used to retrieve input and output variables; however, the application itself is not the object of the analysis carried out within this paper.

- the selection of better traffic flow organisation efficiency results in economic, environmental and other impacts expressed as better permeability (congestion avoidance) and a shorter stay of traffic subjects in the network.
- in the process of generating variant solutions, the construction of new roads is not considered as an option. The modifications were related to the traffic flows organisation and regulation within the existing infrastructure, with the variations of traffic flow loads;  $p$ ,  $q$ .
- the identification of the aforementioned optimum was accomplished through the comparison of the DMUs according to the set of uniform criteria, thus also enabling the identification of the deficiencies for the less efficient DMUs in comparison to the best one, all based on the analysis which can be input- or output-oriented.
- the results have proven as good indicators (of variant solutions relative efficiency), pointing to the differences between variant solutions (on the specific example of reducing the unnecessary traffic flow intersections and the time spent within the transport network), based on their comparison and the predefined traffic flow loads (DEA model represents a radial approach to measuring efficiency).

Therefore, by the application of the suggested method, the traffic flows are being analysed in a systematic way, based on the optimal level of network utilisation. The key mechanism to reach this objective is related to the evaluation of the achieved minimisation of the unnecessary TFIs in the network during peak hour load at intersections and/or increasing the capacity by applying the traffic solutions that include traffic routing and regulation.

The proposed model is tested and verified through the research carried out in the City of Makarska, on a part of the transport network. The model used is essentially oriented to inputs in terms of the analysis in relation to the number of users of the transport network, and in particular coming to the peak in hours of traffic load. The scientific contribution is recognised within the new approach to the evaluation of the variant traffic solutions efficiency, on the account of unnecessary TFI avoidance, considering their intensity. The research opens new possibilities of exploring the relations of traffic flows.

Dr. sc. **DAMIR BUDIMIR**<sup>1</sup>

E-mail: damir.budimir@fpz.hr

Doc. dr. sc. **MARKO ŠOŠTARIĆ**<sup>1</sup>

E-mail: marko.sostaric@fpz.hr

Dr. sc. **KREŠIMIR VIDOVIĆ**<sup>2</sup>

E-mail: kresimir.vidovic@ericsson.com

<sup>1</sup> Sveučilište u Zagrebu, Fakultet prometnih znanosti

Vukelićeva 4, 10000 Zagreb, Hrvatska

<sup>2</sup> Ericsson Nikola Tesla

Krapinska 45, 10000 Zagreb, Hrvatska

## **PRIMJENA METODE DATA ENVELOPMENT ANALYSIS (DEA) ZA IZRAČUN UČINKOVITOSTI VARIJANTNIH RJEŠENJA ORGANIZACIJE I USMJERENJA PROMETNIH TOKOVA**

### **SAŽETAK**

Mali broj empirijskih modela vezan je za izračun učinkovitosti varijantnih rješenja s aspekta održive mobilnosti u urbanim sredinama. U praksi je često neophodno, naročito u slučajevima gradskih prometnica, valorizirati rješenja organizacije i usmjerenja prometnih tokova, kako bi se primijenila ona kod kojih je manja mogućnost da dođe do zagušenja u vršnim satima opterećenja prometne mreže. Rad analizira problem određivanja učinkovitosti varijantnih rješenja (alternativna rješenja) kroz analizu učinkovitosti prometnog sustava na konkretnom primjeru (Makarska). Aspekt efikasnosti organizacije i usmjerenja prometnih tokova u varijantnim rješenjima razjašnjen je kroz razvoj modela za analizu iz premisa metode DEA (Data Envelopment Analysis) te načela teorije bespotrebnog presijecanja prometnih tokova. Predloženi model razvijen je na ideji analize omeđivanja podataka koji se pripisuju varijantnim rješenjima, na temelju izračuna presijecanja prometnih tokova i mogućnosti DEA metode. Uključena je usporedba i određivanje relativne učinkovitosti usmjerenja svakog od prometnih tokova varijantnog rješenja nasuprot granice učinkovitosti, kako bi se dobila relativna učinkovitost naspram svih rješenja.

### **KLJUČNE RIJEČI**

data envelopment analysis; modeliranje prometnih tokova; organizacija prometnih tokova; presijecanje prometnih tokova; analiza učinkovitosti; prometna mreža; održiva mobilnost;

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