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TRANSPORTATION COST ASSESSMENT BY MEANS OF A MONTE CARLO SIMULATION IN A TRANSSHIPMENT MODEL

ABSTRACT

The task of transport management is to organize the transport of goods from a number of sources to a number of destinations with minimum total costs. The basic transportation model assumes direct transport of goods from a source to a destination with constant unit transportation costs. In practice, however, goods are frequently transported through several transient points where they need to be transshipped. In such circumstances transport planning and organization become increasingly complex. This is especially noticeable in water transport. Most of the issues are directly connected to port operations, as they are the transshipment hubs. Since transportation is under a number of influences, in today's turbulent operating conditions the assumption on fixed unit transportation costs cannot be taken as realistic. In order to improve decision making in the transportation domain, this paper will present a stochastic transshipment model in which cost estimate is based on Monte Carlo simulation. Simulated values of unit costs are used to devise an adequate linear programming model, the solving of which determines the values of total minimum transportation costs. After repeating the simulation for a sufficient number of times, the distribution of total minimum costs can be formed, which is the basis for the pertinent confidence interval estimation. It follows that the design, testing and application of the presented model requires a combination of quantitative optimization methods, simulation and elements of inferential statistics, all with the support of computer and adequate software.

KEY WORDS

transportation costs, stochastic transshipment model, Monte Carlo simulation, quantitative methods, confidence interval, t-distribution, computer analysis

1. INTRODUCTION

The advancement of means of transport and traffic infrastructure has a significant influence on overall social and economic developments, allowing companies to improve their operations to a large degree. Among other possibilities, they can increase their profitability by putting their products on the markets that have become more accessible after road construction, or by purchasing raw materials and other items needed for the production under more favourable conditions. At the same time, with new traffic connections companies are beginning to face new challenges, since they are exposed to more and tougher competition. In such circumstances, company management needs to pay particular attention to organizing an adequate transportation system.

An intuitive approach to devising a transportation plan can yield satisfactory results only when solving quite simple distribution problems of routine character. Taking into account that a non-optimum transport organization significantly increases operating costs, trial-and-error method is unjustified in more complex situations. Successful transport management therefore strongly depends on using adequate quantitative methods, aimed at resolving problems in transportation and improving the decision making process.

In its basic form, the transportation model is a special case of linear programming. Initial model assumptions can rarely be taken as realistic in practice. In many cases the goods have to be transshipped at certain transient points, which is particularly noticeable in water transport. In this type of transport, the goods have to be unloaded after arrival to the port, and then transshipped onto railway wagons or trucks. Since transportation is under a number of influences, in today's turbulent operating conditions the assumption on fixed unit transportation costs is also questionable. The increase of unit costs can, among other things, be a consequence of waiting at the transshipment point; it can also be a consequence of including the losses arising from delivery delays into transportation costs. On the other hand, unit transportation costs can be reduced owing to the discount given for transporting larger quantities of a product. In recent years, fluctuating prices of fuel have played an important role in setting transportation costs.

In the conditions described above, transport planning and organization require the usage of more complex model constructions. With this aim in mind, in this paper we have formulated a stochastic transshipment model in which cost estimate is based on the Monte Carlo simulation. Its application can be based on the data obtained through managers' estimates, or on real data about unit costs arising from goods transportation. Simulated values of unit costs are then used to set up an adequate linear programming model, which is solved by the simplex method. In this way, the values of total minimum transportation costs are determined. In the model presented here, repeating the simulation process and solving the model allows us to form a distribution of total minimum costs. Based on such distribution, we can finally establish the interval estimation of total minimum costs, which is an important information basis for making business decisions.

The importance of using the computer and adequate software for this model is particularly emphasized. Their usage has significantly simplified the procedure of generating random values of unit costs from the given interval in the Monte Carlo simulation. In addition, computer analysis has facilitated the process of determining the solution of total minimum transportation costs, as well as establishing the associated interval estimation.

2. TRANSPORTATION MODEL AND TRANSSHIPMENT MODEL

In the transportation model (D. R. Anderson, D. J. Sweeney, T. A. Williams [1], F. S. Hillier, G. J. Lieberman [4], H. A. Taha [8]) it is assumed that a homogeneous quantity of goods needs to be transported from m shipping points to n destinations at minimum total costs. There is also a requirement for the sending location i that total available quantity of goods a_i be sent to places which expressed the need for such goods. It is also assumed for the receiving point j that it should get the demanded quantity of goods b_j, regardless of the origin they come from. Unit transportation costs from the source i to the destination j are constant in the initial model and amount to c_{ij}. The quantity of goods transported in the model from the source i to the destination j is marked by x_{ij}. A precondition for solving the transportation problem is equality of the total quantity offered and demanded. In case when this condition is met we speak of the balanced transportation model. If the stated equality condition is not fulfilled, the model has to be supplemented with a fictitious shipping or receiving point with the offered or demanded quantity that is missing.

Figure 1 shows the transportation model as a network with m sources and n destinations. In this figure, sending and receiving locations are represented by nodes. By multiplying unit costs and quantities transported from the source i to the destination j we get transportation costs for that route.

The described transportation problem can be formulated as a linear programming model with the general form:

$$\text{minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \ i = 1,2,\ldots,m$$

$$\sum_{j=1}^{m} x_{ij} = b_j, \ j = 1,2,\ldots,n$$

$$x_{ij} \geq 0, \ i = 1,2,\ldots,m, \ j = 1,2,\ldots,n$$

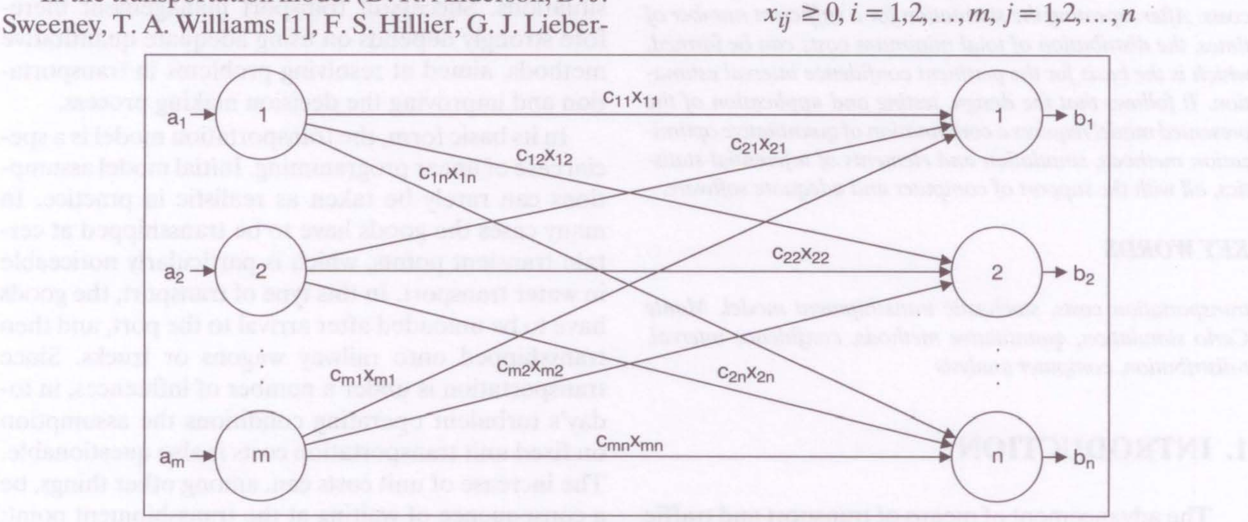

Figure 1 - Network diagram of the transportation model

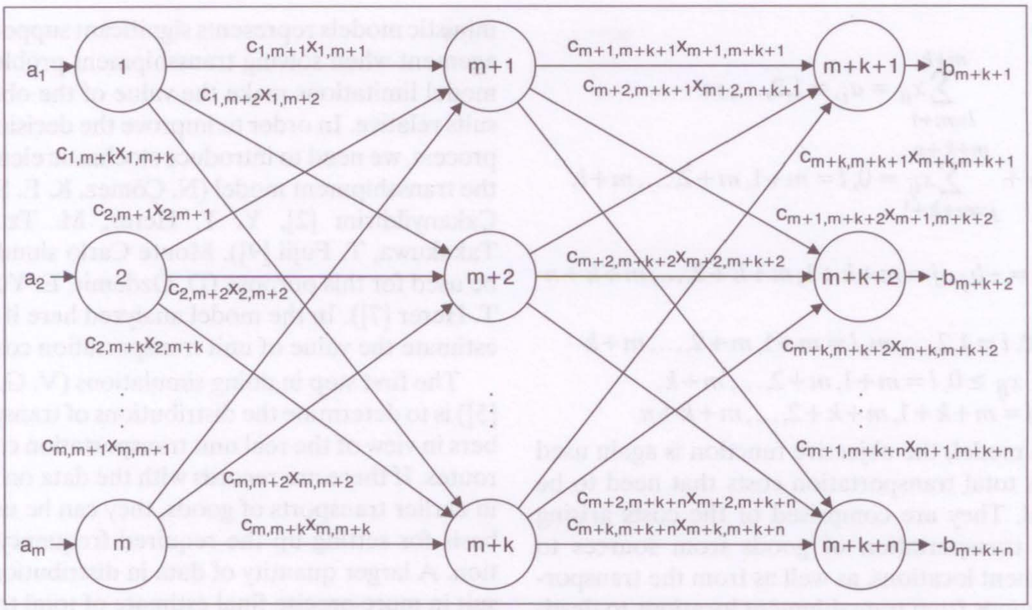

Figure 2 - Network diagram of the transshipment model

In the stated model the objective function is used to express total transportation costs that need to be minimized. The first constraint reflects the requirement that the total quantity offered has to be dispatched from shipping points, whereas the other requires that total quantity demanded has to be delivered to receiving points. For all quantities being transported the condition of non-negativity is valid. Since the sum of all the offered quantities has to be equal to the sum of demanded quantities it can be written:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

The stated mathematical formulation of the transportation problem is valid for the case when the goods are transported directly from sources to destinations. It happens frequently, however, that in the course of transportation goods need to be reloaded. By introducing into the analysis the places where such reloading is done, the multi-phase transportation problem arises, i. e. the transshipment problem.

In formulating the transshipment problem (D. R. Anderson, D. J. Sweeney, T. A. Williams [1], F. S. Hillier, G. J. Lieberman [4], H. A. Taha [8]) let it be assumed that the goods arriving to the transshipment nodes can be transported from those nodes only towards receiving points. This means that in the analyzed model it will not be possible to transport goods between different transshipment locations. Furthermore, it will be assumed that the goods cannot circulate between sending or receiving locations, and that they cannot be transported directly from sources to destinations. When formulating the transshipment problem, it would be possible to define a linear programming model without the above limitations. In that case, however, it would be necessary to devise a model construction different from the one presented in this paper.

The aim of the transshipment model also is to minimize total transportation costs. For each of the m origins there is a requirement to direct all the disposable quantities of goods towards transient nodes. It follows from the above that the total quantity of goods a_i that is located at the source i has to be transported to one or more transshipment locations, which represent intermediate points in the transport of goods. At the intermediate point l, whose capacity is determined by the total supplied, i. e. demanded quantity d_l , the goods are transshipped and sent off to their destinations. It should be noted that the first transient node in the model is marked by $m+1$, whereas the last one is marked by $m+k$. The receiving point j can receive the demanded quantity b_j, which in the model assumes the negative sign, from any transshipment location. The first destination in the model is marked by $m+k+1$, and the last one is marked by $m+k+n$. The total demanded quantity of goods has to be delivered to those places, i. e. destinations.

To gain a clearer insight into the relations being established in the analyzed model, in Figure 2 the transshipment model is shown as a network diagram consisting of source nodes, transient nodes, and destination nodes. Total transportation costs for each route are also given in the diagram.

For the assumed transshipment problem the following linear programming model can be formulated:

$$minimize\ z = \sum_{i=1}^{m} \sum_{l=m+1}^{m+k} c_{il} x_{il} + \sum_{i=m+1}^{m+k} \sum_{j=m+k+1}^{m+k+n} c_{lj} x_{lj}$$

subject to

$$\sum_{l=m+1}^{m+k} x_{il} = a_i, = 1,2,\ldots,m$$

$$-\sum_{i=1}^{m} x_{il} + \sum_{j=m+k+1}^{m+k+n} x_{lj} = 0, l = m+1, m+2, \ldots, m+k$$

$$-\sum_{l=m+1}^{m+k} x_{lj} = -b_j, j = m+k+1, m+k+2, \ldots, m+k+n$$

$$x_{il} \geq 0, i = 1,2,\ldots,m, l = m+1, m+2, \ldots, m+k$$

$$x_{lj} \geq 0, l = m+1, m+2, \ldots, m+k,$$
$$j = m+k+1, m+k+2, \ldots, m+k+n$$

In this model, the objective function is again used to express total transportation costs that need to be minimized. They are comprised of the costs arising from the transportation of goods from sources to transshipment locations, as well as from the transportation of goods from transshipment locations to destinations.

The transshipment model analyzed here requires us to define the constraints for each of the three node types. The first constraint thus refers to the nodes of supply, the second constraint refers to transshipment nodes, and the third to the nodes of demand. In all this, variables defined as the quantity of goods leaving a node have a positive sign. Variables that represent the quantity of goods entering a node have a negative sign. Similarly to the linear programming model defined for the transportation problem, the first constraint reflects the requirement that the total quantity of goods to be transported has to be delivered from the origin. According to the second restriction, all the goods that have reached transshipment locations have to be transported out of them. The third restriction refers to the requirement that the total demanded quantity has to be delivered to the destinations. For all the decision-making variables the condition of non-negativity is valid here as well.

The transshipment problem formulated as a linear programming model can be solved by means of the simplex method. Using the computer and adequate applications can simplify and accelerate the whole procedure of determining the solution.

3. APPLYING COMPUTER ANALYSIS IN THE SIMULATION OF UNIT COST VALUES AND THE ESTIMATE OF TOTAL TRANSPORTATION COSTS

In the transshipment model defined above it is assumed that functional relations exist between the variables identified, and that model parameters are known with certainty. Such assumptions are characteristic of deterministic models. Although using deterministic models represents significant support to management when solving transshipment problems, their model limitations make the value of the obtained results relative. In order to improve the decision making process, we need to introduce stochastic elements into the transshipment model (N. Çömez, K. E. Stecke, M. Çakanyildirim [2], Y. T. Herer, M. Tzur [3], S. Takakuwa, T. Fujii [9]). Monte Carlo simulation can be used for this purpose (D. Özdemir, E. Yücesan, Y. T. Herer [7]). In the model analyzed here it is used to estimate the value of unit transportation costs.

The first step in doing simulations (V. G. Kulkarni [5]) is to determine the distributions of transport numbers in view of the real unit transportation costs for all routes. If there are records with the data on unit costs in earlier transports of goods, they can be used as the basis for setting up the required frequency distribution. A larger quantity of data in distributions will result in more precise final estimate of total transportation costs. It is not mandatory to have an equal number of data for each route. In case the manager does not have unit costs records, frequency distributions required for simulations will have to be set up on the basis of own estimates.

Once the required distributions have been determined, we need to establish for each one their relative frequency and cumulative relative frequency. After that, a random number from the interval 0 to 1 is determined. This is carried out by computer, which generates a pseudo-random number. Simulated value of unit costs is read off in the first class whose cumulative relative frequency is equal or larger than the value of random number.

When the values of all unit costs have been simulated in the way described here, the linear programming model is set up. By solving the model we arrive at the value of total minimum transportation costs.

The procedure described above has to be repeated at least 125 times. For a larger number of unit cost simulation sets the critical value of the t-distribution, which is used in determining the confidence interval of expected total costs, changes by less than 1% and thus has no significant effect on the estimate. Nevertheless, given that the standard error of the mean is decreased with increasing number of simulations, the manager has to make a decision on their optimum number.

In the transshipment model, the confidence interval of mean total transportation costs (m) is finally calculated in the following way:

$$P\left(\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}\right) = (1-\alpha),$$

Where

$\bar{x}$ – Mean value of total costs obtained through simulations,

s – Standard deviation,

$t_{a/2}$ – Critical value of the t-distribution,

n – Number of simulation set.

Special emphasis in model design and implementation should be given to building an integrated database which would bring together all the indicators relevant in the analysis of the transshipment problem. It is essential to use such data when the model is controlled, evaluated and possibly adjusted or corrected.

4. AN EXAMPLE OF DEVISING AND SOLVING A STOCHASTIC TRANSSHIPMENT MODEL ON THE BASIS OF HYPOTHETICAL DATA

Let us assume that goods need to be transported from three manufacturing plants, through two distribution centres, into three sales outlets. Let it further be assumed that the stock in the first plant is a_1=800, in the second plant it is a_2=1000, and in the third there are a_3=1200 products in stock. On the other hand, the first outlet has demanded b_6=900, the second b_7=600, and the third outlet b_8=1500 of products (in the model, the first outlet is represented by node 6, the second by node 7, and the third by node 8). Since goods had already been transported along those routes, it was possible to determine the distribution of transport numbers for each route, regarding real unit costs. The first two columns of Table 1 contain such hypothetical data for the first route. The table also provides calculations of class midpoint, relative frequency and cumulative relative frequency.

By generating the first pseudo-random number by computer we arrived at the value of 0.7217. Considering cumulative relative frequencies, the corresponding value of unit costs in the hypothetical distribution is c_{14}=6. This value can also be calculated as follows:

Table 1 - Hypothetical data required for conducting a Monte Carlo simulation of unit costs for the first route (route 1-4)

values of unit costs	number of transportations	class midpoint	relative frequency	cumulative relative frequency
1-3	3	2	0.15	0.15
3-5	7	4	0.35	0.50
5-7	9	6	0.45	0.95
7-9	1	8	0.05	1.00
total	20	–	1.00	–

$$c_{14} = 5 + \frac{0.7217 - 0.50}{0.95 - 0.50} \cdot 2 = 5.9853$$

The difference will be bigger if the value of 0.8452 is generated by computer:

$$c_{14} = 5 + \frac{0.8452 - 0.50}{0.95 - 0.50} \cdot 2 = 6.5342$$

The manager has to decide which approach is better.

The amounts of unit transportation costs need to be determined in one of the described ways for other routes as well. Let us assume that this yields unit cost values c_{15}=2, c_{24}=3 c_{25}=1, c_{34}=4, c_{35}=6, c_{46}=7, c_{47}=2, c_{48}=8, c_{56}=4, c_{57}=5 and c_{58}=3. These values, together with the data on the quantities offered and demanded, are given in the network diagram in Figure 3.

The transshipment problem posed here can be formulated as the following linear programming model:

minimize i

$$z = 6x_{14} + 2x_{15} + 3x_{24} + x_{25} + 4x34 + 6x_{35} + \\ + 7x_{46} + 2x_{47} + 8x_{48} + 4x_{56} + 5_{57} + 3x_{58}$$

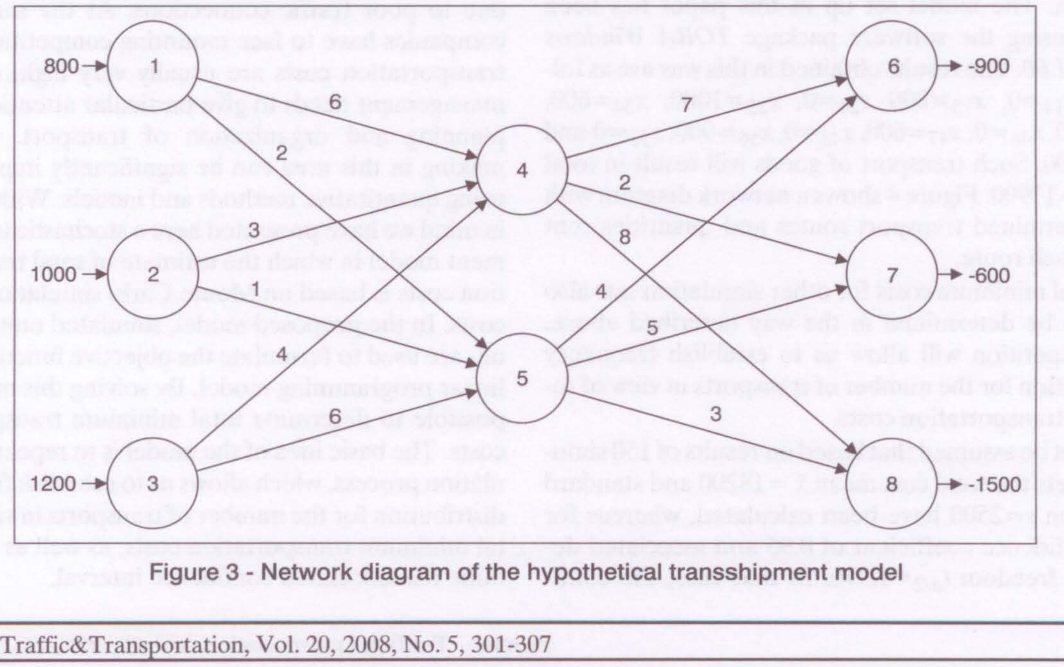

Figure 3 - Network diagram of the hypothetical transshipment model

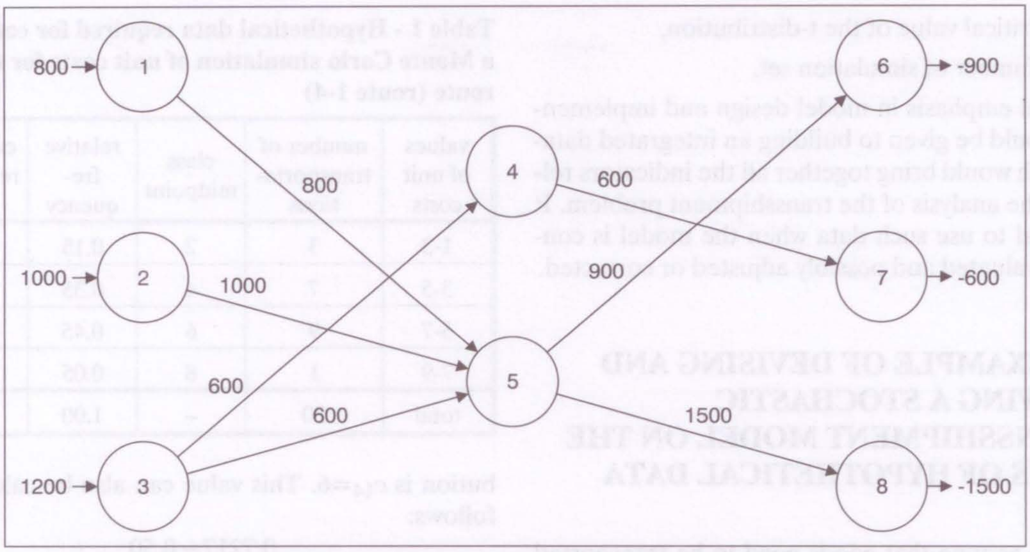

Figure 4 - Network diagram with solutions of the hypothetical transshipment model

subject to

$$x_{14}+x_{15}=800$$
$$x_{24}+x_{25}=1000$$
$$x_{34}+x_{35}=1200$$
$$-x_{14}-x_{24}-x_{34}+x_{46}+x_{47}+x_{48}=0$$
$$-x_{15}-x_{25}-x_{35}+x_{56}+x_{57}+x_{58}=0$$
$$-x_{46}-x_{56}=-900$$
$$-x_{47}-x_{57}=-600$$
$$-x_{48}-x_{58}=-1500$$
$$x_{il}\geq 0,\ i=1,2,3,\ l=4,5$$
$$x_{lj}\geq 0,\ l=4,5,\ j=6,7,8$$

The first three constraints refer to source nodes, the next two refer to transient nodes, and the last three to destination nodes.

A number of applications have been developed which are targeted at solving the linear programming problem. The model set up in this paper has been solved using the software package *TORA Windows version 1.00*. The results obtained in this way are as follows: $x_{14}=0$, $x_{15}=800$ $x_{24}=0$, $x_{25}=1000$, $x_{34}=600$, $x_{35}=600$, $x_{46}=0$, $x_{47}=600$, $x_{48}=0$, $x_{56}=900$, $x_{57}=0$ and $x_{58}=1500$. Such transport of goods will result in total costs $z=17900$. Figure 4 shows a network diagram with the determined transport routes and quantities sent along each route.

Total minimum costs for other simulation sets also have to be determined in the way described above. Such repetition will allow us to establish frequency distribution for the number of transports in view of total real transportation costs.

Let it be assumed that based on results of 150 simulation sets the total cost mean $\bar{x}=18200$ and standard deviation $s=2500$ have been calculated, whereas for the confidence coefficient of 0.95 and associated degree of freedom $t_{\alpha/2}=1.976$. In that case, the confidence interval of the expected total transportation costs in the analyzed transshipment model is as follows:

$$P(17796.651<\mu<18603.349)=0.95$$

It follows that the management in charge of transport can be 95% confident that average total transportation costs in the analyzed transshipment problem will be higher than 17796.651, and lower than 18603.349.

5. CONCLUSION

Advances in traffic have had significant influence on company operations. Accelerated development of traffic infrastructure and improved means of transportation allow companies to put their products on the markets which were previously difficult to access due to poor traffic connections. At the same time, companies have to face mounting competition. Since transportation costs are usually very high, company management needs to give particular attention to the planning and organization of transport. Decision making in this area can be significantly improved by using quantitative methods and models. With this aim in mind we have presented here a stochastic transshipment model in which the estimate of total transportation costs is based on Monte Carlo simulation of unit costs. In the proposed model, simulated unit cost values are used to formulate the objective function of the linear programming model. By solving this model it is possible to determine total minimum transportation costs. The basic idea of the model is to repeat the simulation process, which allows us to establish frequency distribution for the number of transports in view of total minimum transportation costs, as well as to determine the associated confidence interval.

In the presented model, special attention is given to computer analysis. Without computers and adequate software it would be impossible to perform simulations of unit transportation costs and generate all the solutions of the model within reasonable time limit. Similarly, determining the confidence interval of total minimum transportation costs presupposes using one of the statistical programme packages. Possible modifications of the proposed model should be based on computer analysis as well.

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SAŽETAK

PROCJENA TROŠKOVA TRANSPORTA POMOĆU MONTE CARLO SIMULACIJE U MODELU PREKRCAJA

Zadatak je managementa transporta osigurati prijevoz robe iz ishodišnih u odredišna mjesta uz ukupne minimalne troškove. Osnovni model transporta pretpostavlja da se roba prevozi direktno iz ishodišta u odredišta uz konstantne transportne jedinične troškove. No, često se u praksi prijevoz robe obavlja preko određenog broja prijelaznih točaka gdje je potrebno izvršiti njezin prekrcaj. U takvim okolnostima planiranje i organizacija transporta postaju još složeniji. Ta problematika posebno dolazi do izražaja u vodnom prometu, a neposredno je vezana uz djelatnost luka, kao čvorišta u kojim se prekrcaj obavlja. Obzirom da na transport utječu brojni čimbenici, u današnjim turbulentnim uvjetima poslovanja pretpostavka o fiksnim jediničnim troškovima prijevoza također se ne može smatrati realnom. S ciljem unapređenja odlučivanja iz domene transporta, u ovom je radu prezentiran stohastički model prekrcaja u kojem se procjena troškova temelji na upotrebi Monte Carlo simulacije. U njemu se simulirane vrijednosti jediničnih troškova koriste za postavljanje odgovarajućeg modela linearnog programiranja, čijim rješavanjem se determinira vrijednosti ukupnih minimalnih troškova transporta. Nakon što je simulacija ponovljena dovoljan broj puta pristupa se formiranju distribucije ukupnih minimalnih troškova te se na temelju nje sastavlja pripadajući interval procjene. Iz navedenog proizlazi da je u izgradnji, testiranju i primjeni prezentiranog modela potrebno objediniti kvantitativne metode optimizacije, simulaciju i elemente inferencijalne statistike, uz podršku računala i odgovarajućih programa.

KLJUČNE RIJEČI

troškovi transporta, stohastički model prekrcaja, Monte Carlo simulacija, kvantitativne metode, interval procjene, t-distribucija, računalna analiza

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