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A SIMPLE THROW MODEL FOR FRONTAL VEHICLE-PEDESTRIAN COLLISIONS

ABSTRACT

This paper discusses a simple theoretical throw model for frontal vehicle-pedestrian collisions. The model is based on the simple assumption that pedestrian movement after impact can be approximated by the movement of a mass point. Two methods of vehicle-pedestrian collision reconstruction are discussed: one knowing only the throw distance and the other when also impact to ground contact distance is known. The model is verified by field data available in the literature and by comportment with full scale numerical simulation.

KEYWORDS

traffic accidents, vehicle-pedestrian collision, Searle equation

1. INTRODUCTION

The investigation of vehicle-pedestrian collisions must have begun in the middle of the sixties mainly for the purpose of accident reconstruction. From that time several models describing the motion of pedestrians after impact with vehicles was developed ([1], [2], [3], [4], [5], [6], [8], [9], [10], [11], [13], [14], [15], [17], [18]). Basically there are two types of models: theoretical, based on laws of mechanics, and empirical. Theoretical models yield reliable results; however, considerable input data from real world collisions is needed to solve the equations. On the other hand empirical models, usually consisting of a single regression formula which connects the vehicle impact speed with pedestrian throw distance ([5], [17]), need no particular data; however, their application is limited only to well defined scenarios and the accuracy of models is within, say, ± 10 km/h ([7]). Typically, the empirical models do not include road grade, which can be an influence factor when one determines vehicle impact velocity from throw distance. The hybrid models try to combine features of both basic models ([3]).

In the present paper the model of frontal vehicle-pedestrian collision closely following the Han-Brach approach ([3], [6]) is developed. The details of derivation of equations are included for comprehensiveness. In addition to Han-Brach equations the equations for total flying time and total throw time and throw distance are also given. The basic equation of reconstruction, the equation for calculation of pedestrian launch velocity, is then obtained by inverting the equation for total throw distance. This equation is, in the special case of a horizontal road, reduced to the so-called Searle equation ([15], [16]). The four methods of reconstruction are then discussed: the method when one knows the pedestrian launch angle and friction between pedestrian and road; the Searle method ([15]) where the launch angle is estimated on the basisof extreme of launch velocity; and two new methods where in addition to throw distance the distance from impact to ground contact is also known.

2. THE MODEL

2.1 Assumptions

Only the frontal impact of the vehicle with the pedestrian is considered. In the case when the vehicle has enough speed or is braking the pedestrian will, after impact, be thrown from the vehicle hood, fly through the air, impact the ground and then slide/ /roll/bounce on the ground to a rest. The possible impacts of pedestrian with the road obstacles in the last phase are excluded from consideration. To describe these events mathematically the following assumptions are made:

- the car-pedestrian impact is symmetric so all events happen in a single plane;
- the initial velocity of the pedestrian is zero;
- after launch the pedestrian is considered as a mass point;
- the ground is flat;
- the pedestrian-ground friction is constant;
- all air resistance is neglected.

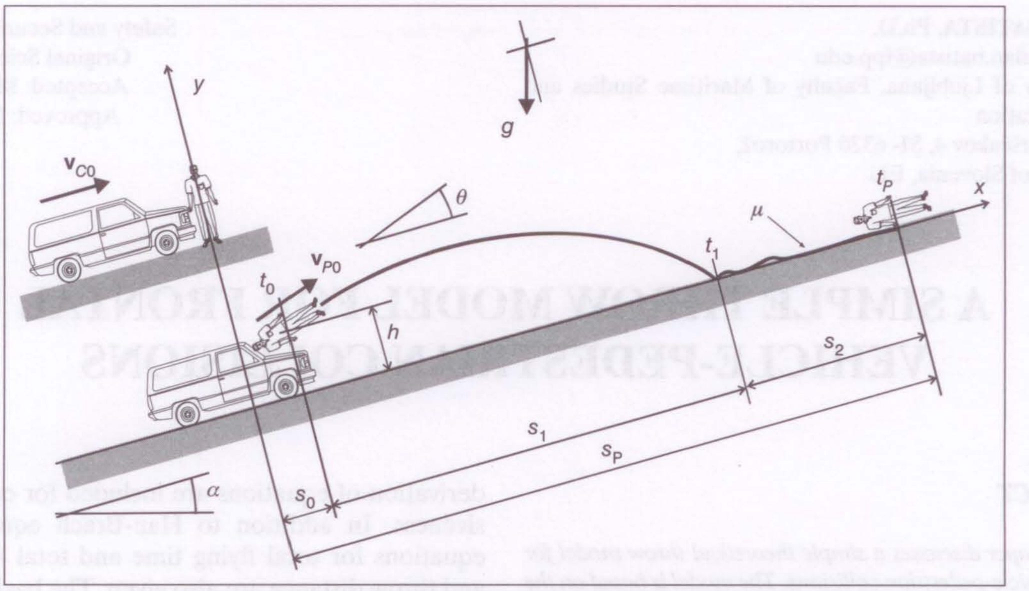

Figure 1 - Vehicle-pedestrian collision variables and events

According to events description and the above assumptions the following basic variables are included in the model (Figure 1):

- gravity acceleration $g = 9.8$ m/s^2,
- mass of the vehicle m_C and mass of the pedestrian m_P,
- initial pedestrian launch height h (not pedestrian centre of gravity, COG),
- total pedestrian throw distance s_P; i. e., the distance the pedestrian travels from impact to the rest position on the ground,
- total pedestrian throw time t_P,
- vehicle impact velocity v_{C0},
- pedestrian launch velocity v_{P0},
- road gradient angle α,
- pedestrian launch angle θ,
- coefficient of friction μ between the pedestrian and the ground.

It is further assumed that the total throw distance s_P and the total throw time t_P are expressed as the sum of three phases: contact phase, flying phase and sliding/rolling/bouncing phase ([4], [11]). The total throw distance is therefore

$$s_P = s_0 + s_1 + s_2 \quad (1)$$

and the total throw time is

$$t_P = t_0 + t_1 + t_2 \quad (2)$$

where indices 0, 1, 2 belong consecutively to contact, flying and sliding distance/time.

2.2 Contact phase

This phase roughly consists of ([4])

- vehicle-pedestrian contact;
- impulse of the pedestrian's body;
- movement on the vehicle hood.

In the scope of the present paper, the movement of the body onto the vehicle can be roughly of two types:

- wrap trajectory - here the pedestrian is wrapped over the front of vehicle , usually involving a decelerating vehicle,
- forward projection - in this case COG of the pedestrian is below the leading edge of the vehicle at impact.

The main goal in this phase is to connect vehicle impact velocity v_{C0} with pedestrian launch velocity v_{P0} and also to determine the contact path length s_0 and contact time t_0. This last is beyond the scope of this paper and therefore will not be discussed. However, in the case of forward projection one can approximately take $s_0 = 0$ and $t_0 = 0$. More detailed analysis of impact and future references can be found in [4], [6] and [18].

Despite the fact that this phase of throw influences others, only a simple model will be presented: it is assumed that impact between vehicle and pedestrian is plastic. In this case from conservation of momentum $m_C v_{C0} = (m_C + m_P) u_{C0}$ one obtains the vehicle/pedestrian post-impact velocity u_{C0}

$$u_{C0} = \frac{v_{C0}}{1 + m_P / m_C} \quad (3)$$

The case of non-plastic impact is discussed in [8].

Because the velocity u_{C0} and the pedestrian launch velocity v_{P0} differ for the case of wrap trajectory, a coefficient η called pedestrian impact factor is introduced to relate them ([6], [15], [18]):

$$v_{P0} = \eta u_{C0} = \frac{\eta v_{C0}}{1 + m_P / m_C} \quad (4)$$

In general, the coefficient η cannot be constant and it is in general dependant on various factors, including vehicle impact velocity, geometry of vehicle front, pedestrian height, etc. ([18]).

2.3 Flying phase

Following Figure 1 and Newtonian 2nd Law the equations of motion of pedestrian COG are the well known equation of a projectile in a vacuum:

$$\frac{dx}{dt} = v_x \quad m_P \frac{dv_x}{dt} = -m_P g \sin\alpha$$
$$\frac{dy}{dt} = v_y \quad m_P \frac{dv_y}{dt} = -m_P g \cos\alpha \tag{5}$$

where t is time, x, y are coordinates of COG of pedestrian, and v_x, v_y its velocity components. The equation is completed with the following initial conditions

$$x(0) = 0 \quad v_x(0) = v_{P0} \cos\theta$$
$$y(0) = h \quad v_y(0) = v_{P0} \sin\theta \tag{6}$$

Carrying out the integration and imposing initial conditions one finds velocity

$$v_x(t) = v_{P0} \cos\theta - g \sin\alpha \cdot t$$
$$v_y(t) = v_{P0} \sin\theta - g \cos\alpha \cdot t \tag{7}$$

and position coordinates

$$x(t) = v_{P0} \cos\theta \cdot t - g \sin\alpha \cdot \frac{t^2}{2}$$
$$y(t) = h + v_{P0} \sin\theta \cdot t - g \cos\alpha \cdot \frac{t^2}{2} \tag{8}$$

At time t_1, the time from launch to impact with the ground, the following conditions are reached: $y(t_1) = 0$ and $x(t_1) = s_1$. From these, by using (7) and (8), one obtains the flying time

$$t_1 = \frac{v_{P0} \sin\theta + \sqrt{v_{P0}^2 \sin^2\theta + 2gh\cos\alpha}}{g\cos\alpha} \tag{9}$$

and the flying distance

$$s_1 = v_{P0} \cos\theta \cdot t_1 + g \sin\alpha \cdot \frac{t_1^2}{2} \tag{10}$$

2.4 Impact with the ground

At pedestrian impact with the ground the Newtonian dynamical equations take the following impulse form

$$m(v_y^+ - v_y^-) = I_y \qquad m(v_x^+ - v_x^-) = -I_x \tag{11}$$

where superscripts - and + denote velocities before and after impact, and I_x and I_y are impulses in road horizontal and vertical direction, respectively. Here one needs further assumptions about the nature of impact. The simplest are:

- if the impact is plastic then $v_y^+ = 0$
- the Coulomb friction law: $I_x = \mu \cdot I_y$

On the basis of these assumptions one can from the first of (11) find impulse in vertical direction $I_y = -m \cdot v_y^-$ and from the second the horizontal velocity after impact $v_x^+ = v_x^- - I_x / m$. From these, by using friction law, one obtains $v_x^+ = v_x^- + \mu \cdot v_y^-$. By using (7) this becomes

$$v_x^+ = v_{P0}(\cos\theta + \mu\sin\theta) - g(\sin\alpha + \mu\cos\alpha)t_1 \tag{12}$$

2.5 Sliding phase

After impact with ground the pedestrian slides to rest. The dynamical equations governing this motion are

$$\frac{dx}{dt} = v_x$$
$$m\frac{dv_x}{dt} = -mg\sin\alpha - N_x \tag{13}$$
$$0 = -mg\cos\alpha + N_y$$

where N_x and N_y are horizontal and normal reaction of the ground, respectively, and the initial conditions are

$$x(0) = 0 \quad v_x(0) = v_x^+ \tag{14}$$

By assuming that the Coulomb friction law is valid: $N_x = \mu N_y$. After integration and imposing the initial conditions they obtain the velocity

$$v_x(t) = v_x^+ - g(\sin\alpha + \mu\cos\alpha)t \tag{15}$$

and the distance

$$x(t) = v_x^+ t - g(\sin\alpha + \mu\cos\alpha)\frac{t^2}{2} \tag{16}$$

At the end of pedestrian sliding one has $v_x(t_2) = 0$ and $x(t_2) = s_2$. From these, by using (15) and (16), the sliding time t_2 is

$$t_2 = \frac{v_x^+}{g(\sin\alpha + \mu\cos\alpha)} \tag{17}$$

and the sliding distance s_2 is

$$s_2 = \frac{(v_x^+)^2}{2g(\sin\alpha + \mu\cos\alpha)} \tag{18}$$

2.6 Total throw time and throw distance

The total time and distance can be obtained, after calculation, by summing the times and distance of flying and sliding phases. However, it turns out that a simple formula exists; by using (2) and by eliminating v_x^+ from (12) and (17) one obtains the total pedestrian throw time t_P

$$t_P = t_0 + \frac{v_{P0}(\cos\theta + \mu\sin\theta)}{g(\sin\alpha + \mu\cos\alpha)} \tag{19}$$

By introducing (10) and (18) into (1) and then from the resulting expression eliminating v_x^+ and t_1 by means of (12) and (9), one finds the total throw distance

$$s_P = s_0 + \mu h + \frac{v_{P0}^2(\cos\theta + \mu\sin\theta)^2}{2g(\sin\alpha + \mu\cos\alpha)} \tag{20}$$

	Analysis of Pedestrian Throw Distance		
CONSTANTS			
g	9,81	m/s2	acceleration of gravity
π	3,1416		
INPUT			
mc	1460	kg	mass of vehicle
mp	80	kg	mass of pedestrian
vc0	50	km/h	initial speed of vehicle
η	0,90		car-pedestrian launch factor
t0	0,20	s	contact time
s0	2,00	m	contact distance
h	1,00	m	height of pedestrian COG at launch
θ	16,00	deg	theta - angle of launch relative to x axis
	0,96		cos(theta)
	0,28		sin(theta)
p	8,00	%	road grade =tan(alpha)
	1,00		cos(alpha)
	0,08		sin(alpha)
μ	0,60		coefficient of friiction over sliding distance
OUTPUT			
v0	42,66	km/h	launch speed of pedestrian
	11,85	m/s	
t1	0,90	s	flying time
s1	9,90	m	flying distance
t0+t1	1,10	s	impact time
s0+s1	11,90	m	impact distance
vx	38,49	km/h	horizontal velocity at imapct with ground
vy	10,78	km/h	vertical velocity at impact with ground
vxp	7,39	m/s	initial horizontal velocity after impact with ground
	26,61	km/h	
t2	1,11	s	sliding time
s2	4,11	m	sliding distance
tp	2,21	s	total time
sp	16,00	m	total distance

Figure 2 - Spreadsheet program for analyzing vehicle-pedestrian collision

2.7 Analysis of vehicle-pedestrian collision

On the basis of the above formulas one can analyse the pedestrian motion from the contact with vehicle to rest on the ground if the following ten data are available

$$m_C, m_P, \mu, \alpha, v_{C0}, \eta, t_0, s_0, h, \theta$$

Among these the last five can only be estimated in practice. Nevertheless, since all the equations describing the collision are algebraic and explicit they can be easily implemented into a spreadsheet program. An example of this is shown in Figure 2. The data used in the example of Figure 2 will be used in all the following examples.

3. RECONSTRUCTION

In the reconstruction of pedestrian accidents one usually knows the throw distance and asks for launch velocity on which the vehicle impact speed can be estimated. Two cases will be considered: the case of known throw distance and the case where the flying distance is also known.

3.1 The known throw distance

If the pedestrian throw distance s_P is known, then from (20) one can express pedestrian launch velocity

$$v_{P0} = \frac{\sqrt{2g(\sin\alpha + \mu\cos\alpha)(s_P - s_0 - \mu h)}}{\cos\theta + \mu\sin\theta} \tag{21}$$

By denoting

$$p \equiv \tan\alpha \tag{22}$$

the formula (21) can be written in the following form

$$v_{P0} = \frac{\sqrt{2g(p+\mu)(1+\tan^2\theta)(s_P - s_0 - \mu h)}}{(1+\mu\tan\theta)\sqrt[4]{1+p^2}} \tag{23}$$

The confidence analysis of the formula is given in Appendix A. For $p = 0$ and $s_0 = 0$ one obtains the Searle formula ([15]). If the pedestrian launch velocity v_{P0} is known, the vehicle impact velocity is, from (4)

$$v_{C=} = (1 + m_P / m_C)\frac{v_{P0}}{\eta} \tag{24}$$

The formulas (23) and (24) can be used for reconstruction of vehicle-pedestrian collision if the following data are available

$$m_C, m_P, \eta, \mu, p, s_P, s_0, h, \theta$$

The spreadsheet program for reconstruction of vehicle-pedestrian collision on the basis of (23) and (24) is shown in Figure 3. Note that by taking the throw distance 16m and data from the previous example, the impact velocity of the vehicle is 50km/h, as it should be.

		Reconstruction of Pedestrian Throw Distance				
CONSTANTS						
g	9,81	m/s2	acceleration of gravity			
π	3,1416					
INPUT						
mc	1460	kg	mass of vehicle			
mp	80	kg	mass of pedestrian			
η	0,90		car-pedestrian launch factor			
s0	2,00	m	contact distance			
h	1,00	m	height of pedestrian COG at launch			
θ	16,00	deg	theta - angle of launch relative to x axis			
	0,96		cos(theta)			
	0,28		sin(theta)			
p	8,00	%	road grade =tan(alpha)			
	1,00		cos(alpha)			
	0,08		sin(alpha)			
μ	0,60		coefficient of friiction over sliding distance			
sp	16,00	m	total distance			
OUTPUT				SENSITIVITY		
v0	11,85	m/s	launch speed of pedestrian	h	-0,022	0,001
	42,66	km/h		sp	0,597	0,356
vc0	49,99	km/h	initial speed of vehicle	s0	-0,075	0,006
	46,77	km/h	min	p	0,056	0,003
	53,22	km/h	max	θ	-0,075	0,006
				μ	0,213	0,046
					in	out
				COV	0,1	0,065

Figure 3 - Spreadsheet program for reconstruction of a vehicle-pedestrian collision

Before proceeding, the effect of grade on pedestrian launch velocity will be discussed in some details. Let $v_{P0} = v_{P0}(\alpha, \theta)$. In the special case $\alpha = 0$ eq. (21) becomes

$$v_{P0}(0,\theta) = \frac{\sqrt{2g\mu(s_P - s_0 - \mu h)}}{\cos\theta + \mu\sin\theta} \qquad (25)$$

The quotient of (21) and (25) is

$$\frac{v_{P0}(\alpha,\theta)}{v_{P0}(0,\theta)} = \sqrt{\sin\alpha/\mu + \cos\alpha}$$

and is independent of θ. In order to study the analytical properties of this expression the following coefficient is introduced

$$u_\alpha(\mu, p) \equiv \sqrt{\frac{\mu + p}{\mu\sqrt{1+p^2}}} \qquad (26)$$

The diagram of (26) for various values of μ is shown in Figure 4.

Now it follows that $u_\alpha(\mu, 0) = 1$ and

$$\lim_{p\to\infty} u_\alpha(\mu, p) = \frac{1}{\sqrt{\mu}}.$$

If $\mu \le 1$, then the launch velocity for reaching the distance s_P is greater than that of horizontal ground in the case $p > 0$ (uphill launching). For $\mu > 1$ this is true only for grades satisfying

$$p \le \frac{2\mu}{\mu^2 - 1}.$$

But since the grades of the roads are limited to approximately $p < 0.3$ and the friction coefficient is limited approximately to $\mu < 2$ this limit can practically never be reached. In the case $p < 0$ (downhill launching) one should have $\mu > -p$ in order for the pedestrian to attain the rest position. In this case the launch velocity for reaching the distance s_P is lesser than that of horizontal ground.

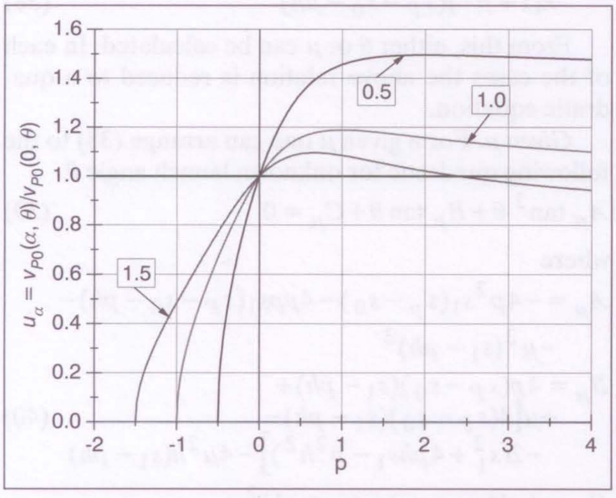

Figure 4 - Effect of grade on normalized launch velocity for various values of μ

3.2 Known throw distance - Searle method

The difference between grade p of the road and the angle θ in Eq (23) is practical: while the grade of the road can be measured, the launch angle θ can only be estimated. Here it was Searle's idea that the formula (23) can be used to determine lower and upper boundaries, by considering the launch angle θ that will minimize and maximize the expression (see [15]). Therefore, the effect of launch angle on pedestrian launch velocity will now be considered.

If the launch is horizontal then $\theta = 0$ and (21) reduces to

$$v_{P0}(\alpha, 0) = \sqrt{2g(\sin\alpha + \mu\cos\alpha)(s_P - s_0 - \mu h)} \qquad (27)$$

The quotient

$$\frac{v_{P0}(\alpha,\theta)}{v_{P0}(alpha,0)} = \frac{1}{\cos\theta + \mu\sin\theta}$$

is independent of grade angle α so one can introduce the coefficient

$$u_\theta(\mu,\theta) \equiv \frac{1}{\cos\theta + \mu\sin\theta} \qquad (28)$$

The graph of this function for various values of μ is shown in Figure 5, where for practical reason θ is in degrees.

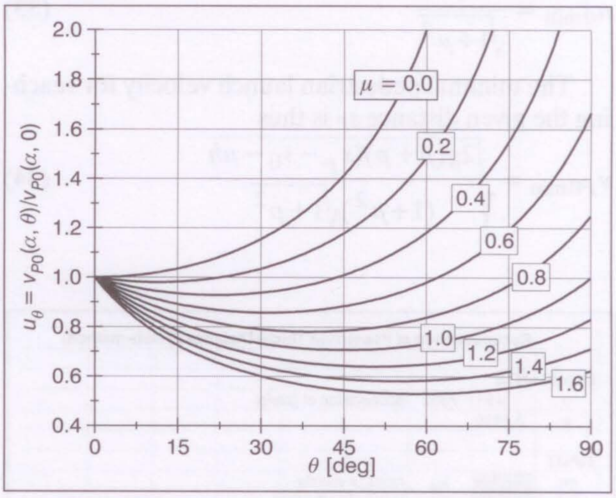

Figure 5 - Velocity coefficient as a function of launch angle for various values of friction coefficients

Since the launch angle is limited to

$$0 \le \theta < \frac{\pi}{2}$$

the two limits of the functions are

$$u_\theta(\mu, 0) = 1 \text{ and } u_\theta\left(\mu, \frac{\pi}{2}\right) = \frac{1}{\mu}.$$

First the maximal pedestrian launch velocity for reaching the distance s_P will be considered. If $\mu > 1$, then it follows from (28) that $u_\theta(\mu,\theta) < 1$. The function in this case reaches its maximum at $\theta = 0$. For $\mu \le 1$ one sees from Figure 5 that $u_\theta(\mu, 0) =$

$= u_\theta(\mu, \theta_*) = 1$ where θ_* is the critical launch angle ([15]) given by

$$\theta_* = \arctan \frac{2\mu}{1-\mu^2} \tag{29}$$

In both cases the maximal pedestrian launch velocity is given by expression

$$v_{P0\max} = \sqrt{2g \frac{(\mu+p)}{\sqrt{1+p^2}} (s_P - s_0 - \mu h)}$$

$$(\mu > 1 \text{ or } \mu \leq 1 \text{ and } \theta \leq \theta_*) \tag{30}$$

If $\theta > \theta_*$ then the maximal pedestrian launch velocity is obtained in limit $\theta \to \pi/2$ with

$$v_{P0\max} < \frac{1}{\mu} \sqrt{2g \frac{(\mu+p)}{\sqrt{1+p^2}} (s_P - s_0 - \mu h)}$$

$$(\mu \leq 1 \text{ and } \theta > \theta_*) \tag{31}$$

Now, the minimal pedestrian launch velocity is obtained from the necessary condition

$$\frac{du_\theta}{d\theta} = 0,$$

which gives ([15])

$$\tan\theta = \mu \tag{32}$$

For this value of angle (28) has the minimal value

$$u_{\theta\min} = \frac{1}{\sqrt{1+\mu^2}} \tag{33}$$

The minimal pedestrian launch velocity for reaching the given distance s_P is thus

$$v_{P0\min} = \sqrt{\frac{2g(\mu+p)(s_P - s_0 - \mu h)}{(1+\mu^2)\sqrt{1+p^2}}} \tag{34}$$

	Reconstruction of Pedestrian Throw Distance (Searle method)		
CONSTANTS			
g	9,81	m/s2	acceleration of gravity
π	3,1416		
INPUT			
mc	1460	kg	mass of vehicle
mp	80	kg	mass of pedestrian
η	0,90		car-pedestrian launch factor
s0	2,00	m	contact distance
h	1,00	m	height of pedestrian COG at launch
p	8,00	%	road grade =tan(alpha)
	1,00		cos(alpha)
	0,08		sin(alpha)
μ	0,60		coefficient of friiction over sliding distance
sp	16,00	m	total distance
OUTPUT			
θ	30,96	deg	angle of launch relative to x axis
θ*	61,93	deg	critical launch angle
v0min	11,45	m/s	launch speed of pedestrian
	41,21	km/h	
vc0min	48,30	km/h	initial speed of vehicle
v0max	13,35	m/s	launch speed of pedestrian
	48,06	km/h	
vc0max	56,32	km/h	initial speed of vehicle

Figure 6 - Spreadsheet program for reconstruction of a vehicle-pedestrian collision by the Searle method.

The above formulas generalize the Searle formulas ([15]) in a way that they include road grade p and contact distance s_0. They provide means to estimate velocity bounds if throw distance is known. The spreadsheet program for reconstruction of vehicle-pedestrian collision based on these formulas is shown by Figure 6. Note that if one considers the example in Figure 2 as reference, the Searle method estimates the impact velocity between 48.3km/h to 56.3km/h, while the 'true' value is 50km/h.

3.3 Known flying and throw distance

In the case when besides throw distance the flying distance of the pedestrian is also known, from physical evidence for example, we can calculate both launch angle and launch velocity. If s_* is impact to ground contact distance which is measured from the first contact of vehicle and pedestrian, then the flying distance is

$$s_1 = s_* - s_0 \tag{35}$$

So when s_1 is known from (8) and (9), for $y(t_1) = 0$ and $x(t_1) = s_1$, one can express the pedestrian launch velocity

$$v_{P0} = (s_1 + ph)\sqrt{\frac{g(1+\tan^2\theta)}{2(s_1\tan\theta + h)(1 - p\tan\theta)\sqrt{1+p^2}}} \tag{36}$$

and the flying time

$$t_1 = \frac{s_1 \sin\theta + h\cos\theta}{v_{P0}(\cos\alpha\cos\theta - \sin\alpha\sin\theta)} \tag{37}$$

Since the pedestrian launch velocity is also given by (21) one can find the compatibility equation by (36) and (21)

$$\begin{aligned}(s_1 p + h)^2 (1+\mu\tan\theta)^2 &= \\ &= 4(p+\mu)(s_1\tan\theta + h)(1 - p\tan\theta) \times \\ &\times (1+p^2)(s_P - s_0 - \mu h)\end{aligned} \tag{38}$$

From this, either θ or μ can be calculated. In each of the cases the above relation is reduced to a quadratic equation.

Given μ. For a given μ one can arrange (38) to the following quadratic for unknown launch angle θ

$$A_\mu \tan^2\theta + B_\mu \tan\theta + C_\mu = 0 \tag{39}$$

where

$$\begin{aligned} A_\mu &= -4p^2 s_1(s_P - s_0) - 4\mu p s_1(s_P - s_0 - ph) - \\ &\quad -\mu^2(s_1 - ph)^2 \\ B_\mu &= 4p(s_P - s_0)(s_1 - ph) + \\ &\quad +\mu\left[4(s_P - s_0)(s_1 - ph) - \right. \\ &\quad \left. -2(s_1^2 + 4phs_1 - p^2h^2)\right] - 4\mu^2 h(s_1 - ph) \\ C_\mu &= 4ph(s_P - s_0) - (s_1 + ph)^2 + \\ &\quad +4\mu h(s_P - s_0 - ph) - 4\mu^2 h^2 \end{aligned} \tag{40}$$

	Reconstruction of Pedestrian Throw Distance				
	(known flying distance)				
CONSTANTS					
g	9.81	m/s2	acceleration of gravity		
π	3.1416				
INPUT					
mc	1460	kg	mass of vehicle		
mp	80	kg	mass of pedestrian		
η	0.90		car-pedestrian launch factor		
s0	2.00	m	contact distance		
h	1.00	m	height of pedestrian COG at launch		
p	8.00	%	road grade =tan(alpha)		
	0.08		p/100		
μ	0.60		coefficient of friiction over sliding distance		
s0+s1	11.90	m	distance to impact with ground		
sp	16.00	m	total distance		
OUTPUT					
s1	9.90	m	flying distance		
θ	16.03	deg	launch angle	Solve quadratic	
v0	11.85	m/s	launch speed of pedestrian	A	-64.72
	42.65	km/h		B	238.4
vc0	49.99	km/h	initial speed of vehicle	C	-63.15
				D	40484
				tan(th)	0.2873

Figure 7 - Results of calculation of vehicle-pedestrian collision. Known impact to ground contact distance with estimated μ

The spreadsheet program for reconstruction of vehicle-pedestrian collision based on equation (39) is shown in Figure 7.

Given θ. On the other hand, for given θ one can arrange (38) to the quadratic equation for an unknown μ

$$A_\theta \mu^2 + B_\theta \mu + C_\theta = 0 \tag{41}$$

where

$$A_\theta = 4h(s_1 \tan\theta + h)(1 - p\tan\theta)(1+p^2) + $$
$$+ (s_1 p + h)^2 \tan^2\theta$$
$$B_\theta = -4(s_1 \tan\theta + h)(1 - p\tan\theta)(1+p^2)(s_P - s_0) +$$
$$+4ph(1-p\tan\theta)(s_1\tan\theta + h)(1+p^2)h + \tag{42}$$
$$2(s_1 p + h)^2 \tan\theta$$
$$C_\theta = -4p(1+p^2)(1-p\tan\theta)(s_1\tan\theta + h)$$
$$(s_P - s_0) + (s_1 p + h)^2$$

Since the launch angle θ can practically only be estimated, the case is interesting for the extremes, which give maximum and minimum value of launch velocity. The first maximum value is forward projection. So, if $\theta = 0$ then from (41)

$$\mu = \frac{s_P - s_0 - ph}{2h} \pm$$
$$\pm \frac{\sqrt{(s_P - s_1 - s_0)(s_P + s_1 - s_0 + 2ph)}}{2h} \tag{43}$$

This formula has h, which is usually small compared to s_P, in denominator, so one can expect that it is very sensitive to its variation. The other extreme for maximal projection velocity is $\theta = \pi/2$. In this case (41) reduces to

$$(s_1 - ph)^2 \mu^2 + 4ps_1(s_P - hp)\mu + 4p^2 s_P s_1 = 0 \tag{44}$$

However, this equation does not have any positive roots for $p > 0$.

Now by taking $\theta = \mu$ for minimum launch velocity then the quadratic equation (41) transforms to the quadratic equation of the form

$$a_0\mu^4 + a_1\mu^3 + a_2\mu^2 + a_3\mu + a_4 = 0 \tag{45}$$

where

$$a_0 = (s_1 - hp)^2$$
$$a_1 = 4h\left[(1-p^2)s_1 - ph\right] + 4ps_1(s_P - s_0)$$
$$a_2 = 4h\left[(1-p^2)s_1 + ph\right] + 2s_1(s_1 + 4ph) +$$

	Reconstruction of Pedestrian Throw Distance							
	(known flying distance - Searle method)							
CONSTANTS								
g	9.81	m/s2	acceleration of gravity					
π	3.14159							
INPUT								
mc	1460	kg	mass of vehicle					
mp	80	kg	mass of pedestrian					
η	0.90		car-pedestrian launch factor					
s0	2.00	m	contact distance					
h	1.00	m	height of pedestrian COG at launch					
p	8.00	%	road grade =tan(alpha)					
	0.08		p/100					
s0+s1	11.90	m	distance to impact with ground					
sp	16.00	m	total distance					
OUTPUT				Solve quartic equation by iteration				
s1	9.90	m	flying distance					
μ	0.43		coefficient of friction over sliding dist.	a0	a1	a2	a3	a4
θ	23.16	deg	launch angle	96.432	83.379	-340	-99.67	95.12
v0min	10.67	m/s	launch speed of pedestrian	1	0.8646	-3.526	-1.034	0.9864
	38.43	km/h						
vc0min	45.04	km/h	initial speed of vehicle		x	f(x)	f'(x)	
vc0max	13.61	m/s		0	0.5	-0.241	-3.411	
	48.99	km/h		1	0.4292	-0.005	-3.266	
μ	1.99		coefficient of friction over sliding dist.	2	0.4278	-3E-06	-3.263	
v0max	22.07	m/s	launch speed of pedestrian	3	0.4278	-9E-13	-3.263	
	79.44	km/h		4	0.4278	0	-3.263	
vc0max	93.1076	km/h	initial speed of vehicle	5	0.4278	0	-3.263	

Figure 8 - Results of calculation of vehicle-pedestrian collision. Searle method with known flying distance.

$$+2(2-p^2)h^2 \qquad (46)$$

$$a_3 = -4h\left[ps_1 + (1-p^2)h\right](s_P - s_0) + 4ph^2$$

$$a_4 = (s_1 + hp)^2 - 4ph(s_P - s_0)$$

The roots of this equation can be calculated numerically by the Newton iteration method. The example of the spreadsheet program implementing the above formulas is shown in Figure 8. The estimated boundary for data from Figure 1 is 45km/h to 49km/h. The first maximal value of vehicle impact velocity is calculated by using (30) and calculated μ for minimum value of vehicle impact velocity. The horizontal launch based on (43) gives the unrealistic friction coefficient 1.99 which leads to the upper velocity limit 93km/h. Here one can recommend that in general the maximal value of vehicle impact velocity based on (43) can be used in the cases when $\mu < 1$.

4. VERIFICATION

4.1 Comportment with field data

The first comportment is made for test data given in [4] (Table 1). The test was performed by a vehicle of a mass of 1542kg. Since the mass of the pedestrian was not reported, a value of 80kg was taken into calculation. The results of the calculation using the Searle method are shown in Table 1. Note the huge discrepancy of the calculated and the measured vehicle impact velocity for Test No. 69, performed by a non-braking vehicle. All other results are within acceptable limits of 15%.

Table 1 - Results of calculation on test data [4]. Searle method. (s_0 = 2m, η = 0.95, η = 1.1m, μ = 0.6) *Brake off

Test No.	v_{C0} [km/h]	Input s_P [m]	Output $v_{C0\min}$ [km/h]	$v_{C0\max}$ [km/h]	$\bar{v}_{C0}$ [km/h]	Err [%]
65	32.2	7.9	27.96	32.61	30.29	5.95
67	48.3	13.4	38.27	44.63	41.45	14.18
68	48.3	17.4	44.28	51.64	47.96	0.70
69*	64.4	61.9	86.51	100.89	93.70	-45.50
72	48.3	20.7	48.69	56.78	52.74	-9.18

For verification of the proposed reconstruction method also the data from [12] were used (Table 2). Besides the pedestrian throw distance these data also include the impact to ground distance. The lack of data from a real pedestrian accident means that there is no mass of vehicle and pedestrian included in the report and that the vehicle impact velocity is below 36km/h (Table 2). The mass of 1500kg for the vehicle and 80kg for the pedestrian was thus assumed in the calculation. The results of comportment are shown in Table 2. It can be seen from the table that in trial case 1 the calculated velocity overestimates the impact velocity by about 25%, and in case 3 the calculated values underestimate the velocity by about 50%. In the other four cases the calculated impact velocity is between the calculated limits within an error maximum of 13%.

4.2 Comportment with full scale numerical simulation

The full scale numerical simulation of vehicle-pedestrian collision was done by the PC-Crash 7.1 computer program. The following data were used:

- vehicle bumper height: 0.5m;
- vehicle front height: 0.8m;
- distance from vehicle front to windshield: 1.02m;
- vehicle mass: 1460 kg;
- coefficient of tire-road friction : 0.8;
- coefficient of car-pedestrian friction: 0.2;
- coefficient of road-pedestrian friction (μ): 0.6;
- coefficient of restitution for pedestrian impact: 0.1.

The first numerical experiment was conducted for a pedestrian of mass 80kg and height 1.83m. The path and speed of COG of pedestrian torso for various im-

Table 2 - Comportment with field data [12]. Values marked with * are not included in calculation of $v_{C0\max}$. (h = 1, s_0 = h = 1m)

Trail No.	v_{C0} [km/h]	Input s_* [m]	s_P [m]	$\mu_{\min}$	$\mu_{\max}$	Output $v_{C0\min}$ [km/h]	$v_{C0\max}$ [km/h]	$\bar{v}_{C0}$ [km/h]	Err [%]
1	26.99	8.23	10.38	0.50	1.70*	31.67	35.38	33.53	24.21
3	25.13	2.59	5.42	0.13	0.15	12.30	13.37	12.84	48.93
4	20.19	3.53	7.41	0.19	0.26	17.83	21.28	19.56	3.15
10	20.28	3.65	4.73	0.34	0.55	17.08	22.29	19.69	2.93
11	27.85	5.52	8.52	0.35	0.76	25.09	38.01	31.55	13.29
14	32.13	8.24	8.51	0.71	2.76*	30.13	36.96	33.55	4.40

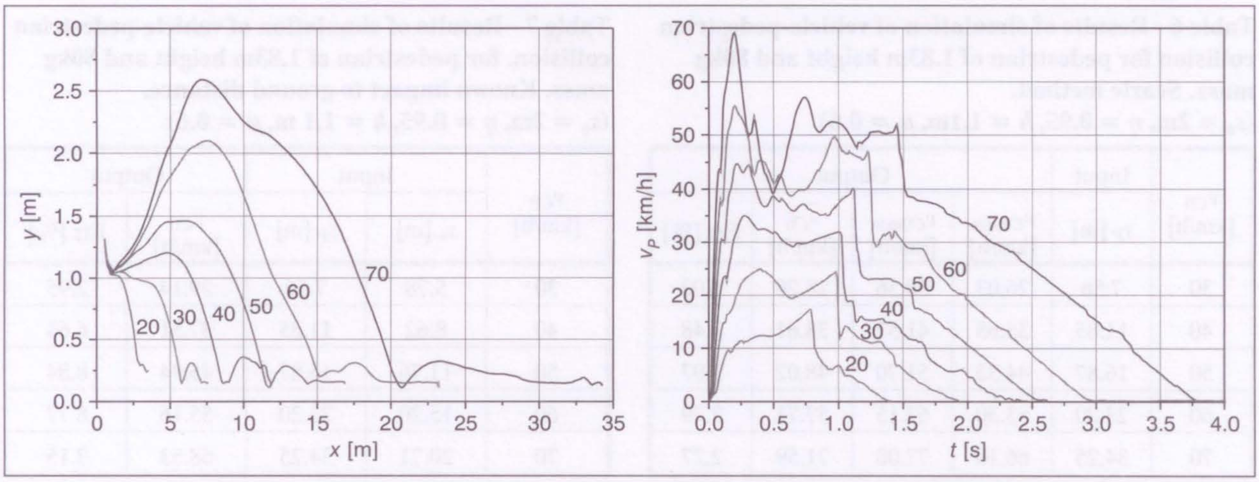

Figure 9 - Path and time dependence of velocity for pedestrian torso COG for various vehicle impact speeds. Pedestrian height is 1.83 m and mass 80 kg. (scales of x and y are different)

Table 3 - Results of simulation of vehicle-pedestrian collision with PC-Crash for pedestrian of 1.83m height and 80 kg mass

v_{C0} [km/h]	v_{P0} [km/h]	θ [°]	t_0 [s]	s_0 [m]	h [m]	t_* [m]	s_* [m]	s_P [m]	η
20	17.30	-44.16	0.79	2.72	0.39	-	-	-	0.92
30	24.96	-6.10	0.40	2.31	1.01	1.00	5.78	7.56	0.87
40	34.99	3.52	0.27	2.07	1.08	1.05	8.62	11.85	0.92
50	44.77	7.42	0.22	2.14	1.15	1.16	11.76	16.87	0.95
60	55.46	9.88	0.20	2.37	1.20	1.26	15.20	23.20	0.97
70	69.08	14.16	0.20	2.88	1.35	1.51	20.71	34.25	1.04

Table 4 - Results of calculation of vehicle-pedestrian collision with present method for pedestrian of 1.83m height and 80kg mass

Input					Output				
v_{C0} [km/h]	η	θ [°]	t_0 [s]	s_0 [m]	h [m]	v_{P0} [km/h]	t_* [m]	s_* [m]	s_P [m]
20	0.92	-44.16	0.79	2.72	0.39				
30	0.87	-6.10	0.40	2.31	1.01	24.47	0.79	4.94	6.39
40	0.92	3.52	0.27	2.07	1.08	34.89	0.80	7.23	11.26
50	0.95	7.42	0.22	2.14	1.15	45.03	0.9	10.53	18.02
60	0.97	9.88	0.20	2.37	1.20	55.18	1.03	14.91	26.72
70	1.04	14.16	0.20	2.88	1.35	69.02	1.39	24.96	42.60

pact speeds are displayed in Figure 9 and numerical values are given in Table 2.

For the comportment of results of numerical simulation with PC-Crash and the present model the calculated values given in Table 3 are taken as input to the present spreadsheet programs. The results of calculations are shown in Tables 4, 5, 6, 7 and 8. The results of analysis of throw distance are within 20%. Among reconstruction methods the Searle method is dominant with maximum error in impact vehicle velocity of about 6% (Table 6). When impact to ground distance is included in input data the error rises to about 9%

Table 5 - Comportment of results from Tables 3 and 4. Relative error in % for various variables

v_{P0}	t_*	s_*	s_P
1.96	21.00	14.53	15.48
0.29	23.81	16.13	4.98
0.58	22.41	10.46	6.82
0.50	18.25	1.91	15.17
0.09	7.95	20.52	24.38

Table 6 - Results of simulation of vehicle-pedestrian collision for pedestrian of 1.83m height and 80kg mass. Searle method.
($s_0 = 2$m, $\eta = 0.95$, $h = 1.1$m, $\mu = 0.6$)

v_{C0} [km/h]	Input	Output			
	s_P [m]	$v_{C0\min}$ [km/h]	$v_{C0\max}$ [km/h]	$\bar{v}_{C0}$ [km/h]	Err [%]
30	7.56	26.03	30.36	28.20	6.02
40	11.85	35.65	41.57	38.61	3.48
50	16.87	44.33	51.70	48.02	3.97
60	23.20	53.30	62.15	57.73	3.79
70	34.25	66.10	77.08	71.59	2.27

Table 7 - Results of simulation of vehicle-pedestrian collision. for pedestrian of 1.83m height and 80kg mass. Known impact to ground distance.
($s_0 = 2$m, $\eta = 0.95$, $h = 1.1$ m, $\mu = 0.6$)

v_{C0} [km/h]	Input		Output	
	s_* [m]	s_P [m]	v_{C0} [km/h]	Err [%]
30	5.78	7.56	29.14	2.95
40	8.62	11.85	37.51	6.64
50	11.76	16.87	45.94	8.84
60	15.20	23.20	55.16	8.77
70	20.71	34.25	68.53	7.15

(Table 7). When using the Searle method with known impact to ground distance the error rises to about 20% (Table 8).

The second numerical experiment was conducted for vehicle impact velocity 60km/h with various pedestrians. The following pedestrian height/mass pairs were used for calculation: 1.2/25, 1.4/40, 1.6/60, 1.83/80. The results of the calculations are shown in Figure 10 and results of calculation are given in Table 9. Note that the pedestrian of 1.25m height is subject to forward projection while all the others follow wrap trajectory. The comportment with results of calculation shows similar discrepancies as in the previous case (Table 10 and 11).

Table 8 - Results of simulation of vehicle-pedestrian collision for pedestrian of 1.83m height and 80kg mass. Searle method. ($s_0 = 2$m, $\eta = 0.95$, $h = 1.1$m)

v_{C0} [km/h]	Input		Output				
	s_* [m]	s_P [m]	μ	$v_{C0\min}$ [km/h]	$v_{C0\max}$ [km/h]	$\bar{v}_{C0}$ [km/h]	Err [%]
30	5.78	7.56	0.37	22.95	24.48	23.715	20.95
40	8.62	11.85	0.42	32.32	35.02	33.67	15.83
50	11.76	16.87	0.44	40.70	44.42	42.56	14.88
60	15.20	23.20	0.43	48.73	53.13	50.93	15.12
70	20.71	34.25	0.42	59.85	65.01	62.43	10.81

Table 9 - Results of simulation of vehicle-pedestrian collision with PC-Crash for vehicle collision velocity 60km/h for various pedestrians heights h_P

h_P [m]	v_{P0} [km/h]	θ [°]	t_0 [s]	s_0 [m]	h [m]	t_* [m]	s_* [m]	s_P [m]	η
1.25	70.39	12.94	0.22	3.74	1.49	1.34	21.96	36.30	1.19
1.40	63.38	8.74	0.21	3.18	1.24	1.18	17.24	30.55	1.09
1.60	57.81	12.15	0.20	2.60	1.27	1.33	16.76	25.61	1.00
1.83	55.46	9.88	0.20	2.37	1.20	1.26	15.20	23.20	0.97

Table 10 - Results of simulation of vehicle-pedestrian collision with present method for various pedestrians at 60km/h

h_P [m]	Input					Output			
	h	θ [°]	t_0 [s]	s_0 [m]	h [m]	v_{P0} [km/h]	t_* [m]	s_* [m]	s_P [m]
1.25	1.19	12.94	0.22	3.74	1.49	70.2	1.37	25.66	44.36
1.40	1.09	8.74	0.21	3.18	1.24	63.66	0.85	17.97	34.88
1.60	1.00	12.15	0.20	2.60	1.27	57.63	1.16	17.58	29.89
1.83	0.97	9.88	0.20	2.37	1.20	55.90	1.04	15.16	27.34

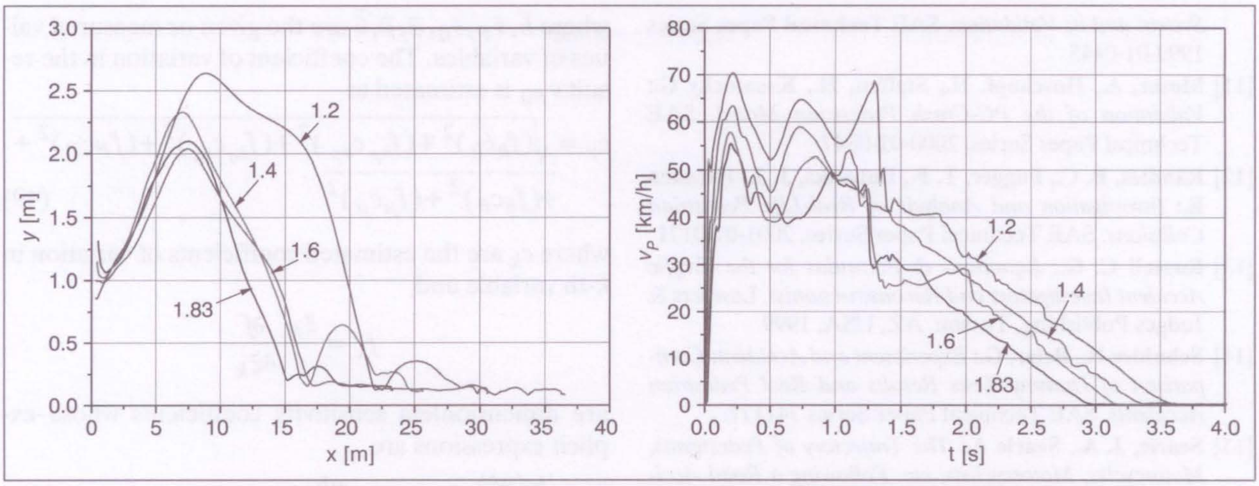

Figure 10 - Path and speed of COG of pedestrian torso for various heights of the pedestrian. Vehicle impact speed 60km/h.

Table 11 - Comportment of results between Tables 9 and 10. Relative error in % on various variables

v_{P0}	t_*	s_*	s_P
0.27	2.24	16.85	22.20
0.44	27.97	4.23	14.17
0.31	12.78	4.89	16.71
0.79	17.46	0.26	17.84

5. CONCLUSION

The vehicle-pedestrian collision is a complicated event which cannot be exactly modelled. The simple model presented in this paper is comparable with the empirical models and tested full scale model within an error of about 20%. The model has the advantage over empirical models when the road has a grade or when the pedestrian impact to ground contact distance is available as data. The model can therefore be used for reconstruction purposes; however, one should be aware of its limitations and accuracy. All the present spreadsheet programs are available from

www.fpp.edu/~milanb/pedestrian

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POVZETEK

PREPROST MODEL ODBOJA PEŠCA PRI ČELNEM NALETU VOZILA

V članku je podan preprost teoretični model odboja pešca pri čelnem naletu vozila. Model temelji na predpostavki, da se da gibanje pešca po trku opisati kot gibanje masne točke. Obravnavana sta dva primera rekonstrukcije tovrstnega trka: pri prvem je znana samo celotna dolžina odboja pešca, pri drugem pa je poleg tega znana tudi dolžina leta pešca. Uporabnost model je verificirana s podatki, ki so dostopni v literature in s primerjavo z rezultati celovite tridimenzionalne numerične simulacije trka.

KLJUČNE BESEDE

prometne nesreče, trk vozila in pešca, Searlova enačba

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APPENDIX A - Confidence analysis for Eq. (21)

To estimate the errors in parameters that influence the pedestrian launch velocity Eq. (21) is viewed as a function of six variables

$$v_{P0} = f(h, s_P, s_0, p, \mu, \theta) \tag{47}$$

The expected value of this function is

$$\bar{v}_{P0} = f(\bar{h}, \bar{s}_P, \bar{s}_0, \bar{\alpha}, \bar{\mu}, \bar{\theta}) \tag{48}$$

where $\bar{h}, \bar{s}_P, \bar{s}_0, \bar{\alpha}, \bar{\mu}, \bar{\theta}$ are the given or measured values of variables. The coefficient of variation in the result v_{P0} is estimated as

$$c_v = \sqrt{(f_h c_h)^2 + (f_{s_P} c_{s_P})^2 + (f_{s_0} c_{s_0})^2 + (f_P c_P)^2 + (f_\theta c_\theta)^2 + (f_\mu c_\mu)^2} \tag{49}$$

where c_k are the estimated coefficients of variation in k-th variable and

$$f_k = \frac{\xi_k}{f}\frac{\partial f}{\partial \xi_k}$$

are dimensionless sensitivity coefficients whose explicit expressions are

$$f_h = \frac{h}{f}\left(\frac{\partial f}{\partial h}\right) = \frac{\mu h}{2(s_P - s_0 - \mu h)} \tag{50}$$

$$f_{s_P} = \frac{s_P}{f}\left(\frac{\partial f}{\partial s_P}\right) = \frac{s_P}{2(s_P - s_0 - \mu h)} \tag{51}$$

$$f_{s_0} = \frac{s_0}{f}\left(\frac{\partial f}{\partial s_0}\right) = -\frac{s_0}{2(s_P - s_0 - \mu h)} \tag{52}$$

$$f_p = \frac{p}{f}\left(\frac{\partial f}{\partial p}\right) = \frac{p}{2}\frac{1 - \mu p}{(1+p^2)(p+\mu)} \tag{53}$$

$$f_\theta = \frac{\theta}{f}\left(\frac{\partial f}{\partial \theta}\right) = \theta\frac{\tan\theta - \mu}{1 + \mu\tan\theta} \tag{54}$$

$$f_\mu = \frac{\mu}{f}\left(\frac{\partial f}{\partial \mu}\right) = \frac{\mu}{2}\frac{[s_P - s_0 - (2\mu + p)h] - [(2\mu + p)(s_P - s_0) - p\mu h]\tan\theta}{(\mu + p)(s_P - s_0 - \mu h)(1 + \mu\tan\theta)} \tag{55}$$