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INCORPORATION OF DUFFING OSCILLATOR AND WIGNER-VILLE DISTRIBUTION IN TRAFFIC FLOW PREDICTION

ABSTRACT

The main purpose of this study was to investigate the use of various chaotic pattern recognition methods for traffic flow prediction. Traffic flow is a variable, dynamic and complex system, which is non-linear and unpredictable. The emergence of traffic flow congestion in road traffic is estimated when the traffic load on a specific section of the road in a specific time period is close to exceeding the capacity of the road infrastructure. Under certain conditions, it can be seen in concentrating chaotic traffic flow patterns. The literature review of traffic flow theory and its connection with chaotic features implies that this kind of method has great theoretical and practical value. Researched methods of identifying chaos in traffic flow have shown certain restrictions in their techniques but have suggested guidelines for improving the identification of chaotic parameters in traffic flow. The proposed new method of forecasting congestion in traffic flow uses Wigner-Ville frequency distribution. This method enables the display of a chaotic attractor without the use of reconstruction phase space.

KEY WORDS

road traffic; congestion prediction; dynamic system; Wigner-Ville distribution; chaotic identification pattern;

1. INTRODUCTION

Traffic flow congestion has become a social predicament, which demands increasing national resources. Road traffic flow prediction plays an important role for traffic managers and in planning the improvement of traffic flow management. Hence, collecting data and using it to predict future traffic patterns in a way that enables the estimation of traffic flow congestion is of most importance. With the use of intelligent transport systems, traffic estimation and prediction has become more reliable for traffic managers [1].

Another predicament is the fact that some traffic congestions cannot be predicted in either space or time. The cause of the congestion of traffic flow is researched in various states of traffic flow.

Losses (costs) for temporary and occasional congestion of traffic flow can also lead to long-term congestion on the roads, most often measured by the external costs of transport. These are the costs of traffic accidents, due to delays in traffic flow and due to the impact of traffic flow on the environment (exhaust emissions, noise, and dust).

In 2014 the European Commission [2] published a paper in which external costs in EU road transport were estimated to between 140 and 240 million euros annually. Of these, the largest share was incurred by delays in road traffic. Traffic flow uncertainty reflects its causation through the influence of various elements. These elements can be divided into categories based on their features. In terms of environment they can be divided into external and internal elements. External elements could be weather etc. while the internal elements could be road conditions and car movement conditions. Further, based on the features, they could be divided into subjective and objective elements. Passenger's destination, the reason for travel, etc. are all objective elements, while driver's skills, habits, character etc. all fall under subjective elements. With this in mind, it can be easily seen that the elements affecting traffic flow are not certain. For example, human subjective mobility. Humans predict the reason for going somewhere at some point in the future. Similarly, due to differences between drivers in skills and character, it is not possible to accurately predict the drivers' behaviour. All of these factors combine to create the uncertainty of traffic flow. These uncertainties are reflected in traffic flow, through congestion or irregular movements.

Therefore, the presented paper is a research which seeks to investigate and find a method or methodology which can in a relatively short time calculate a large number of key parameters of traffic flow that would enable us to predict road congestion faster and more accurately.

2. LITERATURE OVERVIEW

Traffic flow is a complex random system, which is affected by numerous elements that feature uncertainty and complexity. If we take the traffic flow as a flow which has cases of mobility, with observation time-sequence can be derived and further research conducted.

At the earliest stage three models were used – moving average model [3], autoregressive moving average model [4], and regressive model [5]. These models considered traffic flow impact elements in a very simple manner. The parameters were usually derived from the least square real-time estimates which can be derived through simple calculations and are suitable for real-time data updates. But these prediction models could not reflect non-linear relations in a traffic flow system, and the impact of other elements in the system were not sufficiently assessed. So, the disadvantages are fairly clear; when prediction data intervals are shortened, the predictive precision is largely influenced by these shortcomings.

It is exactly for this reason that researchers have developed more complex and more precise prediction models and methods which can be divided into two categories:

1) Methods based on certain mathematical models: multi-element regressive model, ARIMA model, associative model of self-adaptive load sum, Kalman filter model, model of reference function and smooth component, as well as combined prediction models comprising these methods and models.
2) Non-model calculation methods: non-parameter regression, spectrogram analysis, model of space reconstructs, microwave network, multi-dimensional fractal method, and other prediction models based on micro-wave analysis and reconstruction related to neural networks.

Traffic variables estimation, i.e. volume, speed, density, travel time, headways, etc. are important in traffic planning and design operations. Several methods are presented in the literature for prediction of traffic parameters, such as time series analysis, real-time method, historic method, statistical methods and machine learning, etc. To understand the working process behind each of these methods it should be understood that the methods have both advantages and limitations. For the data-based algorithms and time series model limitations, Smith [6] applied neural networks to short traffic flow prediction. Another model successfully applied to traffic flow was presented by Ledoux [7]. One-minute forecasting was simulated on the gathered data, with the queue lengths and output flows achieving fairly good accuracy. Traffic forecasts using neural networks were also investigated by Kumar and Parida [8].

In the past, the chaos theory was widely used in medicine [9], solid-state physics [10] electronic counter measures [11], surveillance in IT connections [12, 13] and also in seismic investigation. Much of the recent work in this field focuses on enhancement techniques and methods of increasing the signal-to-noise ratio [14–17]. Methods like linear regression, logic regression model, Poisson model and negative binominal model are subject to strong assumption and limitations in applications [18]. Some methods are more commonly used; however, this does not state its accuracy in prediction.

3. NEW METHOD OF CHAOTIC PATTERN IDENTIFICATION

Detailed analysis of various chaotic pattern identifications helped to understand the limits of the methods used for chaotic pattern identification. Wigner-Ville distribution has favourable frequency aggregate since its spectrogram clearly shows the signal frequency and energy. Chaotic time sequence is an appearance of a low bound, non-linear and certain dynamic system, which looks like a random signal but in reality it is not. Wigner-Ville's good frequency aggregate can show internal layers of chaotic time sequence, and it can use Lebesgue measure to perform quantitative analysis on the chaotic features of time sequence. This chapter explains this kind of chaotic identification method, using the Duffing chaotic dynamic system as an example [19].

$$W_s(t,f) = \int_{-\infty}^{\infty} s(t+\frac{\tau}{2})s*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau \tag{1}$$

where t and f are time and frequency components, $s*$ is a complex conjugated form of the original signal and τ is time delay, while $e^{-j2\pi f\tau}$ is a phase function and j is an imaginary number. Wigner-Ville has many favourable features especially for traffic flow prediction [20, 21].

With the use of bilinear transformation, which is often used in digital signal processing and discrete-time control theory, transformation occurs from continuous-time system representations into discrete-time and vice versa. There is no window function. This way, we can avoid short-time Fourier transform in time resolution. It is exactly due to this advantage and a lot of favourable characteristics in its distribution that the Winger-Ville transform is widely used in many real projects.

Discrete Wigner distribution has many calculating methods. Here we have used the fast Fourier transform - FFT calculation method. When it comes to long sequences without cycles, before FFT is performed, a window has to be added to the sequence, i.e. the sequence must be defined as a Wigner discrete distribution. From the definition, it can be seen that Wigner distribution is a form of visual distribution of a signal energy. Theoretically, $W(t,f)$ is a certain share in total energy which is occupied by time and frequency units. Times and frequencies should always be positive, but in reality, it is a different story.

With the selection of various k_0 Lebesgue measures in a Wigner-Ville transform on $x(t)$, frequency intervals can be presented. We can determine the chaotic or periodic state of Lebesgue measure with its frequency intervals ω.

3.1 Duffing's dynamic attributes

Duffing's oscillator holds an important place in non-linear dynamics research. A form of the equation for Duffing's oscillator is simple, but it can derive many characteristics of a non-linear state. This is because Duffing has added on the right side of equation an item of additional force, which for the result has the system's intrinsic frequency and frequency of additional force interacting.

Adjusting the range value of the driving force, which makes the system alternate between chaotic and periodic state, a Runge-Kutta calculation is used to derive an equation on the derived $x(t)$, and with Wigner transform we get $W(t,\omega)$.

A Holmes model of Duffing's oscillator has been chosen in order to identify chaotic patterns. Its equation is as follows:

$$\ddot{x} + k\dot{x} - x^3 + x^5 = \gamma \cos(\omega t) \tag{2}$$

where k=0.5 denotes the ratio of damping, which is the reference signal and also an internal signal in the forced periodic terms. The term is the non-linear recovery force in system 1, the kinematic state of the system depends mainly on this recovery force term $\gamma\cos(\omega t)$. The experiments have proven that damping ratio k ranges from 0.2-0.5 [22]. With k=0.5 and equation for the state above it follows:

$$\begin{cases} \dot{x} = \omega y \\ \dot{y} = \omega[-ky + x^3 - x^5 + r\cos(\omega\tau)] \end{cases} \tag{3}$$

In *Equation 2*, γ is the range of periodic driving force, while k is the damping ratio and $-x^3+x^5$ is the non-linear recovery force in system 1. The system mainly depends on the recovery force $r\cos(\omega\tau)$, where r is an amplitude of the reference signal in the chaotic critical state. *Equation 3* can now be used to model a periodically forced pendulum with a cubic restoring force. Presented *Equation 3* is used to model a periodically forced resistor circuit with non-linear elements where $x(t)$ represents the charge oscillating in the circuit at time t.

In *Figure 1* the Duffing system damping ratio is k=0.5. When k is fixed, the state of the system will change frequently according to the changes of γ (range of periodic driving force) – state of homoclinic orbit, bifurcation orbit, chaotic orbit, critical period orbit, orbits of huge periods. The time-domain waveform and phase plane orbits of various states of this system are shown in the figures below. Analysing the Duffing

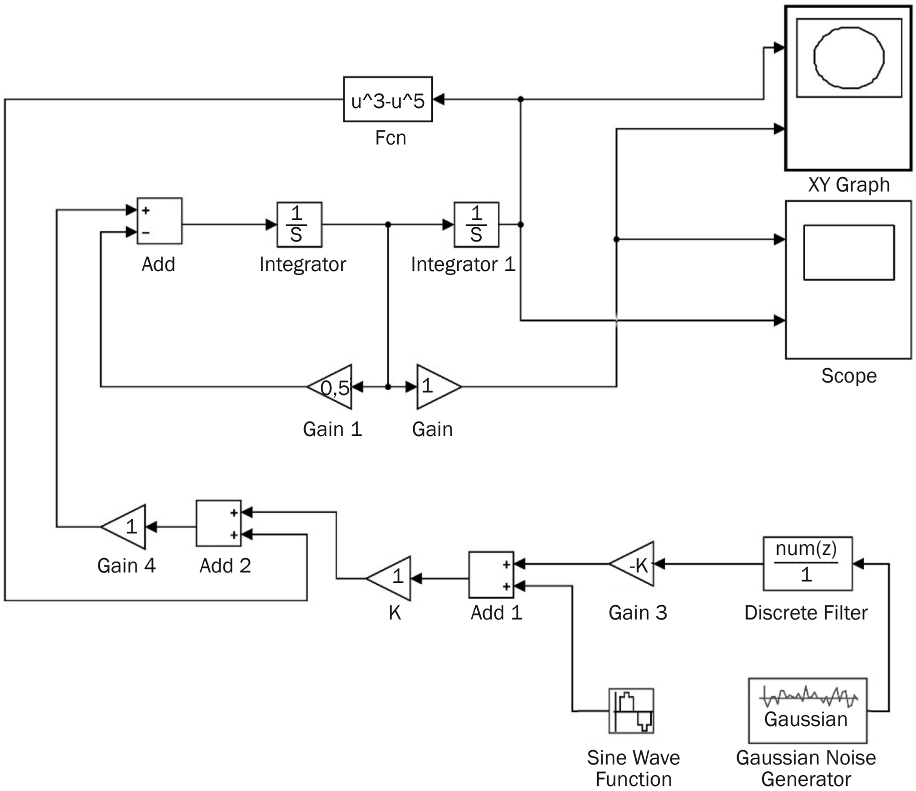


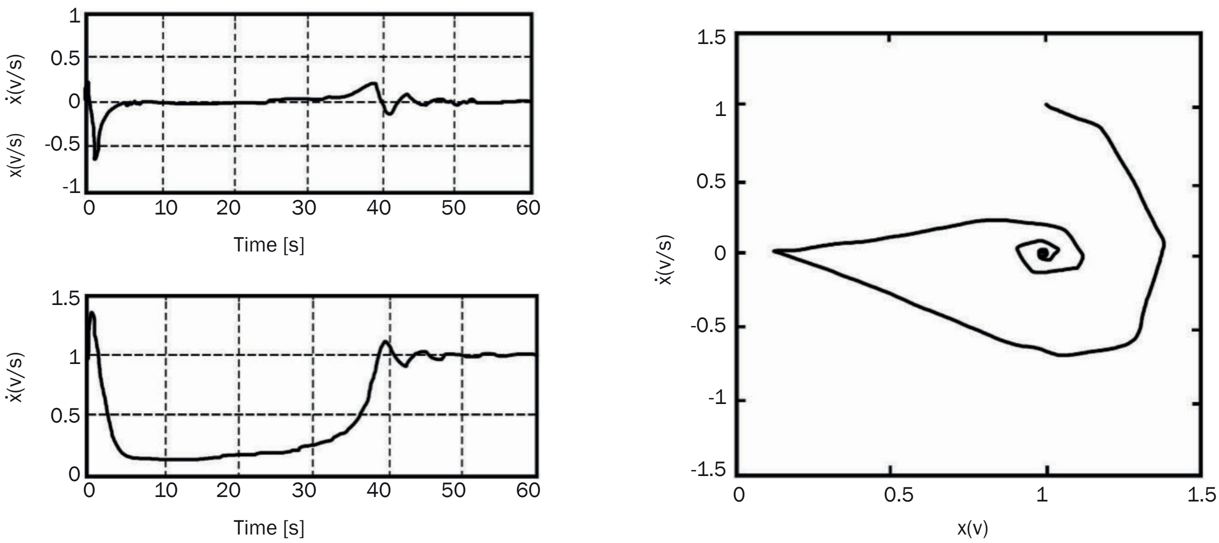
Figure 1 – MATLAB model of Duffing's dynamic system

system presented in *Figure 1*, the time-domain waveform and planetary orbits can be established:

1) when γ=0, the saddle point for the system phase plane is (0,0) and the focal point is (±1,0). Point (1,1) should in the end, stop between two focal points as shown in *Figure 2* and *Figure 3*.
2) when γ<0, the system encounters complicated dynamical states which can be divided into several cases.
3) when γ is relatively small, the phase orbit behaves as attractor. Phase points are circling around the focal points, or another focal point and vibrating when γ exceeds the fixed closing value critical value - γc (size of γc can be derived from Melnikov's method) [23]. Simultaneously as γ rises, the system will go from homoclinic orbit (*Figure 3*), periodical bifurcation and all the way to a chaotic state (*Figure 4*). This process is very fast. If γ is presented for a long time, the system will always be in the chaotic state. Only after a higher closing value of γd appears - the amplitude of the driving power - will the system enter a periodic state (*Figure 5*). At that moment, the phase orbit will encircle focally the saddle point, and in the corresponding Poincare map it will also be a static point.

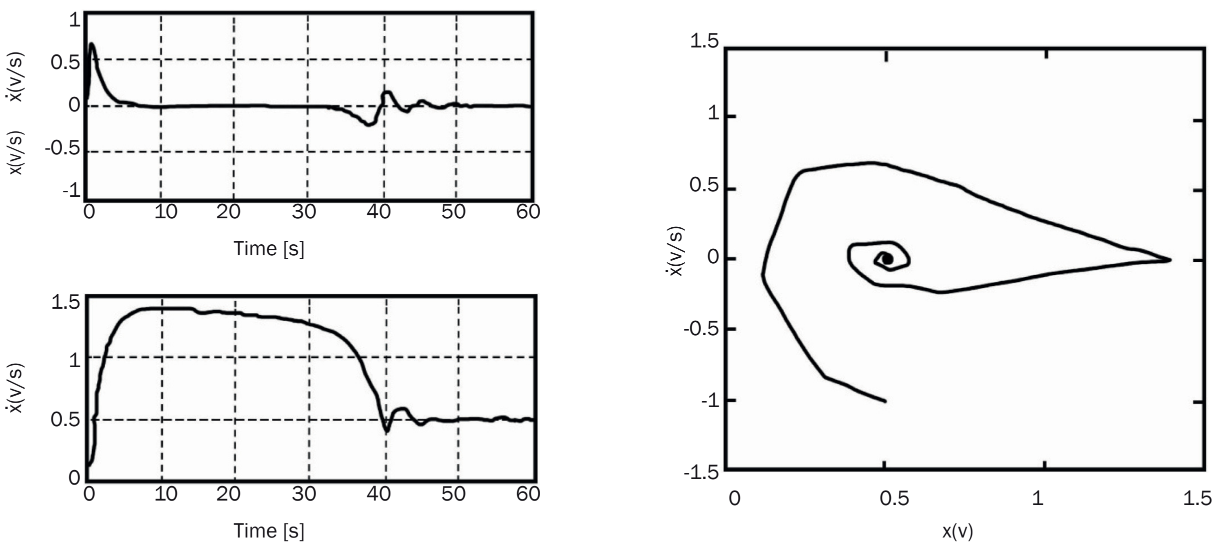
3.2 Identification of chaotic features based on Wigner-Ville transform

Equation $-x^3+x^5$ was taken for the Duffing oscillator restoring force item as follows:



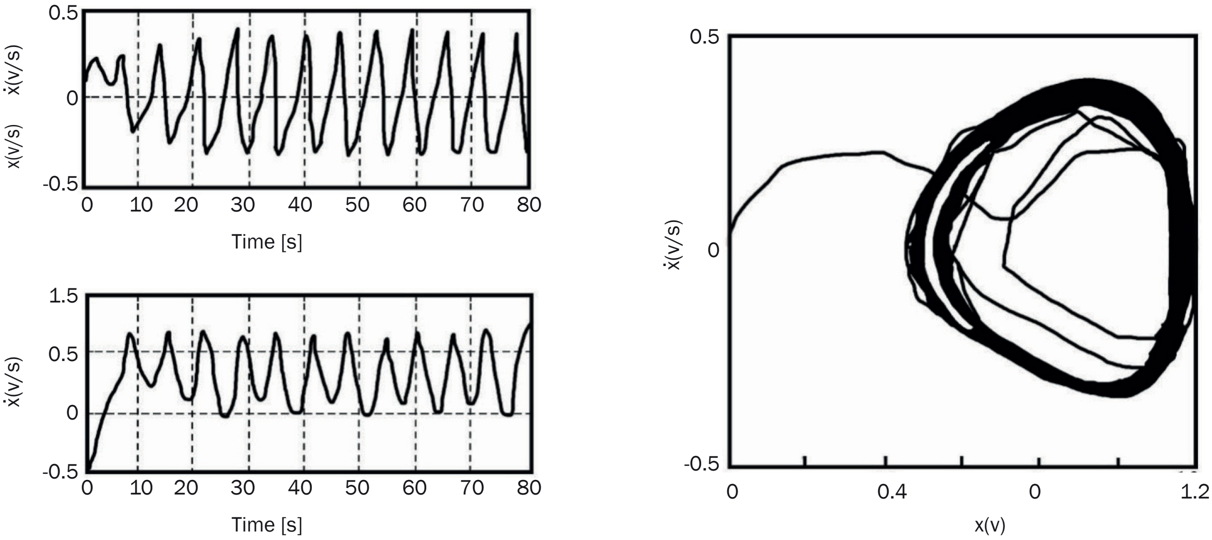
a) Time-domain waveform *b) Phase planar orbit when γ=0 is the starting state*

Figure 2 – Duffing system at $\gamma = 0, [x(0), \dot{x}(0)] = [1, 1]$



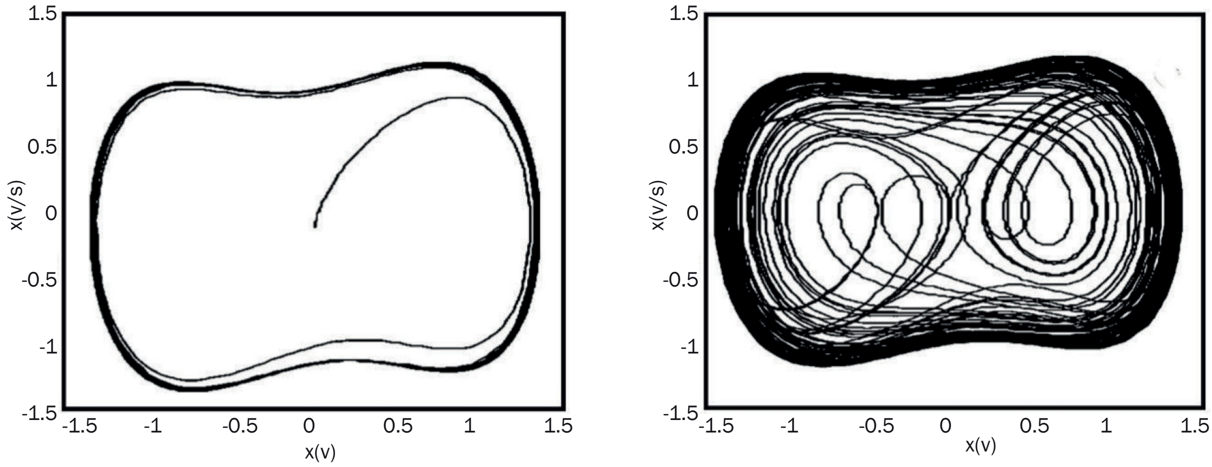
a) Time-domain waveform *b) Phase planar orbit when γ<0*

Figure 3 – Duffing system at $\gamma = 0, [x(0), \dot{x}(0)] = [-1, -1]$



a) Time-domain waveform *b) Phase planetary orbit*

Figure 4 – Duffing system at γ =0.2, ω=1rad/s



a) *b)*

Figure 5 – a) Duffing system at γ =0.75 – periodic state, b) Duffing system at γ =0.69 – chaotic state

$$\ddot{x} + k\dot{x} - x^3 + x^5 = \gamma \cos(\omega t) \tag{4}$$

Adjusting the range value of the driving force, which makes the system alternate between a chaotic state and a periodic state, a four step Runge-Kutta calculation method to derive the new equation was used. On a derived $x(t)$, the Wigner transform was applied, which is written as $W(t,\omega)$.

In the figure, the horizontal coordinate represents the frequency and the vertical one represents the width. *Figure 7* clearly shows differences between the chaotic state and the periodic state. Its Wigner distribution domain is flat, but it still has a certain regularity, and it is exactly that intrinsic regularity mentioned above when discussing chaotic nature. In the periodic state, a graph of Wigner distribution clearly shows wave peaks, while other parts are flat. This explains that at that moment the Duffing system has been entering from one periodic state into another. Still, these frequency spectrograms cannot describe or identify these two types of states. This requires the usage of a Lebesgue measure in order to further explore the system. A Lebesgue measure gives a concrete way of measuring the volume (or area) of subsets of $\mathbb{R}^n$. Here we will use [24]:

$$L(k_0)=\omega \tag{5}$$

Its Wigner distribution domain is flat, but it still has a certain intrinsic regularity, which was mentioned above when discussing chaotic nature. In the periodic state, the Wigner distribution graph clearly shows wave peaks, while other parts are flat. This explanation for this is that the Duffing system has been entering from one periodic state into another. Nevertheless, these frequency spectrograms cannot describe or identify these two types of states; this requires the usage of a Lebesgue measure in order to further explore the system. Wigner-Ville's good frequency aggregate can show internal layers of chaotic time sequence, and with the use of a Lebesgue measure we can perform quantitative analysis on chaotic features of the time sequence. The use of a Lebesgue measure provides us with a finite number of countable intervals. We will use [25]:

$$L(k_0)=\left\| W(t,\omega) \right\| \geq k_0 \tag{6}$$

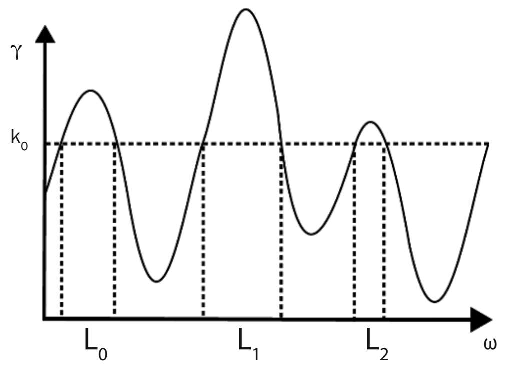


Figure 6 – Visual explanation of $L(k_0)$ size

Its amplitude frequency is shown in *Figure 6*. According to the section above where the Lebesgue measure and the additive of union of all non-overlapping countable interval lengths, the following must be pointed out:

$$L(k_0)=L_0+L_1+L_2 \tag{7}$$

There are many methods of selecting k_0, but the suitability of the selection immediately decides on the accuracy of chaotic pattern identification. Thus, a golden section point was used: [26]:

$$k_0=\max \left\| W(t,\omega) \right\| \cdot 0.618^n \tag{8}$$

In this way, thanks to the change of the selected value n we can observe the situation with a Lebesgue measure $L(k_0)$ when there is a change in size of the chaotic and periodic states, which is represented in *Table 1* and *Figure 7*.

Table 1 – N value chaotic state $L(k_0)$ large period $L(k_0)$

n value	Chaotic state $L(k_0)$	Large period $L(k_0)$
4.4	0.86748	0.024587
4.8	0.96724	0.037868
...		
8.2	1.45823	0.153278
...		
12.2	1.470796	1.315078
...		
13.3	1.593079	1.593079
13.6	1.593079	1.593079

Based on *Table 1* and *Figure 7* one can conclude as follows:

a) The system is in a chaotic and periodic state, while ω of the Lebesgue measure rises in conjunction with the rise of n value.
b) Despite the increase of $L(k_0)$'s n value following the increase of n, the curves still differ. The rise of the curve of the periodic state is smooth and stable, because of the shape of the spectrogram after curve goes through the Wigner transform. In a chaotic state its shape is not smooth and stable; it shows irregular ups and downs, and the curve's ascending speed is faster than the one in the periodic state.
c) When the n value is the same, the $L(k_0)$ value in the large periodic state must be lower than the same values in the chaotic state.

Using the above simulation experiments and conclusions, it can be seen that the method of chaotic pattern identification which is based on Wigner transform

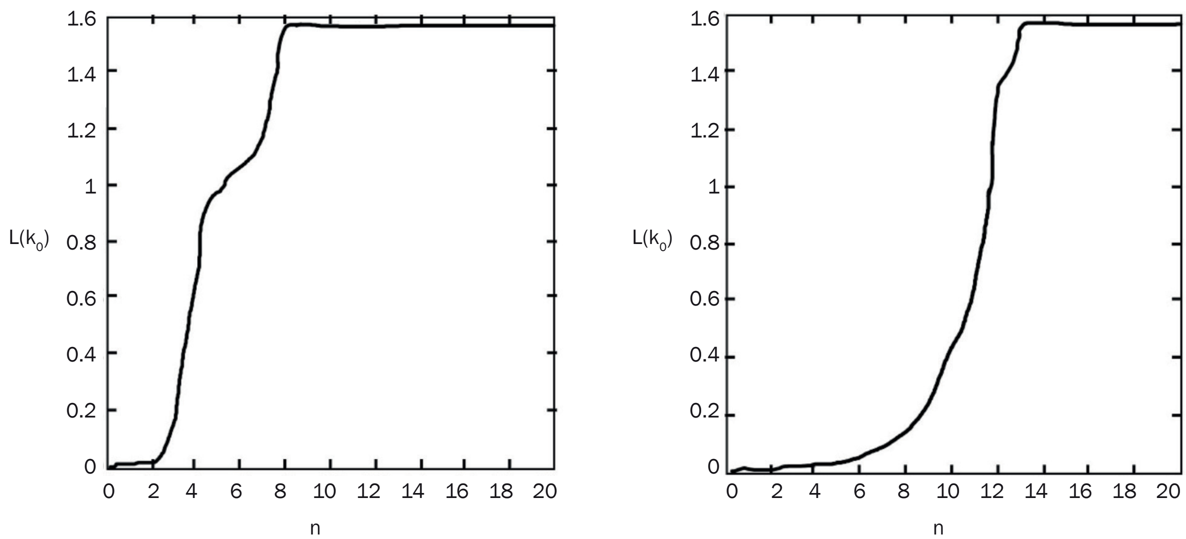


Figure 7 – Change of Lebesgue measure following the change in n (left – chaotic state, right – periodic state)

and Lebesgue measure is actually based on assessing the trends of speed changes in the $L(k_0)$ value when it follows the changes in n value in order to observe a chaotic state. At the same time the Duffing system was used to prove the effectiveness of this method.

4. CHAOTIC PATTERN IDENTIFICATION IN TRAFFIC FLOW-TIME SEQUENCE

The general effects of manmade subjective elements and non-manmade objective elements allow the phenomena of regular traffic flow and chaotic traffic flow to coexist. When there is a low number of cars in the street, they are in a state of free movement. Along with the increase in the number of cars and other external disturbances or characteristics of drivers and their cars which affect the element of uncertainty, there is a greater chance of a chaotic state.

Regarding the observation done in time-sequence, it must be pointed out that although there is a certain correlation between time and traffic flow, there might be many other elements (such as noise) which make a chaotic state less visible. So, in order to make its chaotic state more visible, some preparations regarding time-sequence must be made. With preparation, more precise predictions can be reached for better chaotic pattern identification.

4.1 Pre-processing of time sequence

There are many pre-processing methods, but here the smoothing and differential method will be used.

1) The smoothing method when gathering time sequence data involves a process of "burring". The traffic flow structure is complex, so the time sequences it produces might be affected by various subjective and objective impacts, which could bring about sudden changes. Hence, the result and predictability of chaotic pattern recognition might be affected. We will use the preceding smoothing which uses a real value of the first i point and the smoothing value of the preceding point M-1 as a value of the first point i. The mathematical model regarding time-series $\{x_0, x_1, \ldots x_n\}$ is shown below.

$$\widetilde{x}_1 = \begin{cases} x_i & i < M \\ \frac{1}{M}(x_{i-M+1} + x_{i-M+2} + \ldots x_i) & M \leq i \leq n \end{cases} \quad (9)$$

Among them: $i, M \in Z$, $\check{x}_i$: smoothed value of the i-th point, where x_1 is the value of the first point i, after smoothing. *Equation 7* shows the first smoothing of the time sequence, or in other words the gradual levelling of the time-sequence. The M value shows a time sequence that was smoothed several times by a few points. During computing, the size of the M value must not be too large, otherwise the traffic flow time-sequence could distort and not only affect the precision of criteria for chaotic pattern identification, but also its predictability.

Let us use M=5.

2) Differential operators where the number of columns $\{X_n\}$ defines differential operator $\Delta{:}\Delta X_n = X_{n+1} - X_n$, where X_n is place n of the preceding differential and $\Delta X_n = X_n - X_{n-1}$, where X_n is place n of the latter differential operator. Here we use a differential operator which was taken from the time sequence processed during the smoothing method. So, the value of the first point i of the latter differential in the time sequence $\{\Delta\widetilde{x}_i\}$ is:

$$\Delta\widetilde{x}_i = \widetilde{x}_i - \widetilde{x}_{i-1} \quad 1 < i \leq n \quad (10)$$

It should be noted that the time sequence can be processed by the differential method only once, or it cannot be processed. This is because the differential of the time sequence and the derivation of the continuing functions is the same, and according to the principle of infinitesimal calculus, the equations with a large number of arbitrary items after limited derivation can transform all into constants. Likewise, multiple differentiation might affect the chaotic features of the time sequence and thus affect the result of the criteria for chaotic recognition and there will be problems in restoring the dynamic characteristics of awkward attractors which might occur after the reconstruction of phase space.

4.2 Case study data

The data were collected on a two-lane highway in Slovenia, where in near proximity (1 km) there was no input or output of cars or other vehicles. The data were collected by a pneumatic traffic counter on a Slovenian highway. The locations were randomly selected, where data were collected for three months. However, not all of the collected data were acceptable. A typical working day was observed, with no specific holidays or weather conditions. The proposed method identifies a random occurrence of traffic flow congestion, which is not in any way induced by accident, weather, holidays, etc.

The locations were not in linear connection due to possible interconnection which would distort the actual prediction of chaotic patterns in the time frame.

For the identification of the chaotic phenomena in the time sequence, a non-typical date was used with specific limitations: no difficult weather conditions, weekdays - not Monday or Friday, and there should be no holiday on any chosen date.

4.3 Identification of chaotic phenomena in time sequence

The method for pre-processing time to implement the chaotic pattern recognition was used on a period of time (6:00 – 11:00) at a certain crossroads on an elevated bridge in a traffic flow time sequence (sequence

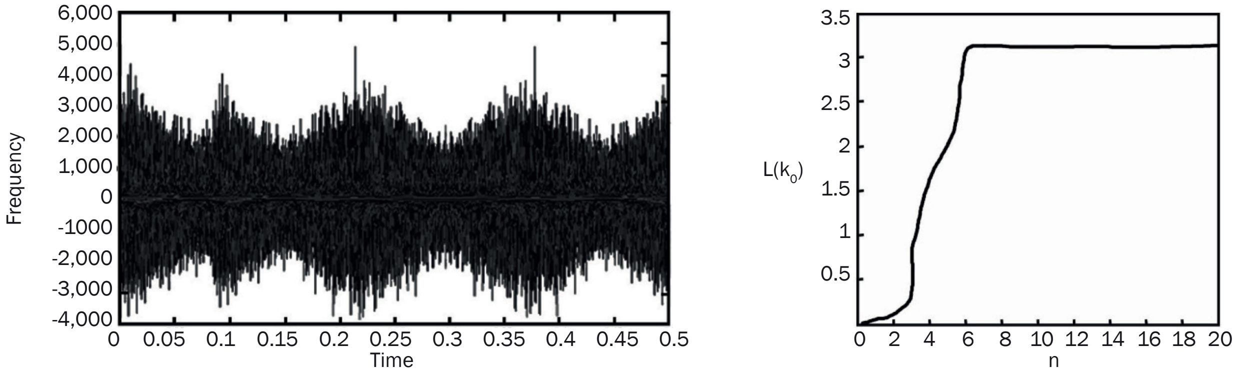


Figure 8 – Amplitude frequency diagram for chaotic time sequence (left), and a diagram of the Lebesgue measure (right)

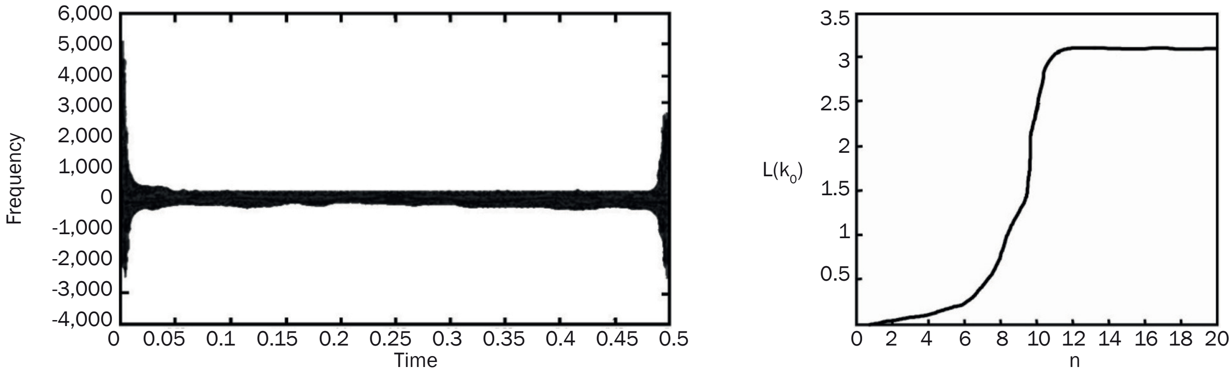


Figure 9 – Amplitude frequency diagram for chaotic time sequence (left), and a diagram of the Lebesgue measure (right)

1, time interval: 1 min, total of 300 data). *Figure 8* - (left side) shows the amplitude frequency diagram for the chaotic time sequence; the right side shows a diagram of the Lebesgue measure.

Figure 8 shows that the shape of the Lebesgue measure curve is not smooth; it shows irregularities and the speed of the ascending curve is very high. So, based on the conclusions in Chapter 3.1 it can be established that the traffic flow time sequence does have chaotic features.

Lyapunov's method was used for chaotic pattern recognition in this time sequence. In this place we used the same day and the same place of this sequence and we gathered data for 24 hours. There was a total of 1,440 data. We have calculated Lyapunov's biggest characteristic exponent as λ=0.015>0. so, this time sequence has chaotic features, and at the same time we have proven the feasibility of our method. The most important reason for chaos is that the car flow at this period was the heaviest, and it was difficult to avoid accidental elements (such as traffic accidents, or traffic light intervals, etc.), which might have even caused overcapacity of the road and even a traffic jam, which as a result made regular traffic movement change to irregular movement, which in the end produced a chaotic phenomenon.

If we use the same method on the same road, but during early morning hours (2:00-7:00) (time interval is 1 min, total of 300 data, recorded as time sequence 2) and implement chaotic pattern recognition, we get the results shown in *Figure 9*.

We can see from *Figure 9* that the curve whose value of $L(k_0)$ produces increases when n starts to increase is clearly different from the one in sequence 1. The growth is smooth and stable. According to the before-mentioned conclusion, it can be established that this traffic flow time sequence does not have chaotic features. This time, the traffic flow time sequence pattern had features similar to those of periodic states found in the Duffing system. But this state is not the periodic state for the traffic flow. Instead, it is a state of regular movement. The reason for this state is a really small number of cars, that is, cars are moving freely, and there are no accidental elements (car accident, manmade traffic control, etc.). During this periodic state of traffic flow, the movement should be described as regular.

5. CONCLUSION

This study presents the importance of good and reliable chaotic pattern recognition in the traffic flow. For the presentation value, three months of data were used at three different locations which had no interconnections. The figures and numbers in the tables present a new possible model for traffic flow prediction. With the use of the Duffing system, a calculation using Lebesgue measure and Wigner-Ville distribution for search of traffic flow patterns was presented. The following can be concluded: the system is in a chaotic and a periodic state, while ω of the Lebesgue measure rises in conjunction with the rise of the n value; despite $L(k_0)$'s n value increase following the increase of n, their curves still differ; when the n value is the same, the $L(k_0)$ value in the large periodic state must be lower than the same values in the chaotic state.

The paper put forward a new way of using the Wigner transform and a Lebesgue measure to perform a chaotic identification of a time series. Firstly, the Wigner-Ville's distribution of frequency aggregation was used, thus making the chaotic characteristics of time sequence in traffic flow emerge with no need of the same spatial reconstruction. A Lebesgue measure was used to perform a qualitative analysis of the traffic flow. With the use of the Duffing chaos system, the feasibility of the presented method was evident. The chaos characteristic identification in the time sequence of the traffic flow also acquired satisfactory results, which also proved that there are both sequential movement and chaotic movement in the traffic flow system. It was estimated now when and where the possible congestion would occur. Further research will incorporate the presented model in a new model of a chaotic neural network. It can be assumed that the model will be faster and more reliable than an ordinary neural network.

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NAPOVEDOVANJE PROMTNEGA TOKA S POMOČJO DUFFINGOVEFGA OSCILATORJA IN WIGNER-VILLE PORAZDELITVE

POVZETEK

Osnovni namen študije je raziskati uporabo metode razpoznavanja kaotičnih vzorcev za uporabo pri napovedovanju zgostitve prometnega toka v cestnem prometu. Prometni tok je spremenljiv, dinamičen in kompleksen sitem, ki je nelinearen nepredvidljiv. Nastanek zgostitve prometnega toka na cestnem odseku ocenjujemo v časovnem intervalu, ko le-ta prične presegati kapaciteto cestnega odseka. Pod določenimi pogojih je mogoče zaznati kaotični vzorec v prometnem toku. Pregled literature teorije prometnega toka in njegovo povezavo s teorijo kaosa je prikazal veliko teoretično in praktično vrednost tovrstne metode. Raziskane metode identifikacije kaotičnega vzorca v prometnem toku so pokazale omejitve, zato je v nadaljevanju opisana smernica za izboljšavo prepoznavanja kaotičnih vzorcev v prometnem toku. Predlagana nova metoda za napovedovanje zgostitev v prometnem toku predlaga uporabo Wigner-Ville frekvenčne porazdelitve. Postopek omogoča prikaz kaotičnega atraktorja brez uporabe rekonstrukcije faznega prostora.

KLJUČNE BESEDE

cestni promet; napoved zgostitve prometa; dinamični sistem; Wigner-ville porazdelitev; identifikacija kaotičnih parametrov;

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