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URBAN COMMUTING AND DAYTIME POPULATION IN SMALL AREAS OF A METROPOLIS: A CASE STUDY OF BRNO, CZECH REPUBLIC

ABSTRACT

A simplified modelling approach to urban commuting patterns is achieved by focusing on daytime populations rather than on commuters, or on the commuting process itself. Whereas past studies were usually economic in nature, and viewed commuting as a process within the continuum of urban space and time, the approach addressing daytime populations transforms the modelling attempt into a demographic deliberation of a binary situation where switching of values between daytime and night-time indicators in each subarea throughout a metropolis is considered. The present study shows that such a focus on diurnal change as a binary concept offers a new paradigm in conceptualizing metropolitan commuting and transportation. Under certain assumptions, rooted in recent observations of metropolitan areas elsewhere, this study conjectures an analytic function for the estimation of daytime populations in small areas throughout the metropolitan region of Brno, Czech Republic. The conjectured relationship is a logistic function that utilizes as its independent variable the average household size in each of the subareas throughout the metropolitan region. Based on the data from the Czech census of 2001, the distributions of average household size and of residential populations throughout the metropolitan region are applied in a case study illustrating the utility of the proposed approach for the estimation of daytime populations throughout the region. The iterative procedure advanced here offers considerable potential for further applications elsewhere.

KEY WORDS

metropolitan commuting, urban transportation, Brno, daytime population, average household size, logistic function, small area demography

1. INTRODUCTION

Over the past half a century, commuting and commuter access have attracted an increasing number of studies. This has been so also due to the escalating problems in urban transport and owing to the increasing costs of metropolitan infrastructure and its maintenance. Accordingly, most studies of commuting were economic in nature, while it appears to be the case that demographic aspects of commuter populations have received only scant attention. The particular issue of daytime population change in subareas throughout a metropolis has not received sufficient consideration at all. Yet, it is precisely the question of small-area daytime populations that holds the promise in identifying useful parameters of commuting, hitherto disregarded. Part of the reason for the lack of studies on small-area daytime populations in North America and Europe seem to be difficulties in the acquirement of relevant data; the other aspect of this paucity is lacking methodology that could bring the issue of daytime populations into the context of urban structure. Both aspects will be addressed in the present study.

For clarity's sake and considering a multitude of subareas throughout a metropolis the main distinction to be made is between daytime population, d, and night-time population, r, of each and every subarea throughout the metropolitan region in question. Evidently, daytime population of a subarea is related to the population of commuters into the subarea, and some of the methodical links between daytime and night-time populations, on the one hand, and commuter flows, on the other hand, have been demonstrated

Figure 1 - Situation of Brno within the Czech Republic and Central Europe

Figure 2 - Forty-eight urban core subareas and 7 rural fringe subareas comprising the Brno Metropolitan Region, 2001

Table 1 - Residential (night-time) population and households in subareas of the urban core and of the rural periphery, Brno Metropolitan Region, 2001

District	Population	Households
Bohunice	16398	6084
Bosonohy	2210	732
Brněnské Ivanovice	1261	457
Bystrc	23805	9439
Černá Pole-total	21591	9483
Černovice	7263	3022
Dolní Heršpice	361	123
Dvorska	335	91
Holásky	935	325
Horní Heršpice	1858	710
Husovice	5473	2211
Chrlice	3176	1062
Ivanovice	999	339
Jehnice	752	259
Jundrov-total	3725	1529
Kníničky	513	193
Kohoutovice	12857	5090
Komárov	5175	1996
Komín	7251	2975
Královo Pole	20205	8860
Lesná	15154	7086
Líšeň	25388	9037
Maloměřice-total	3004	1172
Medlánky	3163	1135
Brno-město	5830	2346
Mokrá Hora	687	241
Nový Lískovec	11400	4094
Obřany	2385	918
Ořešín	452	147
Pisárky-total	2261	853
Ponava	7120	2955
Přízřenice	572	164
Řečkovice	14304	5870
Sadová	404	45
Slatina	8530	3157
Soběšice	1475	491
Staré Brno	13024	5740
Starý Lískovec	14220	5492

District	Population	Households
Stránice	4896	1844
Štýřice	8635	4022
Trnitá-total	3467	1408
Tuřany	2100	708
Útěchov	446	158
Veveří	22366	9690
Zábrdovice-total	11224	4605
Žabovřesky	22280	9663
Žebětín	2070	676
Židenice-total	33172	13027
IVANČICE	22584	7703
KUŘIM	18024	6252
POHOŘELICE	551	173
ROSICE	22817	7993
ŠLAPANICE	50604	16938
TIŠNOV	17490	6102
ŽIDLOCHOVICE	27099	8856

elsewhere.[1] Setting aside the issue of precise relationship between daytime, night-time and commuter populations in subareas throughout a metropolis, we focus here on the formal distinction between night-time and daytime populations. Whereas night-time population counts are easily available from official statistics (referred to, by implication, as residential population counts), daytime populations of metropolitan subareas are usually unknown. This study suggests a direct estimation procedure of the ratio, daytime/night-time population, in subareas comprising a metropolis. This ratio is referred to as the diurnal ratio.

Based upon the estimation of diurnal ratios for subareas, further approximation of daytime populations throughout the metropolis is easily attainable. For the estimation of diurnal ratios two assumptions are used to the effect that (a) the progression of values of the diurnal ratios corresponds to a logistic function, whose formula is offered here; and that (b) ordering of subareas by their diurnal ratios, in descending order throughout the metropolis, is roughly the same as the ordering of the subareas by their average household size, in ascending order. Daytime populations of subareas are then easily approximated by multiplying, for each subarea, its diurnal ratio by its night-time population. The present study examines this approach through the application of measures available for the Brno Metropolitan Region of the Czech Republic (Figures 1 and 2) from the 2001 census. The main input values needed for the estimation of daytime population are as those listed in Table 1 for the Brno Metropolitan Region: Night-time population and average household size for each subarea comprising the metropolitan region (rural or exurban subareas belonging to the Brno Metropolitan Region, in Table 1, are shown in capitals). For all subareas both night-time (i.e. residential) population counts and average household size are attainable through official census statistics and hence the advantage of the proposed methodology, and its potential for application elsewhere.

2. DIURNAL CHANGE AS BINARY PROCESS IN THE METROPOLIS

A binary differentiation between daytime and night-time values in each subarea's population suggests an analytic approach, applicable across a spectrum of empirical situations. The advantage of the binary approach to urban diurnal population change is that it provides a new paradigm to commuting as a critical issue in contemporary metropolitan structure, thus also providing a fresh context to urban and metropolitan transportation patterns.

The acuteness of diurnal population change as a measure for cities and for metropolitan regions emanates from incongruity in contemporary urban transportation: While mechanized and automated means of access were intended to expedite movement within the city, at the turn of the 21st century the very same mechanized and automated means have led to grave obstacles in urban access, manifested mainly in congestion, pollution and traffic hazards. In a case study of Boston and Atlanta, Yang[2] has shown that urban transportation, in fact, defeats its own purpose. Through the last several decades, Yang's study has revealed overall efficiency in urban accessibility has declined, the culprit being, paradoxically, improved urban transportation which leads to spatial decentralization and ultimately to longer commutes. It is the increasing incongruity between the need for urban access and available urban transportation that suggests an interpretation of commuting as access between a night-time location (place of residence) and daytime location (usually place of work or study). Such interpretation also constitutes the basis for discrete, non--continuous approach to the modelling of commuter patterns.

The approach taken in the past studies has conceptualized commuting as a continuum in urban space and time between residence and work. This brought about myriad models of growing complexity. The continuum approach to urban commuting has sometimes been seen as emanating from an analogy between transportation and water flows, the earliest reference possibly being a paper by Spernak and Stevens[3]. But such an approach has raised a number of questions, none the least of them surrounding its lack of simplicity[4]. The adverse effect of complex modelling has been ultimately its inability to sufficiently explain urban or socio-economic phenomena[5], of which commuting is only an example, and its even more rudimentary inability to accurately describe the urban processes in question[6].

In contrast to studies relating to urban space as a continuum, the binary distinction between daytime and night-time populations in small areas throughout cities has another important advantage: simplicity. The non-continuous, binary approach advocated here seeks simplicity over complexity in urban modelling in general, and in the modelling of urban transportation and commuting in particular. The concept of daytime population shift in the modelling of commuter patterns has been introduced through two values attainable for each subarea throughout the metropolis: Night-time and daytime population counts[7]. This approach to urban commuting transforms the modelling attempt into a simplified, binary situation of switching values between daytime and night-time indicators for each subarea throughout the metropolis. Whereas this approach cannot address continuum patterns in metropolitan transportation, it involves on the other hand considerable expediency.

Traditionally, commuting has been considered expression of an urban process, one that links housing with employment. Early works on urban commuting within this context were concentrated on employment densities and distance from Central Business District. Perhaps the most prolific of these was the study by Hamilton[8] analyzing urban commuting by way of these two variables. Hamilton's model, however, adhered strongly to a monocentric urban paradigm, and therefore could not be easily applied to realistic urban structures. Using both employment density as well as population density, Small and Song [9] described the change from a generally monocentric to a polycentric structure that occurred in Los Angeles during 1970s. Significantly, in two subsequent studies Song found that, among nine different measures, distance to CBD was the poorest one to model spatio-temporal links between housing and employment. In the modelling of urban spatial patterns, and urban commuting patterns in particular, gravity-type measures, especially exponential functions, were shown to perform better[10].

In a case study of the San Francisco Bay Area, Cervero[11] confirmed Hamilton's finding by focusing on the link between employment decentralization and residential suburbanization. Decentralization of employment results has been shown to result in no reduction in commuting despite the movement of jobs toward residences, and rather, a slight increase in commuting had been detected in this case. The flip side of the linkage between housing and employment was the Canadian research by Nowlan and Stewart[12] who showed how residential population increase downtown Toronto has triggered a decline in commuter trips (and a corresponding increase in residential quality downtown). As a parameter, night-time population was in fact used in this study, to measure the effect on commuter flows into downtown Toronto.

3. PARAMETERS OF COMMUTER ACCESS

Mainstream studies have viewed commuting as a *process* within which individuals move from their place of origin to their place of destination. At the core of this approach is the notion of individuals attaining varying locations over the period of the working day. The alternative pursued here offers a discrete concept of urban population change. By considering a multitude of single locations, each subarea within the metropolis attaining two values – night-time population counts and daytime population counts – commuter access is addressed here as a binary toggle between the night-time value (such as midnight population) and daytime value (such as noontime population) for the same subarea. This means, inadvertently, that each subarea in the metropolis is considered a place of residence for some people as well as a place of daytime activity for the same or other people. By shifting from continuous parameters of commuting to a binary parameter, such an approach implies considerable simplification in the conceptualization of metropolitan commuting.

A common occurrence in geostatistical division of a metropolis is the very substantial difference in geographic and demographic size of subareas. Comparison between multitudes of subareas is therefore meaningful only by considering the ratios as indicators relevant to the analysis in question. For this reason, the population density has been often employed as a parameter allowing a straightforward comparison between subareas. It is noteworthy that most past studies on urban commuting, to the extent that they have utilized the concept of population density, have often related to night-time population density. One such recent study[13] uses the concept of night-time population density, precisely as a link to commuter patterns within the city. Other studies on commuting, however, reference employment population, i.e. daytime working population, and it is also from this vintage point that the present study attempts to analyze the attributes of commuter access.

McMillen[14] used daytime working population density (or employment density) to point out that while metropolitan areas have become increasingly decentralized over the past decades, daytime destinations of workers have also decentralized, but not in the corresponding or even manner. The same study showed that when a metropolitan area grows and becomes more decentralized, it starts to develop employment subcentres. In identifying such subcentres, as well as patterns of commuting into the central business district, McMillen applied the employment density (as daytime working population density in a subarea) as a parameter.

Similarly, Giuliano and Small[15], and McMillen and Smith[16] have used employment density as an explanatory variable relating to overall urban structure. Employment density and night-time population density were utilised as variables explaining overall urban structure in a study by Baumont, Ertur and Le Gallo[17].

The problem with a relative measure such as population density is that it does not allow for an easy comparison between daytime and night-time populations of a subarea. In the present study, as an alternative relative measure, we advance further the recently introduced notion of diurnal change as a *ratio* between daytime and night-time population in a subarea. While commuting traditionally has been considered a continuous process of movement by individuals from their places of origin to their places of daytime destination, diurnal change offers a binary approach whereby each subarea in the city is considered a place of daytime *and* night-time location of people. Two values, d and r, for daytime and for night-time population, respectively, are attained by each and every subarea throughout the metropolis. The simple measure that follows from this consideration is the diurnal ratio, d/r.

4. DIURNAL CHANGE AND HOUSEHOLD COMMUTER PATTERNS

The usefulness of the diurnal ratio, d/r, is that it is related to average household size. As the arithmetic division of population by the number of households, the average household size, too, is a useful relative measure that alleviates differences in population and geographic size between subareas. A subarea's average household size implies also a linkage to the residential/employment mix of a subarea: A large average household size usually signifies a residential subarea, a small average household size often indicates a high employment district. Subarea ordering by average household size, in ascending order throughout the Brno Metropolitan Region, in Table 2, for example, implies also ordering of subareas from the highest to the lowest diurnal ratio. Subareas with low average household size (implying a high diurnal ratio), at the top of Table 2, suggest non-residential land use (relatively high daytime population to a relatively low night-time population), while subareas with high average household size (implying a low diurnal ratio), towards the bottom of the table, suggest a residential mix. Only three anomalies out of the 55 subareas comprising the Brno Metropolitan Region could be identified from Table 2. The districts of Lesná, Černá pole and Jundrov, which appear in the listing with low average household size, and thus at the upper part of the table suggest a high diurnal ratio, i.e. a relatively high number of persons during daytime to a relatively small number of persons during night-time. The three districts, however, are known to be in fact residential districts. Yet this is a less-than-6% anomaly, and thus, the notion of average household size as a general parameter of an overall land-use mix appears to be fairly indicative.

Table 2 - Subareas ordered by average household size, showing residential (night-time) populations, Brno Metropolitan Region, 2001

Order	District	Population	Households	A.H.S.
1	Lesná	15154	7086	2,14
2	Štýřice	8635	4022	2,15
3	Staré Brno	13024	5740	2,27
4	Černá Pole-total	21591	9483	2,28
5	Královo Pole	20205	8860	2,28
6	Žabovřesky	22280	9663	2,31
7	Veveří	22366	9690	2,31
8	Černovice	7263	3022	2,40
9	Ponava	7120	2955	2,41
10	Jundrov-total	3725	1529	2,44
11	Řečkovice	14304	5870	2,44
12	Komín	7251	2975	2,44
13	Zábrdovice-total	11224	4605	2,44
14	Trnitá-total	3467	1408	2,46
15	Husovice	5473	2211	2,48
16	Brno-město	5830	2346	2,49
17	Bystrc	23805	9439	2,52
18	Kohoutovice	12857	5090	2,53
19	Židenice-total	33172	13027	2,55
20	Maloměřice-total	3004	1172	2,56
21	Starý Lískovec	14220	5492	2,59
22	Komárov	5175	1996	2,59
23	Obřany	2385	918	2,60
24	Horní Heršpice	1858	710	2,62

Order	District	Popula-tion	House-holds	A.H.S.
25	Pisárky-total	2261	853	2,65
26	Stránice	4896	1844	2,66
27	Kníničky	513	193	2,66
28	Bohunice	16398	6084	2,70
29	Slatina	8530	3157	2,70
30	Brněnské Ivanovice	1261	457	2,76
31	Nový Lískovec	11400	4094	2,78
32	Medlánky	3163	1135	2,79
33	Líšeň	25388	9037	2,81
34	Útěchov	446	158	2,82
35	Mokrá Hora	687	241	2,85
36	ROSICE	22817	7993	2,85
37	TIŠNOV	17490	6102	2,87
38	Holásky	935	325	2,88
39	KUŘIM	18024	6252	2,88
40	Jehnice	752	259	2,90
41	IVANČICE	22584	7703	2,93
42	Dolní Heršpice	361	123	2,93
43	Ivanovice	999	339	2,95
44	Tuřany	2100	708	2,97
45	ŠLAPANICE	50604	16938	2,99
46	Chrlice	3176	1062	2,99
47	Soběšice	1475	491	3,00
48	Bosonohy	2210	732	3,02
49	ŽIDLOCHOVICE	27099	8856	3,06
50	Žebětín	2070	676	3,06
51	Ořešín	452	147	3,07
52	POHOŘELICE	551	173	3,18
53	Přízřenice	572	164	3,49
54	Dvorska	335	91	3,68
55	Sadová	404	45	8,98
	TOTAL	535341	205741	2,60

It is for this reason too that a convenient expression of the link between housing and employment in studies on commuting has been the premise to view each person as a household member, and then focus on intra-household patterns of commuter travel[18]. The number of persons unaffiliated with households, within the general population, is usually negligible, and an approach to commuter travel allocation within households, therefore, can potentially address the brunt of urban commuting patterns. Although in very small subareas of a metropolis the proportion of persons unaffiliated with households can occasionally increase due to varying placement of institutions such as jails, hospitals or army barracks throughout the city, such occasions due to both substance and infrequent occurrence have virtually no impact on commuter patterns.

It is also due to the average household size that there appears to be a further advantage to the concept of diurnal change. The simple ordering of subareas by average household size suggests an inverse relation with the ordering of subareas by their diurnal ratios. Such a relationship has been identified, for example, in an earlier study on the Seoul Metropolitan Area[19]. The specific observation that was made for Seoul has been to the effect that, given a large number of small areas throughout a metropolis, the average household size of the small areas generally increases as the diurnal ratios in the same areas decrease, both measures providing an indication to the residential/employment mix of subareas.

Based on this observation, any large multitude of small areas throughout a metropolis can be ordered in a sequence, or a list, according to average household size. Listing of small areas in ascending order of their average household size would then generally, or approximately, correspond to a listing of the same small areas in descending order of their diurnal ratios, d/r.

Such a conjecture is instrumental for the estimation of daytime populations throughout subareas. If for each subarea throughout a metropolis we know the average household size, but not the daytime population, the latter could be estimated through the ordering of small areas by their diurnal ratios which, in turn, correspond to the ordering of the same small areas by average household size, s. Since the average household size, s_i, for each small area i is usually known, the order of small areas by their diurnal ratios can be assumed to approximately correspond to the sequence of values s_i, $i = 1, \ldots, n$.

5. DIURNAL RATIOS AND DAYTIME POPULATIONS

In order to estimate the actual values of the diurnal ratios, we first consider the minimum possible value attained by the diurnal ratio for any subarea. Arguably, the decision of each household throughout a metropolis regarding its subarea choice of residence is largely determined by considerations of commuting travel. Most households will locate so as to minimize the need for travel outside the subarea of residence for at least one person in the household (a child or a homemaker, for example). In the average household (i.e. not necessarily in each and every household), therefore, the number of persons who remain in their subarea of residence during daytime, can be safely assumed to be at least 1. This means that the number of persons remaining during daytime in their subarea of residence, i, is always equal to, or larger than the number h_i of households in that subarea. By definition, the diurnal ratio of any subarea i is the proportion, d_i/r_i, of daytime population, d_i, and night-time (or resident) population, r_i. But if $d_i > h_i$ then

$$d_i/r_i > h_i/r_i = 1/s_i,$$

or

$$d_i/r_i > 1/s_i \,, \qquad (1)$$

and the diurnal ratio for any subarea in the metropolis, then, cannot fall below $1/s_i$, $i = 1, \ldots, n$.

For at least one small-area in our listing of small areas, its daytime population (and thus also its diurnal ratio) is either known or can be estimated with an acceptable level of accuracy. The small area for which such reasonably accurate assumption regarding its diurnal ratio can be made, is some far-away suburb, j. Almost everybody here leaves during daytime. According to the earlier observation, however, there is, on the average, at least one person per household who remains within the subarea during daytime. In this far-away suburb, therefore, we can assume that the approximate number of persons remaining in the subarea approximately *equals* the number of households. Let this far-away subarea be denoted n, as the very last subarea in a sequence of subareas ordered by their diurnal ratios.

Consider then the entire listing of small areas comprising the metropolis, ordered in a sequence, from the subarea with the highest diurnal ratio to the subarea with the lowest diurnal ratio. Let this sequence of subareas thus ordered by their diurnal ratios be referred to as Sequence Q. The lowest value in Sequence Q is $1/s_n$, and since Sequence Q is in the descending order the value $1/s_n$ is actually the value of the very last entry in Sequence Q. The diurnal ratio of the last subarea, n, in Sequence Q, therefore, is estimated as

$$d_n/r_n = 1/s_n \qquad (2)$$

6. DAYTIME POPULATION ESTIMATES: BRNO METROPOLITAN REGION, 2000

Two attributes, (a) and (b), of Sequence Q of subareas ordered by their diurnal ratios, in descending order, could be assumed, both useful for the iterative estimation of daytime populations. The first attribute (a), is the shape of the curve that corresponds to a mathematical function that approximates the progression of values of the diurnal ratios in Sequence Q. Common reflection, as well as the earlier mentioned observation of data for Seoul, South Korea, suggests that diurnal ratios will be high at and near the Central Business District, and at other high employment areas elsewhere in the metropolis. The ratios steeply decline towards residential areas, yet cannot fall below the value $1/s_n$ for reasons outlined earlier. This suggests that a curve approximating the progression of diurnal ratios, in descending order, constitutes a logistic function. Due to the generally centrifugal nature of urban sprawl, the values π or e might also be a useful parameter of such a function. For the simplicity sake and for ease of iterative estimation, consider the logistic function

$$y = a / (1 + \exp(b * (x + c))) + d \qquad (3)$$

as approximating the progression curve of diurnal ratios in a metropolis, where

- x – is the independent variable, average household size;
- y – is the estimated diurnal ratio;
- a – is the arithmetic difference between the maximum and minimum values of the diurnal ratio, i.e. the difference between the value of the diurnal ratio at the city centre and at the city furthest suburb. The minimum value can be set to $1/s_n$ by proposition (2), and the maximum value is an initial estimation or estimation at previous iteration;
- b – is the slope (or the first derivative) of the Function (3) in the vicinity of values $y \approx 1$;
- c – is the value x (average household size) at points $y \approx 1$;
- d – is the minimum value of the diurnal ratio (i.e., $1/s_n$ by earlier assumption).

The logistic function (3) is shown in Fig. 3 for the specific parameters of Brno, 2001. The diurnal ratios and the corresponding daytime population estimates are calibrated through Equation (3) against a control total of the known population of the entire metropolis. For each individual subarea, the product of diurnal ratios and night-time populations yield estimates of the subarea daytime population. In total, these daytime estimates must yield the population of the me-

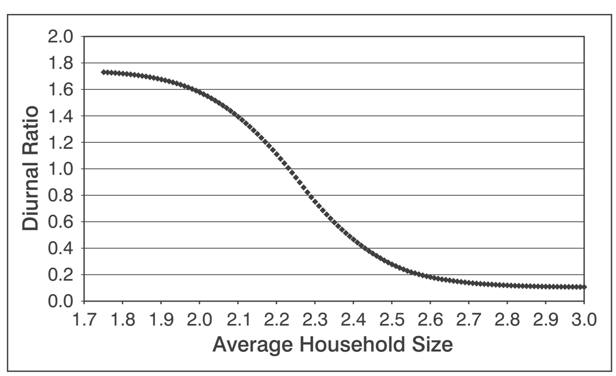

Figure 3 - Diurnal ratio as function of average household size

tropolis. This cannot be achieved through Equation (3) only, but through an approximation procedure whereby some of the coefficients in Equation (3) are varied, to ascertain that the diurnal ratios ultimately conform to the overall population of the metropolis.

The second assumed attribute of Sequence Q ascertains that the calibration procedure applies always to the same ordering of subareas. The second attribute, (b), of Sequence *Q* is that its ordering, i.e. the ordering of subareas by their diurnal ratios, descending, resembles the ordering of subareas by their average household sizes, ascending. The usefulness of attribute (b) is that the ordering of Sequence Q remains constant during the calibration process, since the values of average household size in subareas are given as observable values. The average household size, as a ratio of residential (night-time) population to households is routinely known for each small area, within a standard set, from government statistics or municipal records, and therefore remains constant throughout all iterations.

Utilizing jointly the two attributes (a) and (b), the ordering of subareas by their average household size allows for an approximation procedure that calibrates towards an estimation of values for the diurnal ratios and the corresponding daytime populations.

The procedure varies certain coefficients in Function (3) while retaining other coefficients intact. The coefficients *b* and *d* of Function (3) can be reasonably assumed not to undergo any significant changes during the calibration, and therefore, remain constant. The procedure commences with substituting the value $1/s_n$ of the last entry, *n*, of Sequence *Q*, into Function (3). The value $1/s_n$ is the lowest and the last value of Sequence *Q*, and therefore also the value of the asymptote, or very close to it, of Function (3). Substituting the value $1/s_n$ into Function (3), for y_n we can easily obtain the corresponding, estimated values of Function (3), y_i, $i=1, \ldots, n-1$, as the first approximation, for the remaining entries of Sequence *Q*. If the values y_i were fairly accurate estimates of diurnal ratios,

$$y_i \approx d_i / r_i, \tag{4}$$

then multiplying each value y_i by the value r_i of the corresponding night-time population, would obviously yield an estimate of daytime population d_i in subarea *i* ($i=1, \ldots, n-1$). The accuracy of the estimates can be assessed by calculating values $y_i \times r_i$ for the entire Sequence Q, and summing them up. The daytime population in the metropolis approximates its night-time population, the value of the latter being known from the census. If the sum-total of values $y_i \times r_i$ is smaller or larger than the known (night-time) population of the metropolis, an adjustment in the value of y_1 and, therefore, in the value of *a* in Function (3) must be made. Similarly, a new value of *c* is chosen according to the value of x_i at the point $y_i \approx 1$ of the previous iteration. This procedure is repeated until the sum-total

$$\sum_{i=1}^{n} y_i \times r_i \tag{5}$$

is reasonably close to the known value of the metropolitan population.

We illustrate this approach on the case of 55 subareas comprising the Brno Metropolitan Region. Ordering the 55 subareas by their average household sizes, ascending, provides also an implicit, if only approximate, ordering of the subareas by their diurnal ratios, descending, as in Table 2. The value $1/s_{55}$ is the value of the last entry in Sequence *Q*, and is substituted accordingly in Function (1). The corresponding values of coefficients *a*, *b*, *c*, and *d*, for iterations I1, I2 …, are as follows:

a = 1.647 at I1 (changing at each following iteration according to the newly substituted value for x_1);

b = 8.47335 (assumed to change insignificantly and thus remains constant during calibration);

c = -2.707 at I1 (at each following iteration will attain the value from the previous iteration);

d = 0.104 (will remain constant during calibration).

The first iteration, I1, yields a total for the metropolitan population, which is larger than the true population total. Gradually adjusting the value of the coefficient *c* in Function (3) we calibrate towards the result in the last column of Table 3.

Table 3 shows the base data of population and household distribution by district of the Brno Metropolitan Region, sorted by average household size, in the ascending order. The order is from 1 to 55, as listed in column 1, with the corresponding district and values for population, households, and average household size, in columns 2-5, respectively. The estimates for the diurnal ratio, *d/r*, of Iteration 1, in column 6, are based on the value $1/s_{55}$, as the estimate of the diurnal ratio, d_{55}/r_{55}, for the last district in the ordered sequence of subareas. All the other parameters are best guess initial estimates substituted in Function (3), the results yielding the sequence of initial estimates for diurnal ratios in column 6. Multiplying the diurnal estimate values in column 6 by the corresponding night-time population counts (column 3) yields the initial estimates of daytime population by district, in column 7. Since the corresponding total in column 7 is significantly lower than the true total of the metropolitan population, Iteration 2 calibrates to a better value by substituting closer estimates for the values *b* and *c* of Function (3), based on the results in Iteration 1. The same procedure follows for all subsequent iterations, until the estimated daytime population total reasonably approximates the true count for the entire metropolitan population.

The estimates for daytime populations throughout the 55 subareas of Brno, clearly, are entirely dependent on attributes (a) and (b), along with Function (3), all of which were assumed here as reasonably reflecting the reality of urban access in large Czech cities and in metropolitan areas elsewhere.

Table 3 shows the calibration process towards daytime population estimates for the 55 subareas of Brno from the easily attainable data on residential population and households, utilizing Function (3). The daytime estimates were obtained by interpreting diurnal population change as a recurring binary event. Past

Table 3 - Estimated diurnal ratios by subarea, showing corresponding estimates of subarea daytime populations, Brno Metropolitan Region, 2001

Order	District	Population	Households	A.H.S.	Iteration 1		Iteration 2		Iteration 3		Iteration 4		Iteration 5	
					d/r	dt pop	d/r	dt pop	d/r	dt pop	d/r	dt pop	d/r	dt pop
1	Lesná	15154	7086	2,14	1,731	26225	1,750	26520	1,830	27732	1,840	27883	1,850	28035
2	Štýřice	8635	4022	2,15	1,729	14931	1,725	14897	1,804	15577	1,814	15662	1,824	15747
3	Staré Brno	13024	5740	2,27	1,691	22021	1,682	21906	1,759	22905	1,768	23029	1,778	23154
4	Černá Pole-total	21591	9483	2,28	1,687	36420	1,677	36219	1,754	37870	1,764	38076	1,773	38283
5	Královo Pole	20205	8860	2,28	1,685	34043	1,675	33850	1,752	35393	1,761	35586	1,771	35779
6	Žabovřesky	22280	9663	2,31	1,670	37205	1,659	36951	1,734	38635	1,744	38845	1,753	39056
7	Veveří	22366	9690	2,31	1,668	37313	1,657	37054	1,732	38741	1,742	38952	1,751	39163
8	Černovice	7263	3022	2,40	1,577	11450	1,555	11291	1,625	11804	1,634	11868	1,643	11932
9	Ponava	7120	2955	2,41	1,568	11166	1,546	11005	1,616	11503	1,624	11566	1,633	11628
10	Jundrov-total	3725	1529	2,44	1,528	5692	1,501	5593	1,569	5846	1,578	5877	1,586	5909
11	Řečkovice	14304	5870	2,44	1,527	21845	1,500	21462	1,568	22433	1,577	22554	1,585	22675
12	Komín	7251	2975	2,44	1,526	11067	1,499	10873	1,567	11365	1,576	11426	1,584	11487
13	Zábrdovice-total	11224	4605	2,44	1,526	17131	1,499	16829	1,567	17591	1,576	17686	1,584	17781
14	Trnitá-total	3467	1408	2,46	1,482	5138	1,451	5031	1,516	5258	1,525	5286	1,533	5314
15	Husovice	5473	2211	2,48	1,456	7970	1,423	7789	1,487	8140	1,495	8183	1,503	8227
16	Brno-město	5830	2346	2,49	1,436	8370	1,401	8168	1,464	8535	1,472	8581	1,480	8627
17	Bystrc	23805	9439	2,52	1,348	32095	1,307	31121	1,366	32513	1,373	32687	1,380	32861
18	Kohoutovice	12857	5090	2,53	1,338	17202	1,296	16668	1,354	17413	1,362	17506	1,369	17599
19	Židenice-total	33172	13027	2,55	1,282	42527	1,237	41042	1,292	42869	1,299	43098	1,306	43326
20	Maloměřice-total	3004	1172	2,56	1,233	3704	1,186	3562	1,238	3720	1,245	3740	1,252	3760
21	Starý Lískovec	14220	5492	2,59	1,152	16375	1,101	15662	1,150	16351	1,156	16437	1,162	16523
22	Komárov	5175	1996	2,59	1,140	5901	1,090	5640	1,138	5888	1,144	5919	1,150	5950
23	Obřany	2385	918	2,60	1,123	2678	1,072	2556	1,119	2668	1,125	2682	1,131	2696
24	Horní Heršpice	1858	710	2,62	1,060	1969	1,007	1872	1,051	1953	1,057	1963	1,062	1973
25	Pisárky-total	2261	853	2,65	0,943	2132	0,890	2012	0,928	2098	0,933	2109	0,938	2120
26	Stránice	4896	1844	2,66	0,928	4541	0,874	4281	0,912	4465	0,917	4487	0,921	4510
27	Kníničky	513	193	2,66	0,917	471	0,864	443	0,901	462	0,906	465	0,910	467
28	Bohunice	16398	6084	2,70	0,789	12933	0,738	12098	0,769	12603	0,772	12666	0,776	12729
29	Slatina	8530	3157	2,70	0,766	6536	0,716	6106	0,746	6360	0,749	6392	0,753	6424
30	Brněnské Ivanovice	1261	457	2,76	0,586	739	0,543	685	0,565	712	0,567	715	0,570	719
31	Nový Lískovec	11400	4094	2,78	0,516	5885	0,478	5446	0,496	5654	0,498	5679	0,500	5705
32	Medlánky	3163	1135	2,79	0,510	1615	0,472	1494	0,490	1551	0,492	1558	0,495	1565
33	Líšeň	25388	9037	2,81	0,455	11547	0,421	10678	0,436	11069	0,438	11117	0,440	11166
34	Útěchov	446	158	2,82	0,424	189	0,392	175	0,406	181	0,408	182	0,410	183
35	Mokrá Hora	687	241	2,85	0,368	253	0,341	234	0,352	242	0,354	243	0,355	244
36	ROSICE	22817	7993	2,85	0,360	8224	0,334	7616	0,345	7871	0,346	7903	0,348	7935
37	TIŠNOV	17490	6102	2,87	0,340	5943	0,315	5509	0,325	5688	0,326	5710	0,328	5733

Order	District	Population	Households	A.H.S.	Iteration 1		Iteration 2		Iteration 3		Iteration 4		Iteration 5	
					d/r	dt pop	d/r	dt pop	d/r	dt pop	d/r	dt pop	d/r	dt pop
38	Holásky	935	325	2,88	0,322	301	0,299	279	0,308	288	0,310	289	0,311	291
39	KUŘIM	18024	6252	2,88	0,313	5636	0,290	5233	0,299	5397	0,301	5417	0,302	5437
40	Jehnice	752	259	2,90	0,283	213	0,263	198	0,271	204	0,272	205	0,273	205
41	IVANČICE	22584	7703	2,93	0,248	5602	0,232	5240	0,238	5381	0,239	5398	0,240	5416
42	Dolní Heršpice	361	123	2,93	0,245	88	0,229	83	0,235	85	0,236	85	0,237	85
43	Ivanovice	999	339	2,95	0,232	232	0,218	218	0,223	223	0,224	224	0,225	224
44	Tuřany	2100	708	2,97	0,214	450	0,202	424	0,206	434	0,207	435	0,208	436
45	ŠLAPANICE	50604	16938	2,99	0,197	9963	0,186	9424	0,190	9626	0,191	9651	0,191	9677
46	Chrlice	3176	1062	2,99	0,195	618	0,184	585	0,188	598	0,189	599	0,189	601
47	Soběšice	1475	491	3,00	0,185	273	0,176	260	0,179	265	0,180	265	0,180	266
48	Bosonohy	2210	732	3,02	0,176	389	0,168	371	0,171	377	0,171	378	0,172	379
49	ŽIDLOCHOVICE	27099	8856	3,06	0,156	4218	0,150	4054	0,152	4114	0,152	4121	0,152	4129
50	Žebětín	2070	676	3,06	0,155	320	0,149	308	0,151	312	0,151	313	0,151	314
51	Ořešín	452	147	3,07	0,150	68	0,144	65	0,146	66	0,147	66	0,147	66
52	POHOŘELICE	551	173	3,18	0,122	67	0,120	66	0,121	67	0,121	67	0,121	67
53	Přízřenice	572	164	3,49	0,105	60	0,105	60	0,105	60	0,105	60	0,105	60
54	Dvorska	335	91	3,68	0,104	35	0,104	35	0,104	35	0,104	35	0,104	35
55	Sadová	404	45	8,98	0,104	42	0,104	42	0,104	42	0,104	42	0,104	42
	TOTAL	535341				520019		507280		529229		531972		534716

Column R for Diurnal ratio estimation, is the Function, A / (1 + exp (B * (AHS + C))) + D
where: A = arithmetic difference between values of Diurnal ratio in the city centre and at the city furthest periphery (max - min)
B = Slope of the Function at proximity of Diurnal ratio = 1
C = Average household size at points where Diurnal ratio = 1
D = Minimum value of Diurnal ratio (i.e., at the furthest periphery)
Initial values: A=1.647 (remains constant); B=8.47335 (may change insignificantly during calibration); C= - 2.707 (attains value from previous iteration); D = 0.104 (remains constant)
C at 2nd iteration = -2.64

observations as well as some simple considerations regarding the ensuing concept of diurnal ratios of small area showed their relationship with the corresponding values of the average household size as a logistic function (3). It is also within this context that that diurnal population change emerges as a significant parameter of urban structure, with a particular potential for commuter estimates and urban transportation analysis.

7. CONCLUSION

In the mainstream urban economic literature commuting has been considered a continuous process of movement by individuals from their places of origin to their places of daytime destination. Here, a binary approach has been introduced whereby each subarea in the city is considered a place of daytime and night-time location of people. Each subarea in the metropolis, thus, attains two values: one for the daytime and the other for the night-time population. This approach yields an alternative to the continuous concept of urban commuting. A discrete, toggle notion of only two values – daytime and night-time population – for each and every subarea throughout a city, confers a discrete, binary attribute to each such subarea. The diurnal change of a subarea is the potentially observable and measurable, cyclical variation between the subarea´s night-time and daytime population. Correspondingly, the diurnal ratio is the arithmetic division of daytime population of a subarea by its night-time population.

The usefulness of diurnal ratios is that they allow for an easy estimation of daytime populations throughout the city subareas. The estimates of daytime population in small subareas of a metropolitan region are a good proximate measure that can conveniently balance the need for direct and not easily accessible data on commuter transportation demand. The relationship between daytime population in a subarea and the population of inbound and outbound commuters into and from a subarea is abridged by the notion of daytime and night-time population of the subarea.

Past observations have shown that the ordering of subareas throughout a metropolis by their diurnal ratios in descending order, from a subarea with the highest diurnal ratio to a subarea with the lowest diurnal ratio, approximates the ordering of the same subareas by their average household size in ascending order, from the lowest average household size to the highest. While diurnal ratios – much as data on daytime populations – are not readily available, the average household size is a parameter commonly available from most government census statistics.

In order to estimate the daytime populations throughout the metropolitan subareas, additional two considerations are undertaken here. The first consideration is an independent estimate of the diurnal ratio in the subarea with the highest average household size, usually a far-away suburb. The second consideration is the substitution of a mathematical function to the curve showing values of diurnal ratios for subareas in descending order. In the first consideration we show that the diurnal ratio in the subarea with the highest average household size can be reasonably assumed to be the inverse of the average household size, as shown in Eq. (2).

In the second consideration we show that the diurnal curve follows a logistic or cosine function of average household size approximating a formula, such as Eq. (3), with the resulting calibration of daytime populations as shown in Table 3.

The present study examines this approach against data available from the 2000 Census of the Czech Republic, for 55 subareas of the Brno Metropolitan Region. Based on the two assumptions, as expressed in (2) and (3), an iterative procedure is commenced, calibrating the values of diurnal ratios to their terminal estimates. Each time, throughout the procedure, Sequence Q is estimated, the diurnal ratios, d_i/r_i for each subarea i are multiplied by the known values, r_i , of the night-time populations, $i = 1, \ldots, n\text{-}1$. The control value to which the procedure calibrates is the population size of the metropolitan region, where the total night-time population is assumed to approximately equal the total daytime population of the metropolitan region.

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SOUHRN

DENNÍ MĚSTSKÉ DOJÍŽDĚNÍ OBYVATELSTVA V MALÝCH OKRSCÍCH VESKRZE METROPOLE: PŘÍPADOVÁ STUDIE PRO BRNO, 2000.

Oproti složitému sledování dojíždějících v metropoli či sledováním samotného procesu metropolitního dojíždění, je možné dosáhnout zjednodušeného přístupu k modelování profilu městské dojížďky prostřednictvím sledování denního obyvatelstva. Zatímco většina stávajících studií je ekonomického rázu a sleduje dojížďku coby proces v kontinuitě městského prostoru a času, přístup zde navržený využívá údaje o denním obyvatelstvu, a tím mění modelovací přístup na demografickou úroveň v binární variantě. Namísto kontinuálního sledování dojížďky zde sledujeme jen dvě hodnoty, a to hodnoty denního a nočního ukazatele v každém okrsku veskrze celé metropole. Vnímáním denní proměny v počtu obyvatel v okrscích tato studie poukazuje na novou, zjednodušenou formu uvažování o metropolitní dojížďce a dopravě. Za podmínek odpovídajících podobnému stavu denní proměny obyvatelstva v různých metropolitních regionech světa, sledovaných v předešlých studiích, rozbor zde navržený vede k analytické funkci pro odhad denního obyvatelstva v malých okrscích celého metropolitního regionu Brna. Navržená souvislost je logistická funkce, jejíž nezávisle proměnná veličina je průměrná velikost domácnosti v každém okrsku v rámci celého metropolitního regionu. Na základě údajů z českého sčítání lidu, domů a bytů z roku 2001, jsou zde aplikovány k odhadu denního obyvatelstva distribuce průměrných velikostí domácností a sídelního obyvatelstva metropolitního regionu Brna. Tato případová studie tak poukazuje na možnosti, které přináší navržený přístup k modelování odhadu denního obyvatelstva i pro další metropolitní regiony.

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